LECTURE 8

MON, Feb. 18, 2019

Quiz questions at end

TODAY

1) Collisions (cont'd)

2) Particles in space (vectors)

Collisions (cont'd)

Recall

1-D FBD

$$F \rightarrow (m) \qquad\qquad \hat{i} \longrightarrow +$$

$$F = ma \qquad\qquad a = \frac{dV}{dt}$$

$$\Rightarrow \int_{t_1}^{t_2} F\,dt = m \int_{t_1}^{t_2} a\,dt$$

$$\quad '' \quad = m(V_2 - V_1)$$

$$\quad '' \quad = L_2 - L_1$$

$$\boxed{\text{Impulse} = \Delta L}$$

change in linear momentum

In collisions

1) Collision force so big that other forces are negligible. ⟹ collisional FBD neglects "finite" forces (e.g., mg, springs, etc)

2) We ignore details of $F_{collision}$ & instead only note net impulse.

⟹ From just before coll. to just after;

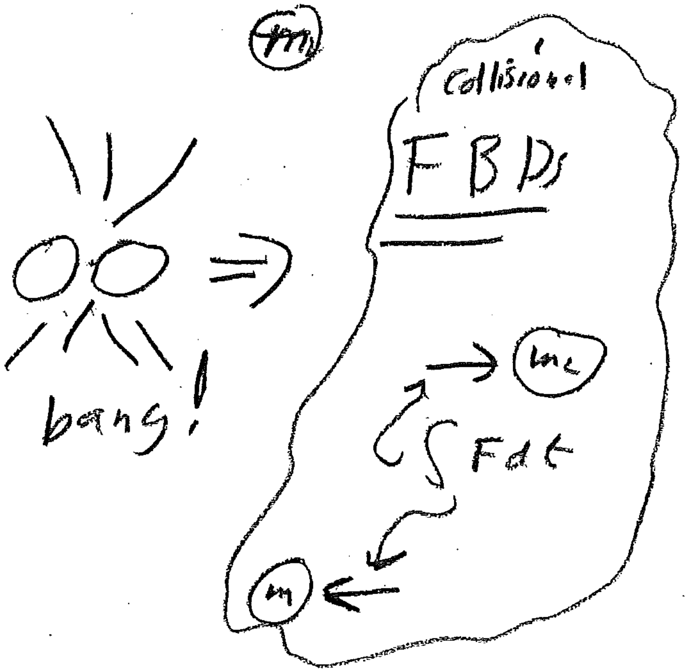
ΔL only due to coll. forces (coll. impulses) ex) 2 balls

— means before coll.

V_1^- $\rightarrow$ m_1 $\qquad$ V_2^- $\rightarrow$ m_2

then

dur. collision



+ means after

V_1^+ $\rightarrow$ $\qquad$ V_2^+ $\rightarrow$

Coll. Eqs

LMB:

$$\boxed{\begin{aligned} m_2 V_2^+ - m_2 V_2^- &= \int F\,dt \\ m_1 V_1^+ - m_1 V_1^- &= -\int F\,dt \end{aligned}}$$

Note

$$L_{tot} = L_1 + L_2$$

$$\Rightarrow \text{const.}$$

$$\boxed{L_{tot}^{+} = L_{tot}^{-}}$$

Cons. of Lin. mom. for system

Constitutive Law

Coeff. of restitution : e

$$\underbrace{(\text{separ. speed})}_{\text{after}} = e \cdot \underbrace{\begin{pmatrix}\text{approach}\\ \text{speed}\end{pmatrix}}_{\text{before}}$$

e = coeff. of restitution

Usually

$$0 \le e \le 1$$

What about energy?

Given V_1^-, V_2^-, e (m_1, m_2)

can calc. V_1^+, V_2^+ $\}$ 2 unknowns

How?

