Lecture 6 Feb. 11, 2019 Monday

TODAY

* Review
* Normal Modes
* "Prof, I was wondering ... units"
* Collisions

Review

What is this course about?

Given info. about force, geometry, motion.

Find other info. about force, geom., motion

So far: 1 D motion of one particle

3) "$F = ma$"

2) $\dot{x} = v$, $\dot{v} = a$, $d = x_2 - x_1$

1) $F = \begin{cases} -kx, & \text{or} \\ -c_1 v, & \text{or} \\ -c_2 v^2, & \text{or} \\ mg, & \text{or} \end{cases}$

Tools: Matlab, (ODEs

Euler's Method

plotting

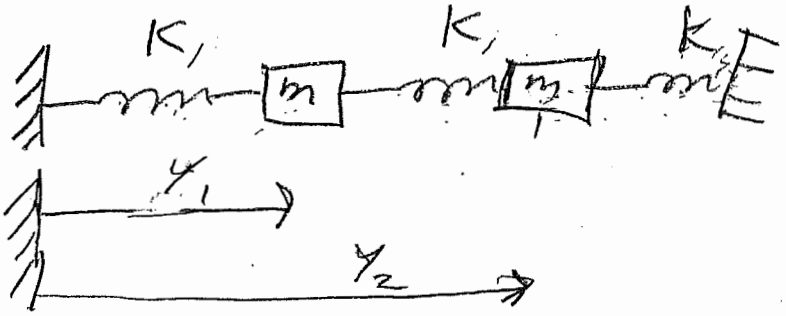
1) $a\ddot{x} + b\dot{x} + cx = f(t)$

2) $\frac{dx}{dt} = f(x)$

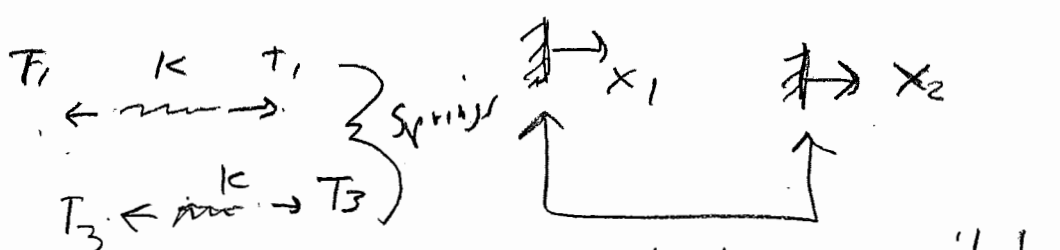
ex) Vibrations: Read text for details ch 9, 10.

Normal Modes

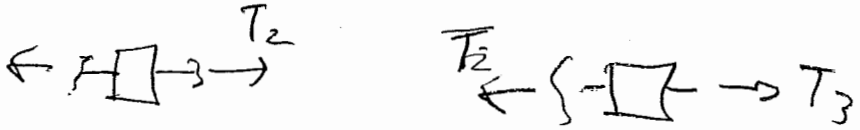
ex)



FBDs



Nodes



$$T_1 = K x_1$$
$$T_2 = K(x_2 - x_1)$$
$$T_3 = K(-x_2)$$

} Constit. Laws

Mechanics

LMB:

$$\left.\begin{aligned} \text{block 1}: \quad m\ddot{x}_1 &= T_2 - T_1 \\ \text{block 2}: \quad m\ddot{x}_2 &= T_3 - T_2 \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} m\ddot{x}_1 &= -2Kx_1 + Kx_2 \\ m\ddot{x}_2 &= -2Kx_2 + Kx_1 \end{aligned}\right\}$$

Matrix form

$$\Rightarrow \left[\begin{array}{c|c} m & 0 \\ \hline 0 & m \end{array}\right] \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = -\left[\begin{array}{c|c} 2K & -K \\ \hline -K & 2K \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\Rightarrow \underset{2\times 2}{M}\ \ddot{X} = -\underset{2\times 2}{K}\ X$$

$$\ddot{X} = \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} \qquad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\ddot{X} = -M^{-1} \cdot K \cdot X$$

$$\exists M^{-1} \ (M)$$

$M^{-1} \cdot M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

mech probs. $\Rightarrow$ M^{-1} exists

$\ddot{x} = -A x$

$A = M^{-1} K$

Solve w/ Matlab.

