TODAY

QUIZ ours 83

* Misc review
* 2 D dyn. of particle (cont'd)
* theorems

Normal Modes review

ANY springs & masses;
Rel. to equilib.;
x_i are displ. rel. to equilib.;

$$M \ddot{X} + K X = 0 \quad *$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

Guess $X(t) = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \sin(\omega t)$

double-u

omega

Plug into *

$\Rightarrow$ Good guess if ω^2 is an e-value of A with e-vector W

$A = M^{-1} \cdot K$

$M^{-1} \cdot M = I$

E-vector eqn.

$$\boxed{A w = \underbrace{\lambda}_{\omega^2} w}$$

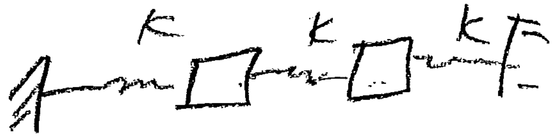
$\omega \neq w$

Vibes probs $\Rightarrow$ allways get A w/ n Lin. Ind. e-vectors

$$\boxed{\omega_i = \sqrt{\lambda_i}}$$

Some special probs $\Rightarrow$ can guess/see soln.

$$\omega = \sqrt{\frac{K_{eff}}{m}}$$

ex) 

e-vector $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$K_{eff} = 3k$

$\Rightarrow \omega_n = \sqrt{3k/m}$

$$\underline{M\ddot{x} + Kx = 0}$$

$$\Rightarrow \quad \ddot{x} = -\underbrace{M^{-1}K}_{A}\, x$$

$$\left[\begin{array}{l} \dot{v} = -\underset{3\times 3}{A}\, x \\ \dot{x} = v \end{array}\right]$$

1st order form

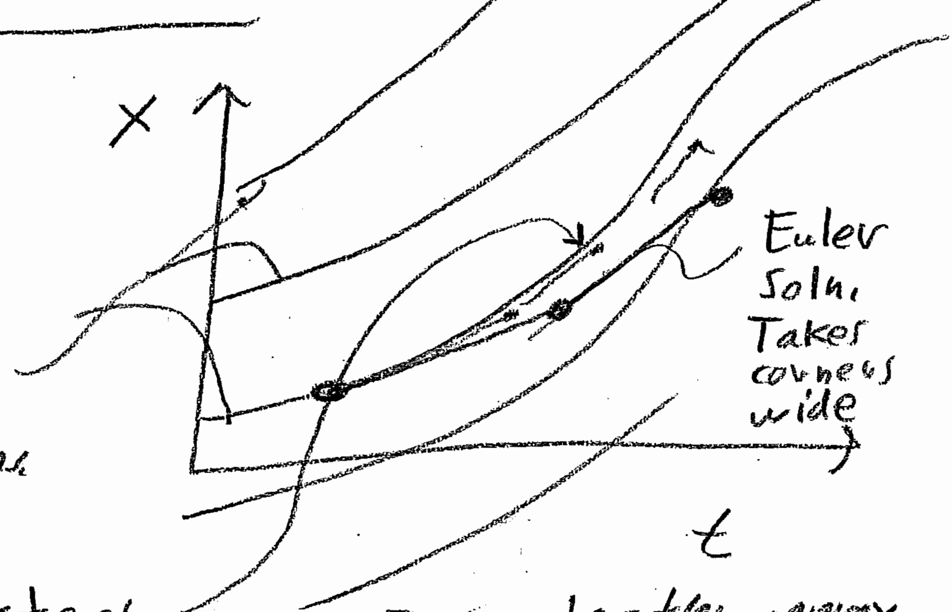
$$\dot{z} = \underset{6\times 6}{B}\, z \qquad B = \left[\begin{array}{ccc|ccc} 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ \hline & -A & & 0&0&0 \\ &&&0&0&0 \\ &&&0&0&0 \end{array}\right]$$

$$\dot{z} = f(t, z)$$

Euler Method

$$\dot{x} = f(t, x)$$

ex)



