TODAY Wed Jan 30, 2019

Quiz $F_D = cv$ vs cv^2

3) Thms in 1D dynamics

1) gen. examples of 1D dyn.

~~Num. methods cont'd~~

2) Examples

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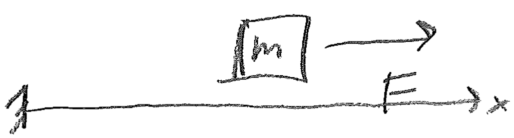
1) Quiz (see next pg)

Gen Ex 1D motion

$\longrightarrow \hat{\imath}$



fill in

I. Force laws : $F = \text{- - - -}$

II. Kin : $\dot{x} = v \; , \; \dot{v} = a$

III. Laws of mech'n : $F = ma$

ex) $F = 0 \Rightarrow \boxed{\dot{v} = 0}$ or $a = 0$

"An object in motion . . . "

Quiz:

$\rightarrow V_0$ (same for both)

$F_R \leftarrow \boxed{m}$ $\qquad F_{D1} = cV$ ①

$m = m$

$F_R \leftarrow \boxed{m}$ $\qquad F_{D2} = cV^2$ ②

V_0 is big

which one goes farther in the end?

a) ①

b) ②

c) depends on c

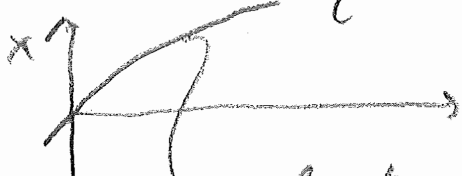
d) depends on V_0

✓ e) depends on comb. of c, V_0 & m

$c_1 + c_2 e^{-t}$

lin. drag

t



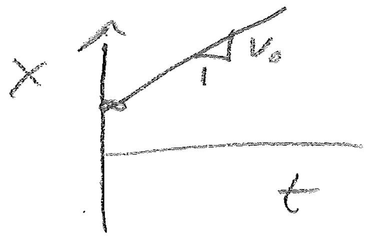
quad drag

$\sim \log t$, goes ∞ distance

$$\Rightarrow \dot{v} = 0 \Rightarrow v = v_0$$

$$\dot{x} = v \Rightarrow \boxed{x = x_0 + v_0 t}$$

$$\ddot{x} = 0 \Rightarrow$$



const.