1) $\Delta L_{TOT} = 0$

2) $(V_2^+ - V_1^+) = -e(V_2^- - V_1^-)$

1) & 2) are 2 eqs in 2 unknowns,

$\Rightarrow V_1^+ \ \& \ V_2^+$

See Matlab Sample from Lect. 7

$\Rightarrow$ cal do cal,

$$\Delta E_k = \left(\frac{1}{2} m_1 V_1^{-2} + \frac{1}{2} m_2 V_2^{-2}\right) - \left(\frac{1}{2} m_1 V_1^{+2} + \frac{1}{2} m_2 V_2^{+2}\right)$$

a bunch of algebra

$\Rightarrow$ $e = 1 \Rightarrow$ const E_k

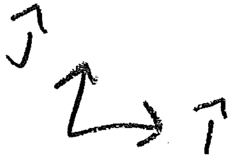
$e = 0 \Rightarrow$ maximizes dissipation

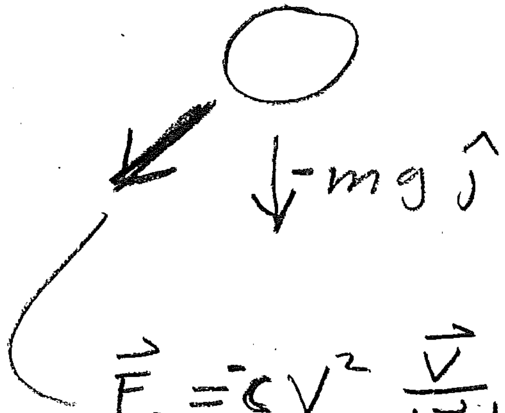
Motion in 2D & 3D

ex) <u>quadratic drag</u>

$\nearrow \vec{V}$

<u>FBD</u>





$$\vec{F}_D = -cV^2 \frac{\vec{V}}{|\vec{V}|}$$

$$= -c\,\vec{V}\,|\vec{V}|$$

[ALTERNATIVE!

<u>Linear Drag</u>

$$\vec{F}_D = -c\vec{V}$$

$\Rightarrow$ Linear ODEs, but not continued in this example] quadratic drag ballistics

not solvable "in closed form"

$\Rightarrow$ Num. Methods

LMB

$$\vec{F}_{TOT} = m\vec{a}$$

$$\boxed{-mg\hat{j} - c\vec{V}|\vec{V}| = m\vec{a}} \quad (*)$$

"EoM"
Eqs. of Motion

$$\dot{z} = f(t, z)$$

$$z = \begin{bmatrix} x \\ y \\ V_x \\ V_y \end{bmatrix}$$

$$\{*\} \cdot \hat{i} \Rightarrow \quad -c V_x \sqrt{V_x^2 + V_y^2} = m \dot{V}_x$$

$$\{*\} \cdot \hat{j} \Rightarrow -mg - c V_y \sqrt{V_x^2 + V_y^2} = m \dot{V}_y$$

$$\underbrace{\begin{aligned} &\left.\begin{aligned} \dot{x} &= V_x \\ \dot{y} &= V_y \end{aligned}\right] \text{Kinematics} \\ &\left.\begin{aligned} \dot{V}_x &= \frac{1}{m}\left(-c V_x \sqrt{V_x^2 + V_y^2}\right) \\ \dot{V}_y &= \frac{1}{m}\left\{\left(-c V_y \sqrt{V_x^2 + V_y^2}\right)\right\} - g \end{aligned}\right\} \text{"Eom"} \end{aligned}}_{\dot{\vec{z}} = \vec{f}(t, \vec{z})}$$