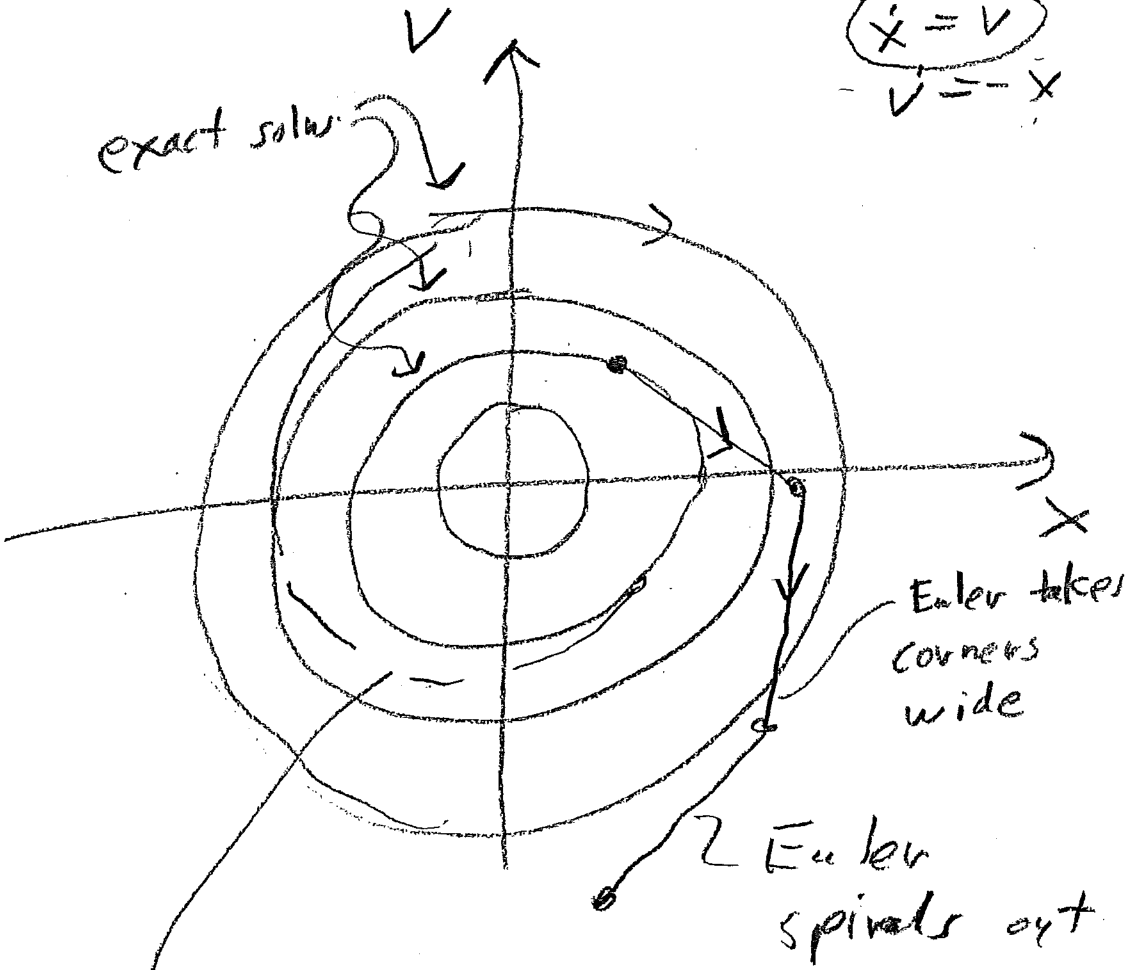
smaller steps $\Rightarrow$ better approx.

ex) Harmonic Osci

$\ddot{x} + x = 0$

$\dot{x} = v$

$\dot{v} = -x$



$$\left.\begin{array}{l}\text{smaller } \Delta t \\ \text{smaller } h\end{array}\right\} \Rightarrow \text{less spiral}$$

$$\boxed{\dot{z} = f(t, z)}$$

MAIN DYN. SYST. EQN

Gen. 1D Vibe prob.

$$m\ddot{x} + cx' + Kx = C_1 + C_2 \sin(\omega t)$$

Ignore forcing & damping $\Rightarrow$ $\boxed{m\ddot{x} + Kx = 0}$

Lots of masses $\Rightarrow$ $\underset{n\times n}{M}\,\underset{n\times 1}{\ddot{x}} + \underset{n\times n}{K}\,\underset{n\times 1}{x} = \underset{n\times 1}{0}$