ex) $F = F_0 \Rightarrow \dot{v} = F_0/m \equiv a_0$

$$\Rightarrow v = v_0 + a_0 t$$

$$x = \frac{1}{2} a_0 t^2 + v_0 t + x_0$$

"$d = \frac{1}{2} a t^2$" ANDY 5th grade report

$\boxed{E = \cancel{mc^2}}$ — Hanley Rising

$\boxed{etc}$

ex) $F = f(t)$

$$a(t) = \frac{f(t)}{m}$$

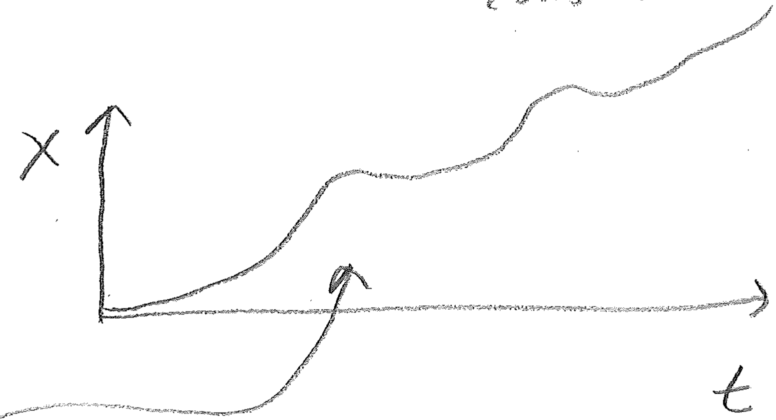
$$\dot{v} = a(t) = f(t)/m$$

$$v = \int f(t)\,dt/m + C_1$$ Math 1910

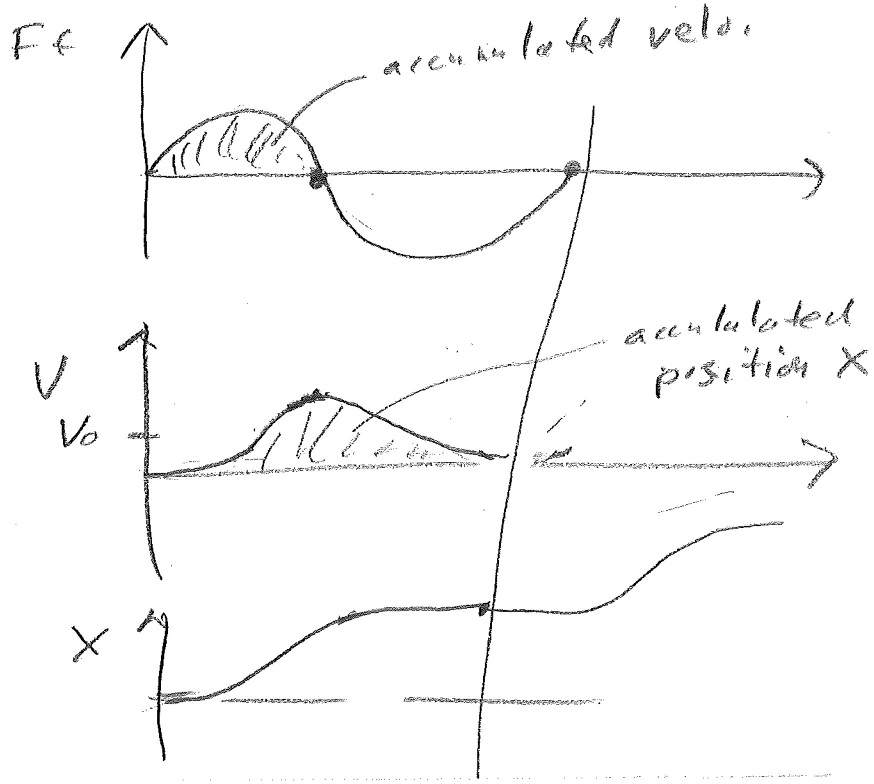
$$x = \int v(t)\,dt/m + C_2$$

sub ex) $f(t) = \underbrace{f_0 \sin \omega}_{\text{const.}} t$

$X_0 = 0$
$v_0 = 0$



a) sine wave ✗
b) " + const ✗
c) " " const + lin fn. ✓



subex) arb. fcs of t

subsubex) $f(t) = \dfrac{c\, e^{-bt}}{t}$

$$\dot{V} = \frac{1}{m}\, c\, \frac{e^{-bt}}{t}$$

$\Rightarrow$ no analytic soln. (even if $t_0 > 0$)

ex.) $F = g(v)$

$\Rightarrow$ $a = \frac{1}{m}\, g(v)$

$$\boxed{\dot{v} = \frac{1}{m}\, g(v)}$$ $\Rightarrow$ separable eqn.

$\dot{x} = v$

$\Rightarrow$ might have anal. soln.

subex) $F = -cv$

$$v = v_0 e^{-ct}$$

$x = \ldots$ (Quiz problem)

subex) (last lecture) Quiz problem

$F = -cv|v|$ $\quad v^2$ if $v > 0$

$$\Rightarrow \boxed{v = v_0 e^{-cx}}$$

$\Rightarrow$ $x \sim \log(t)$

ex) $F = h(x)$ force is fn. of position

$$\left.\begin{aligned}\dot{v} &= \frac{1}{m} h(x) \\ \dot{x} &= v\end{aligned}\right]$$

[Cons of energy
$\Rightarrow v(x)$
↑ (not shown yet)]

subex) $h(x) = -kx$ restoring ~~...~~ spring (linear)

$$\Rightarrow \boxed{m\ddot{x} + kx = 0}$$

or $\begin{bmatrix} \dot{x} = v \\ \dot{v} = -\frac{k}{m} x \end{bmatrix} \Leftrightarrow \dot{z} = f(z)$

