Feb. 27, 2019 WED

TODAY

2D & 3D dynamics

Review (Math)

Lots of probs. reduce to

soln. of * Lin. Alg Eqs $\Rightarrow$ \ (backslash)

* ODEs $\Rightarrow$ Midpoint solver

$\dot{z} = f(t, z)$

1D Mechanics

$\boxed{F = m\underbrace{a}_{\ddot{x}}}$

oscillators, drag, vibrations, etc

(done)

2D & 3D mechanics

Force: $\vec{F} = \nearrow F$ means a scalar F mults by a unit vector

$\vec{F} = F \cdot \hat{a}$ $\qquad \hat{a} \equiv \nearrow$

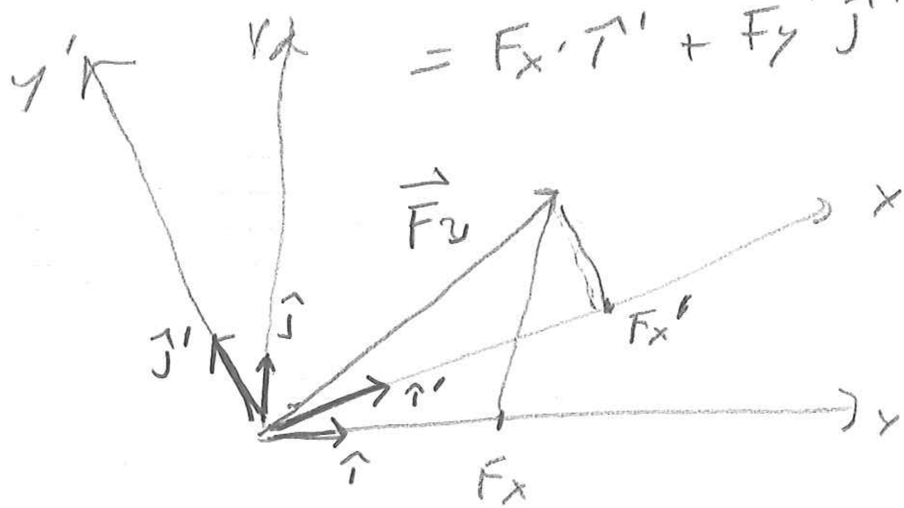
$\hat{j}$, $\hat{i}$

$\vec{F} = \underbrace{F_x \hat{i} + F_y \hat{j} + F_z \hat{k}}_{\text{components times unit vectors}}$

$$[\vec{F}] = \underbrace{[F_x, F_y, F_z]'}_{\text{" } = \vec{F} \text{ "}}$$

List of components

$$\vec{F} = F_x \hat{\imath} + F_y \hat{j}$$
$$= F_{x'} \hat{\imath}' + F_{y'} \hat{j}'$$