IC's: $x_0 = 0, \quad y_0 = 0,$

$V_{x0} =$ ___, $V_{y0} =$ ___

Solve on Computer to get

$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$

Quiz

$$O \xrightarrow{V_A^- = 1} \qquad \xleftarrow{V_B^- = -1} O$$

$$\boxed{e = -1}$$

action on B

a) Impulse > 0

b) Impulse < 0

c) Impulse $= 0$ ✓

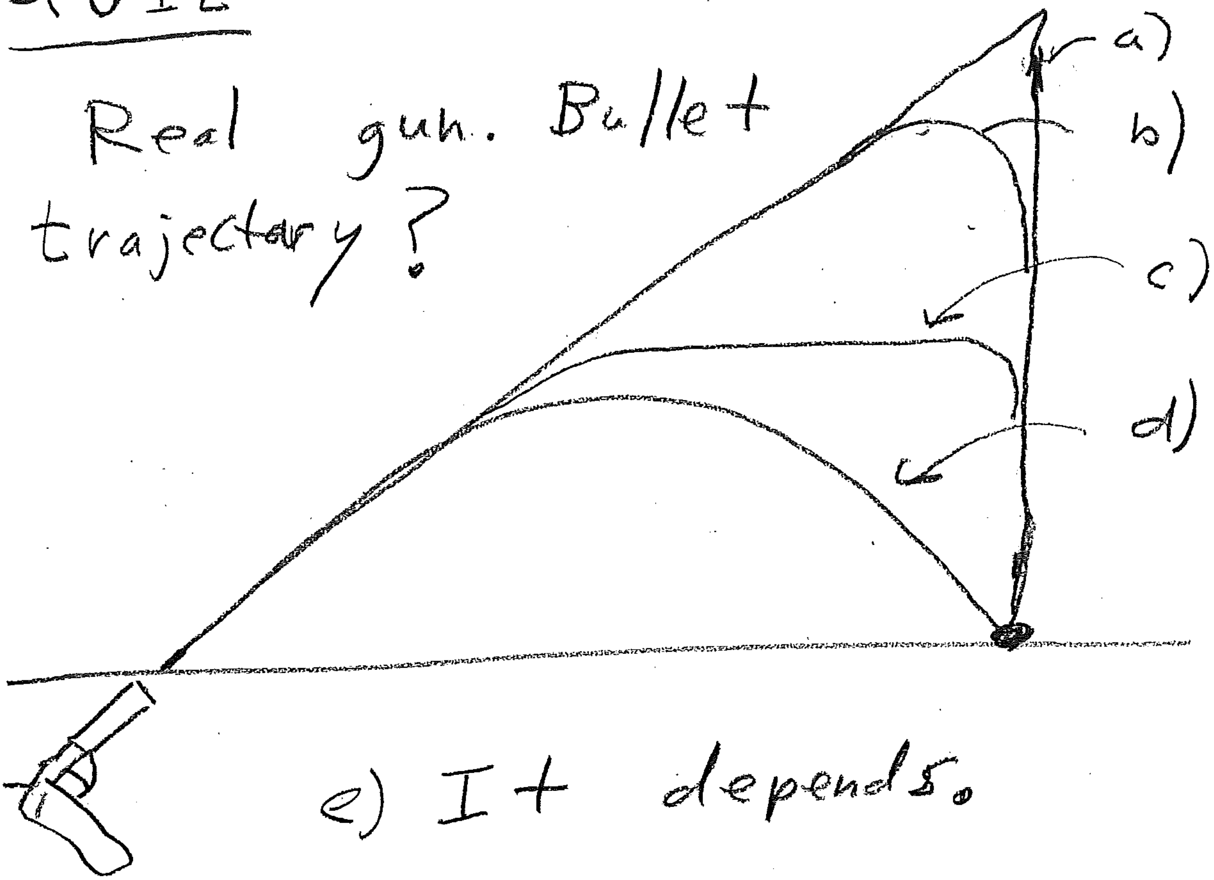
d)

e)

See Matlab code to check

QUIZ

Real gun. Bullet trajectary?



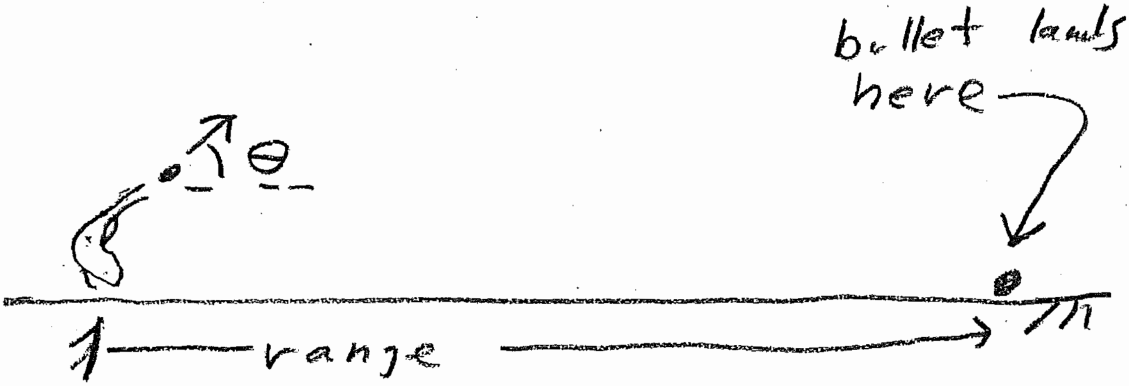
e) It depends.

b ✓ Try it in matlab

Quiz

Shooting some real gun on flat ground. Calm day.

Angle for max range?



$\theta_{max} = ?$

a) $0°$

b) $\theta_m < 45°$ ✓ (try it in matlab)

c) $\theta_m = 45°$

d) $\theta_m > 45°$

e) depends on m, v, etc.