rules of change ← | → state

$$x = A\cos(\omega t) + B\sin\omega t$$
$$\omega = \sqrt{k/m}$$

ex) mix & match

subex) Most famous $\omega_f \neq \omega_n = \omega \neq \sqrt{k/m}$

$$\boxed{F = -kx - cv + F_0 \sin \omega_f t}$$

$\Rightarrow$

$$\dot{x} = v$$

$$\dot{v} = \frac{1}{m}\left(-kx - cv + F_0 \sin \omega t\right)$$

$$\boxed{\boxed{m\ddot{x} + c\dot{x} + kx = F_0 \sin(\omega t)}}$$

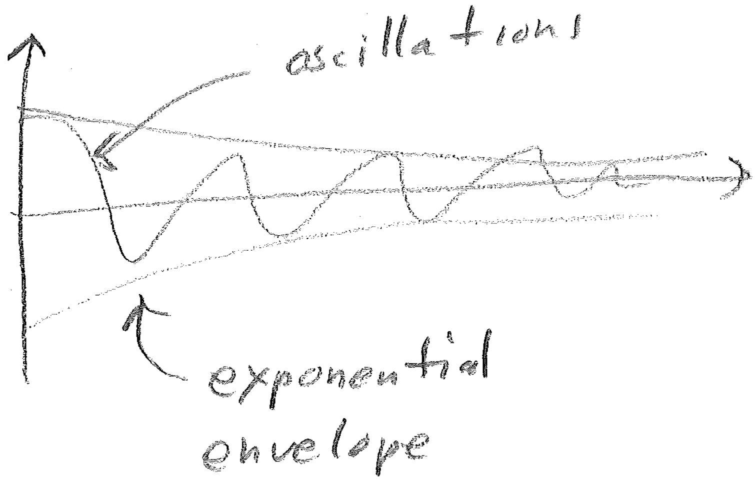
$x(t) = ?$

"Forced, damped harmonic oscillator"

subex) c is small

$F_0 = 0$

underdamped vib. decay



Gen Case of 1D motion

$$F = F(x, v, t)$$

$$\boxed{\begin{aligned} \dot{x} &= v \\ \dot{v} &= \tfrac{1}{m} F(x, v, t) \end{aligned}}$$

gov. eqn. "EoM" "Equations of Motion"

Can solve (at least numerically)

$$\Rightarrow \underbrace{x(t), \quad v(t)}_{\text{solution of EoM}}$$

Can know some things ahead of time! thms / facts. checks ✓✓✓

Thms

$\{\,\}\cdot v$

$$\{F = ma\}$$

$$vF = ma \cdot v$$

$$a = \frac{dv}{dt}$$

$$vF = m \frac{d}{dt}\left(\tfrac{1}{2} v^2\right)$$

$$\underbrace{Fv}_{\text{Power} = P} = \frac{d}{dt}\underbrace{\left(\frac{1}{2} m v^2\right)}_{\text{Kin. En.} = \text{"}\cancel{KE}\text{"}}$$

E_K, $\cancel{T}$, $\cancel{K}$

$$\boxed{\boxed{P = \dot{E}_K} \quad (1) \quad \text{power balance}}$$

<u>Next</u>

$\int (1)\, dt$ $\qquad \int P\,dt = \int \dot{E}_K\, dt$

$$\boxed{\int P\,dt = \Delta E_K} \quad (2)$$

"Power x time = Energy"

$$\int P\,dt = \int P \frac{dt}{dx}\, dx$$

$$= \int F v \frac{1}{v}\, dx$$

$$= \underbrace{\int F\,dx}_{\text{work}}$$

$$\boxed{\underbrace{\int F\,dx}_{W} = \Delta E_K}$$

Work = change in Kin En

$\boxed{W = \Delta E_K}$ Work = change in Kin. Energy

Lin. Mom

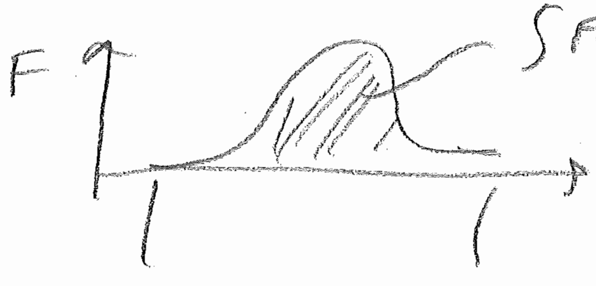
$$\{F = ma\}$$

$\int \{\ \}\,dt$ $\qquad \int F\,dt = \int m \underbrace{a}_{dv/dt}\,dt$

$$\underbrace{\int F\,dt}_{\text{"Impulse"}} = m\,\Delta v$$

$mv = L =$ lin. mom

not. nec. sortly sudden



Summary

* $P = \dot{E}_K$
* $W = \Delta E_K$, $\int P\,dt = \Delta E_K$
* $\int F\,dt = \Delta(mv) = \Delta L$