$$F_x \hat{\imath} + F_y \hat{j} = F_{x'} \hat{\imath}' + F_{y'} \hat{j}'$$

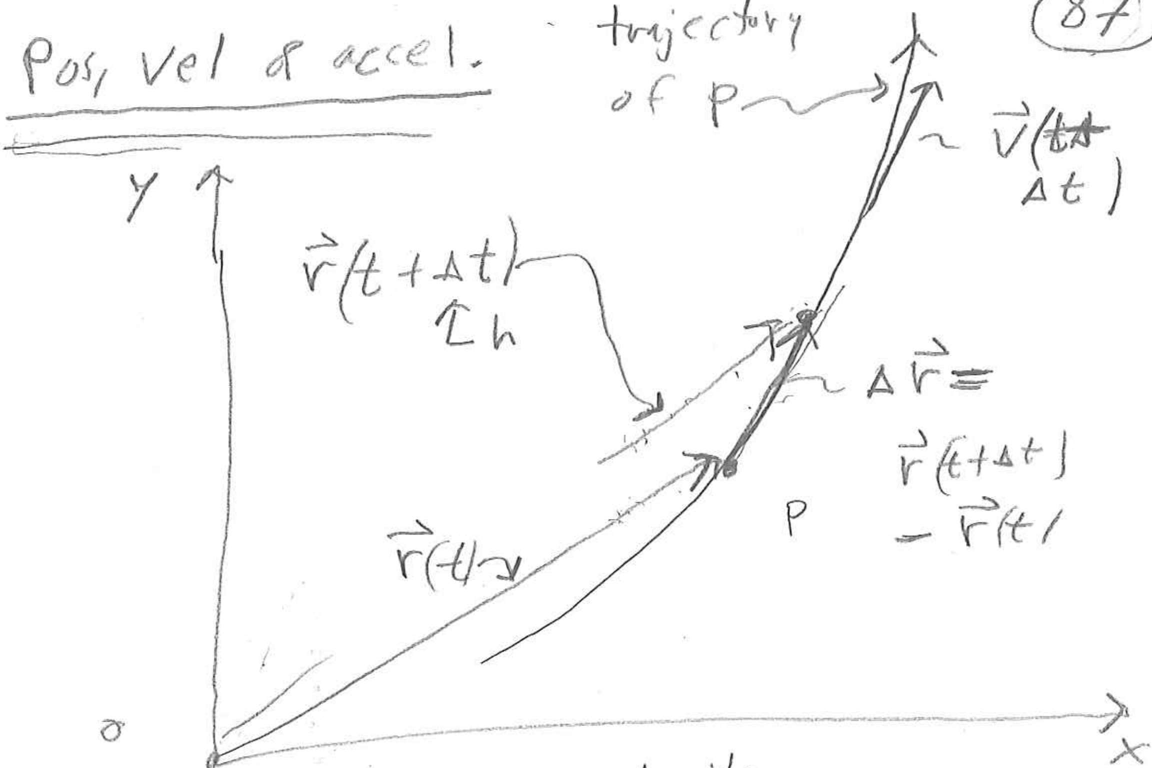
$$\boxed{\vec{F} = \vec{F}}$$

$$[F_x, F_y]' \neq [F_{x'}, F_{y'}]'$$

vector = vector

but list of components ≠ list of components

Pos, vel & accel.

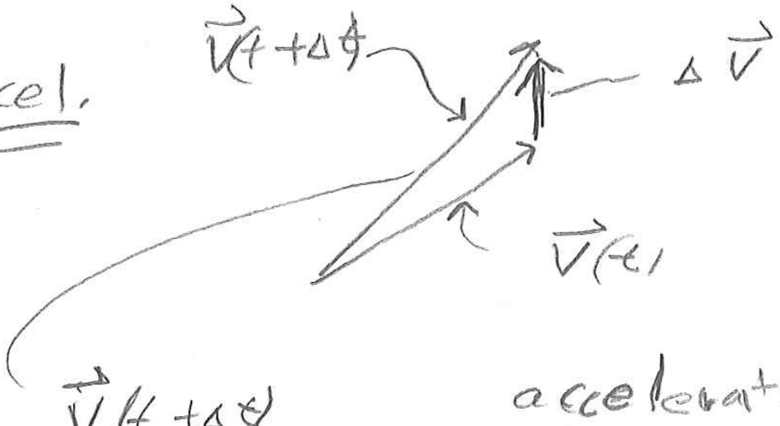


velocity

$$\boxed{\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}}$$

$$\vec{r}_{p/o} = \vec{r}$$

accel.



acceleration

$$\boxed{\vec{a} = \frac{\Delta \vec{V}}{\Delta t}}$$

$\lim \Delta t \to 0$

Components

$$\vec{r} = x\hat{\imath} + y\hat{\jmath}$$

$\hat{\imath}, \hat{\jmath}$ are constant

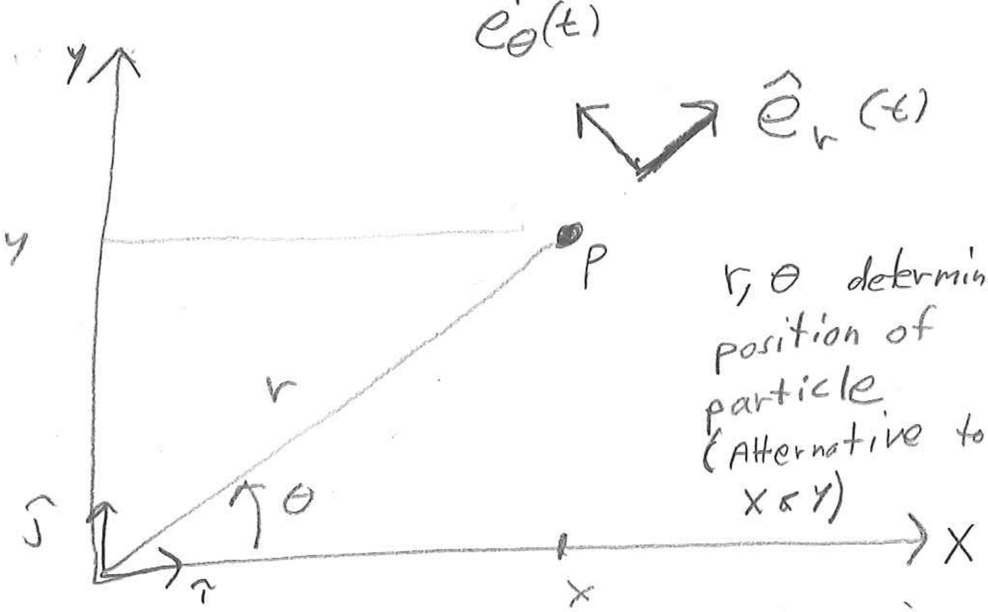
$$\vec{V} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath}$$

$$= V_x\hat{\imath} + V_y\hat{\jmath}$$

$$\vec{a} = \dot{V}_x\hat{\imath} + \dot{V}_y\hat{\jmath}$$

$$= \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}$$

Polar coord.



r, θ determines position of particle (Alternative to x & y)

given $r(t)$ & $\theta(t)$ what are $\vec{V}$ & $\vec{a}$?