$A =$ `M^(-1) * K` , or `M \ K`

backslash

ASIDE

Matlab's backslash

$B x = y$

vectors

`X = B\y`

(Matlab soln. cont'd)

$$\dot{X} = V$$

$$\dot{V} = -A X$$

$$Z = \begin{bmatrix} X \\ V \end{bmatrix}$$

$$\dot{Z} = \begin{bmatrix} \dot{X} \\ \dot{V} \end{bmatrix}$$

} ODEs

ANALYTIC Approach to Soln.

$$\ddot{X} = A X$$

$$\boxed{m\ddot{x} + Kx = 0}$$

Simple Solns:

ex) $x_1 = n_1 \cos \omega t$

$x_2 = n_2 \cos \omega t$

from 2 mass prob! $\Rightarrow$ ex $\begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

eqn

$$M\ddot{x} + Kx = 0 \qquad (*)$$

guess

$$x = \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \end{Bmatrix} \cos(\omega t)$$

$n_1, n_2, \ldots$ } all unknown

ω }

Σ const

$\ddot{x} = -\omega^2 n \cos(\omega t)$

$n = \begin{bmatrix} n_1 \\ n_2 \\ \vdots \end{bmatrix}$

$(*)$ plug into $*$ matrices

$$\Rightarrow M n(-\omega_n^2 \cos(\omega_n t)\, n + K \cdot n \cos(\omega_n t)$$

$n = \begin{Bmatrix} n_1 \\ n_2 \\ \vdots \end{Bmatrix}$

$$-\omega_n^2 M n + K n = 0$$

$$\boxed{(-\omega^2 M + K) \cdot n = 0}$$

algebraic problem

$M^{-1} * \{3\}$

$$\Rightarrow \left(-\omega^2 \underbrace{I}_{\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}} + \underbrace{M^{-1}K}\right) \underbrace{n}_{\begin{bmatrix}n_1 \\ \vdots \\ \vdots\end{bmatrix}} = \begin{Bmatrix}0 \\ 0 \\ 0\end{Bmatrix}$$

$$\boxed{\left[A - \lambda I\right] n = 0}$$

$A = M^{-1}K$

given A find λ, n

$$\boxed{An = \lambda n}$$

$\lambda = \omega^2$

ask Matlab

$$eig(\underbrace{M^{-1} * K}_{A})$$

$\Rightarrow$ n vectors & λ values that solve it

Professor, I was wondering...

UNITS

QUANTITIES

$m_A = 100\,kg$

$v = 2\ m/s$

$L = m \cdot v = (100\,kg) \cdot 2\,(m/s)$

$L = 200\ kg \cdot m/s$

units must match

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3 basic units: mass, length, time

MKS: kg, m, s

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ex) $m = 7g \cdot (1)$

$= 7g \cdot \left(\frac{1\,kg}{1000g}\right) = \frac{7}{1000}\,kg$

Force

$$[F] = N = 1\,kg \cdot \frac{1\,m}{s^2}$$

$$1 = \left(\frac{1\,N}{1\,kg\,m/s^2} \right)$$

$$[W] = [E] = [F] \cdot [d] -$$

$$J = 1\,N \cdot 1\,m$$

$$1 = \left(\frac{1\,J}{N \cdot m} \right)$$

$$E = \frac{1}{2} m v^2$$

$$[E] = [m][V]^2$$

$$\mathbf{J} = 1 \cdot kg \left(\frac{m}{s} \right)^2$$

1 Kg moving at 1 m/s

$$E \neq 1\ kg\ m^2/s^2$$

$m = 100\ kg$

"weighs 100 kg"

"F = 100 kg" $\quad [F] = kgf$

$kgf = g \cdot kg$ $\quad$ Note: $1\,kgf \neq 1\,Kg_m$

English

$[m] = lbm$

$[t] = s$

$[L] = ft$

$1 = \frac{12\,in}{1\,ft}$

$1 = \frac{5280\,ft}{1\,mi}$

$[F] = [m] \cdot [a]$

"Poundal" $= 1\,lbm \cdot 1\,ft/s^2$

$lbf = g \cdot lbm \neq$ poundal

"pound force = gravity force on pound mass

lbm ≠ lbf

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lbm, ft, s, poudal

slug, ft, s, lbf

consistant units

Quiz

A system of m (linear) equations in n unknowns $(x_1, x_2, \ldots)$ can be put in the form

$$Ax = b$$

What are the sizes of A, x & b?

Size	A	x	b
a)	$n \times n$	$n = n \times 1$	$n = n \times 1$
b)	$m \times n$	$m = m \times 1$	$n = n \times 1$
c)	$n \times m$	$m = m \times 1$	$n = n \times 1$
d)	$m \times m$	$m = m \times 1$	$m = m \times 1$
✓ e)	$m \times n$	$n = n \times 1$	$m = m \times 1$