guess: $x(t) = w \sin(\omega t)$ ← const

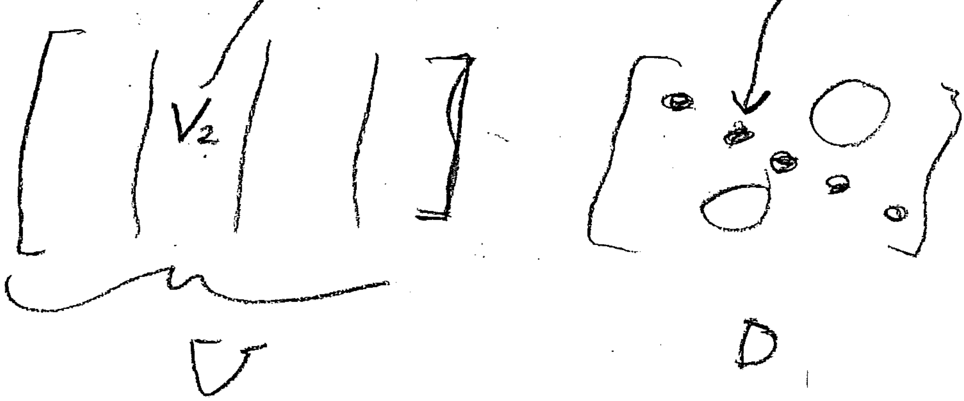
$$A = \left[\ \ \right] = M^{-1}K$$

MATLAB soln

$$[V, D] = \text{eig}(A)$$

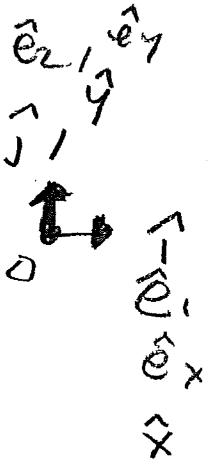
e-vector 2

corresponding e-value 2

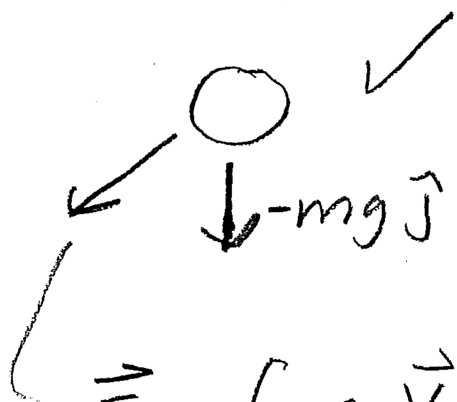


2D motion of particle

Review trajectory



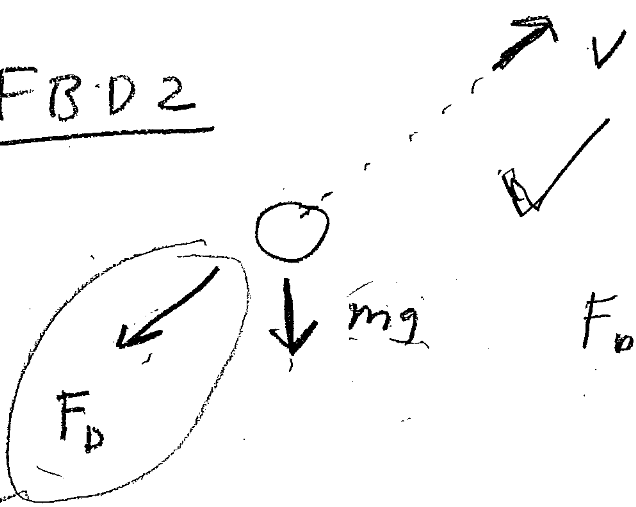
FBD 1 !



FBD should have enough info to write EoM

$$\vec{F}_D = \begin{cases} -c_1 \vec{V} \\ -c_2 |\vec{V}| \vec{V} \end{cases}$$

FBD 2



$$F_D = \begin{cases} cV \\ cV|V| \end{cases}$$

scalar * unit vector drawn

LMB

$$\sum \vec{F}^{ext} = m\vec{a}$$

$$\Rightarrow \quad m\vec{a} = -mg\hat{j} + \vec{F}_D$$

$$\vec{a} = -g\hat{j} + \vec{F}_D/m$$

linear

$$\vec{F}_D = \begin{cases} -c\vec{V} \\ -c\vec{V}|\vec{V}| \end{cases}$$

quadratic

1st order form :
$$\begin{cases} \dot{\vec{r}} = \vec{V} \\ \dot{\vec{V}} = -g\hat{j} + \vec{F}_D/m \end{cases}$$

4 1st order ODEs

Various forces

drag forces: $\vec{F}_D = \begin{cases} -c\vec{v}\,|\vec{v}| \\ -c\vec{v} \end{cases}$

(see prev. pages)

near earth gravity: $-mg\hat{j}$

"Real" gravity: inverse square

$\left.\frac{mMG}{r^2}\right|_{R_e=r} = mg$ at surf. of earth

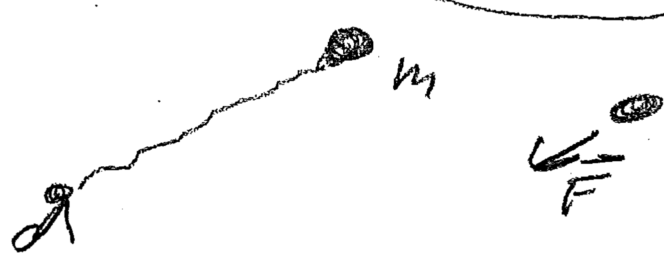
$\Rightarrow \boxed{MG = gR_e^2} \Rightarrow$

$$\vec{F}_g = -\frac{GMm}{r^2}\frac{\vec{r}}{|\vec{r}|}$$

$$= -g\frac{m}{(r/R_e)^2}\frac{\vec{r}}{r}$$

$$= -gm\frac{R_e^2}{r^3}\vec{r}$$

Spring



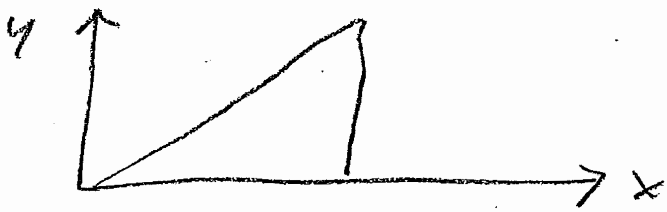
$$\vec{F}_s = -k(|\vec{r}| - l_0)\,\vec{r}/|\vec{r}|$$

QUIZ

Trajectory on Matlab ex?

(a)

(b)



(c)

None of above (d)

Ans: huge drag, huge V_0 $\Rightarrow$ (c)