Lecture 7, Feb. 13, 2019

TODAY

Quiz @ end

UNITS
Normal Modes
Collisions

Systems of units

[t]	[ℓ]	[mass]	[F] (N)	[W] = [E] (J)	[P] (W)
s	m	kg	1 kg m/s²	1 kg m²/s²	1 kg m²/s³
s	ft.	lbm	1 lbm ft/s² pdl poundal	lbm ft²/s²	lbm ft²/s³
s	ft.	slug	lbf slug ft/s²	— —	— —
s	m	mug	kgf = mug m/s²	— —	— —

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2 Rules:

1: Carry units

2: Mult. by 1 as you like

UNITS (cont'd)

Some versions of 1

$1\,ft = 12\,in \Rightarrow 1 = \frac{12\,in}{1\,ft} = \frac{1\,ft}{12\,in}$

$12\,in = 1\,ft$

$3\,ft = 1\,yd$ $\quad\quad$ $1760\,yds = 1\,mi$

$5280\,ft = 1\,mi$

$2.2\,lbm = 1\,kgm$

$lbm \cdot g = lbf$

$$\left[\begin{matrix} 9.8\,m/s^2 \\ 32.2\,ft/s^2 \end{matrix}\right] \quad 1 = \frac{32\,ft/s^2}{9.8\,m/s^2}$$

$Kg \cdot g = 9.8\,N$

$$g = \frac{1\,lbf}{1\,lbm} = \frac{1\,Kgf}{1\,Kgm} = \frac{9.8\,N}{Kgm}$$

$1\,lbf \approx 5.5\,N$

$39.37\,in = 1\,m$

$1\,W = J/s$

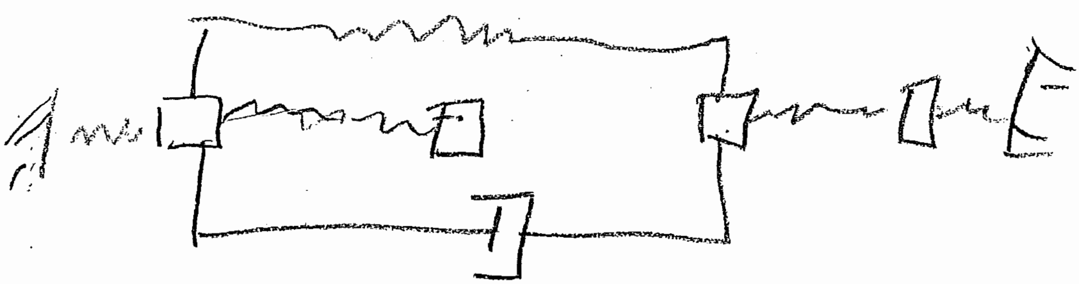
$1\,hp = 550\,ft\,lbs/s$

$\equiv 746\,watts$

Normal Modes/Vibes etc

"General" Case

line of springs, mass & dashpots
w/ forcing $\rightarrow F$



LMB for each mass; set of eqs:

$$m_1 \ddot{X}_1 = K_{12}(X_2 - X_1) + K_{13}(X_3 - X_1)$$
$$c_{14} \quad \dot{X}_2 - \dot{X}_1 + \text{const. stuff}$$
$$+ \text{forcing}$$

$$m_2 \ddot{X}_2 = \ldots$$

$\vdots$

$\Rightarrow$ Linear Eqs $\Rightarrow$ Matrix form

ignore for vibes

$$\left\{ M\ddot{x} + C\dot{x} + Kx = C_1 t \right\}$$

$F_0 \sin \omega t$

$M = n \times n$	$C_1 = n \times 1$
$C = n \times n$	$F_0 = n \times 1$
$K = n \times n$	$\omega = \text{scalar}$
$x = n \times 1$	

$\quad \llcorner x(t)$

$M^{-1}\{\ \} \Rightarrow$

$$\cancel{C_1}$$

$$\ddot{x} = -M^{-1} \cdot \left[C\dot{x} + Kx - F_0 \sin \omega t \right\}$$

$$\begin{cases} \dot{v} = -M^{-1}[\quad\quad\quad] \\ \dot{x} = v \end{cases}$$

$2n$ first order ODEs

2 Key ideas

I: You can solve in matlab

II. Normal Modes

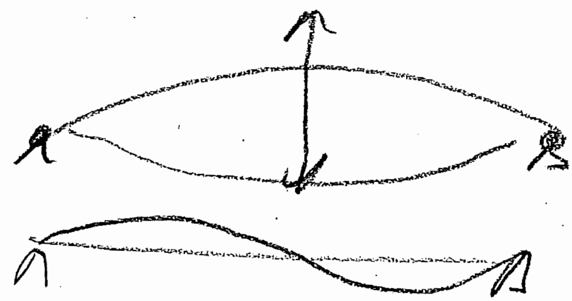
Normal Modes: $C = 0,\ C_1 = 0$
$F_0 = 0$

$$\{ M\ddot{x} + Kx = 0 \}$$

$$M^{-1} \cdot \{\} \Rightarrow \ddot{x} + M^{-1}Kx = 0$$

$$x = \underset{\substack{\uparrow \\ \text{const}}}{W} \sin \underset{\substack{\uparrow \\ \text{scalar}}}{\omega} t = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \end{pmatrix} \sin \omega t$$

(violin string)



$\omega \neq W$ $\llcorner$ double-u omega

<u>eqn:</u> $\ddot{X} + M^{-1} K X = 0$

<u>plug in guess</u>

$$\frac{d^2}{dt^2}\left(W \sin \omega t\right) + M^{-1} K \left[W \sin \omega t\right] = 0$$

$$\left[-\omega^2 + M^{-1} K\right] W = 0$$

$$\left[\underbrace{-\omega^2}_{\lambda} I + M^{-1} K\right] W = 0$$

$$\left[A - \lambda I\right] W = 0$$

$A = M^{-1} K$

$$\boxed{A W = \underbrace{\lambda}_{\omega^2} W}$$

$W = I \cdot W$

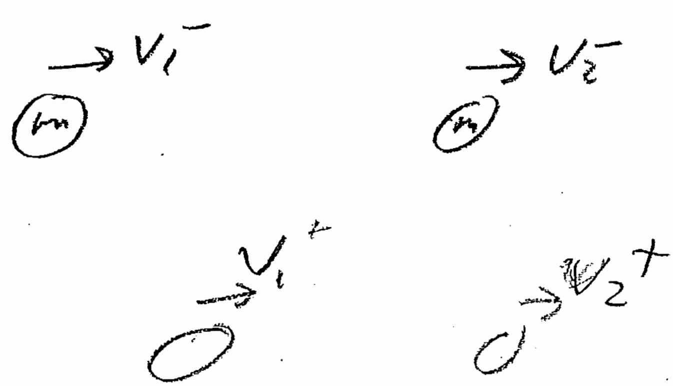
$[V, D] = \text{eig}(A)$

e-vector = V ; $A v = \underbrace{\lambda}_{\text{e-value.}} v$

$$\underbrace{A\cdot(7v) = \lambda\cdot(7v)}_{\text{const * e-vector} \Rightarrow \text{another e-vector}}$$

$\lambda = \omega^2$

Collisions 1D:



$\underbrace{V_2 < V_1}_{\text{collide}}$

collision law:

$$(V_2 - V_1)^+ = -\underbrace{e}_{0 \le e \le 1}(V_2 - V_1)$$

Quiz

$\pm 2\%$

(6)

1 lbm starts from rest.
A const 1 lbf is applied
to it for 1 s.

a) Distance $= X \approx 16$ ft

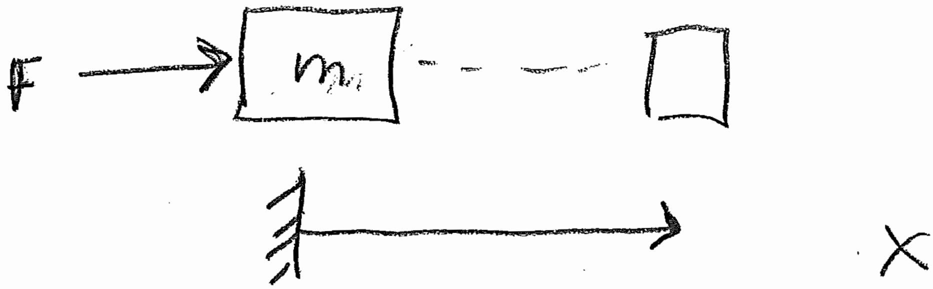
b) $E_K \approx 500$ lbm ft^2/s^2

c) Work done ≈ 500 lbm ft^2/s^2

d) all of above (a, b & c) ✓

e) Any of these:
none of a, b, c
a & b, a & c, b & c

Soln to Quiz [illegible]



LMB

$$F = ma$$

$$\Rightarrow a = F/m$$

$$= 1\,\text{lbf} / 1\,\text{lbm}$$

$$\boxed{1\,\text{lbf} = g \cdot 1\,\text{lbm}}$$

$$= g$$

$$a = 32\,\text{ft}/\text{s}^2$$

at $t = 1\,\text{s}$

$$V = at = \left(32\,\frac{\text{ft}}{\text{s}^2}\right) \cdot (1\,\text{s})$$

$$= 32\,\text{ft}/\text{s}$$

$$x = \frac{1}{2} a t^2 = (32\,\text{ft}/\text{s}^2)\,(1\,\text{s})^2$$

$$= 16\,\text{ft}.$$

$$L = m\,V = (32\,\text{ft}/\text{s}) \cdot (1\,\text{lbm}) = 32\,\text{lbm ft}/\text{s}$$

$$E_K = \frac{1}{2} m V^2 \approx 500\,\text{lbm ft}/\text{s}^2 \;(= F \cdot x)$$