$\hat{e}_\theta \perp \hat{e}_r$

$\hat{e}_\theta = \hat{k} \times \hat{e}_r$

$\hat{k}$ unit vect. in z dir.

$\hat{e}_\theta$ in dir. part. moves if θ changes & r does not.

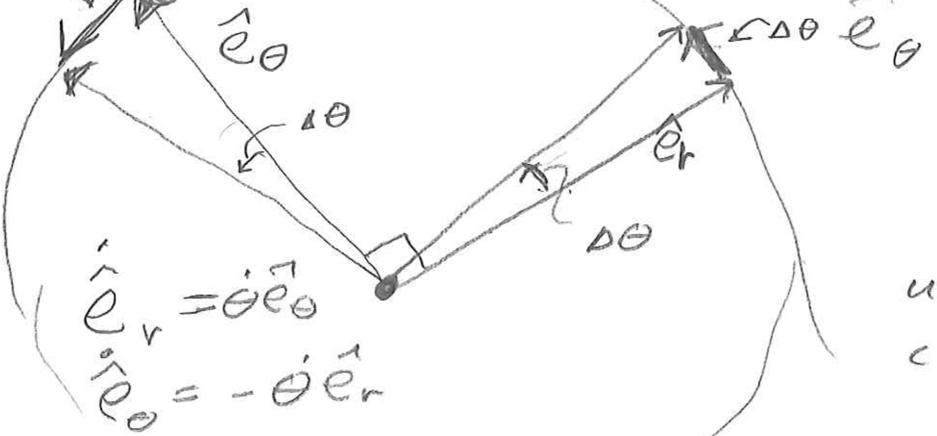
position in polar coords

$$\vec{r} = r\hat{e}_r$$

vel.

$$\vec{v} = \frac{d}{dt}(\vec{r})$$

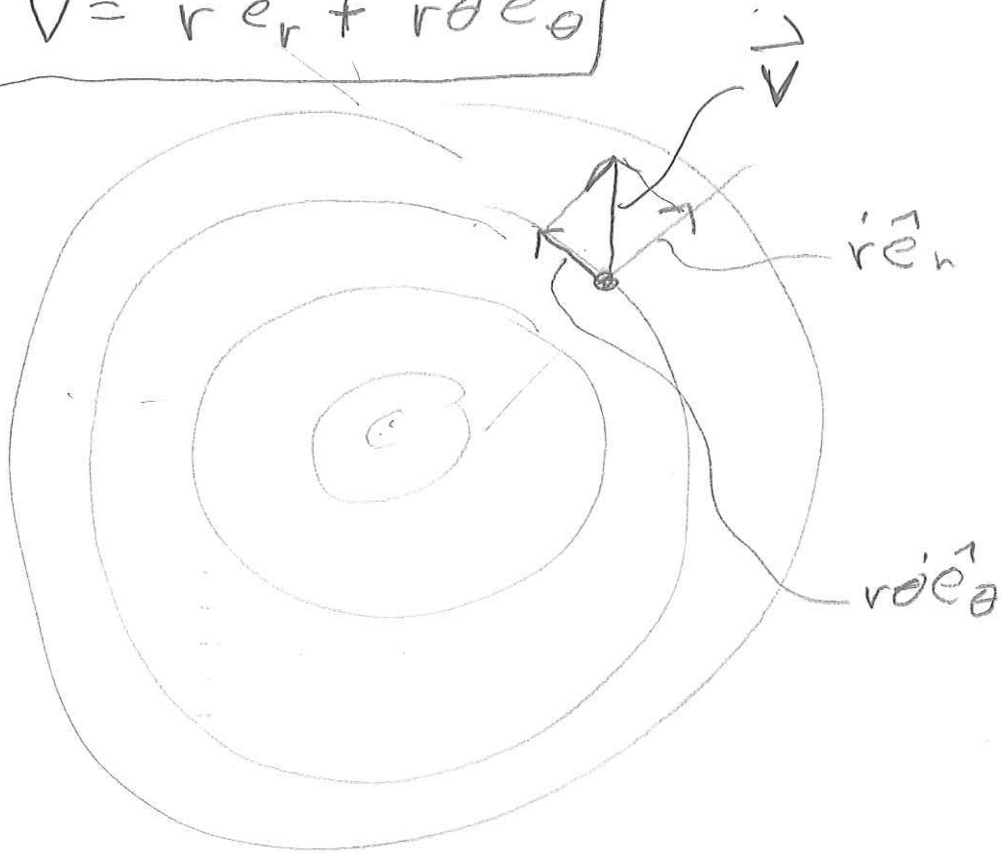
$$= \dot{r}\,\hat{e}_r + r\underbrace{\dot{\hat{e}}_r}_{\dot{\theta}\hat{e}_\theta}$$



$\dot{\hat{e}}_r = \dot{\theta}\hat{e}_\theta$

$\dot{\hat{e}}_\theta = -\dot{\theta}\hat{e}_r$

$$\boxed{\vec{V} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$$



$$\vec{a} = \frac{d}{dt}(\vec{v}) = \frac{d}{dt}\left(\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta\right)$$

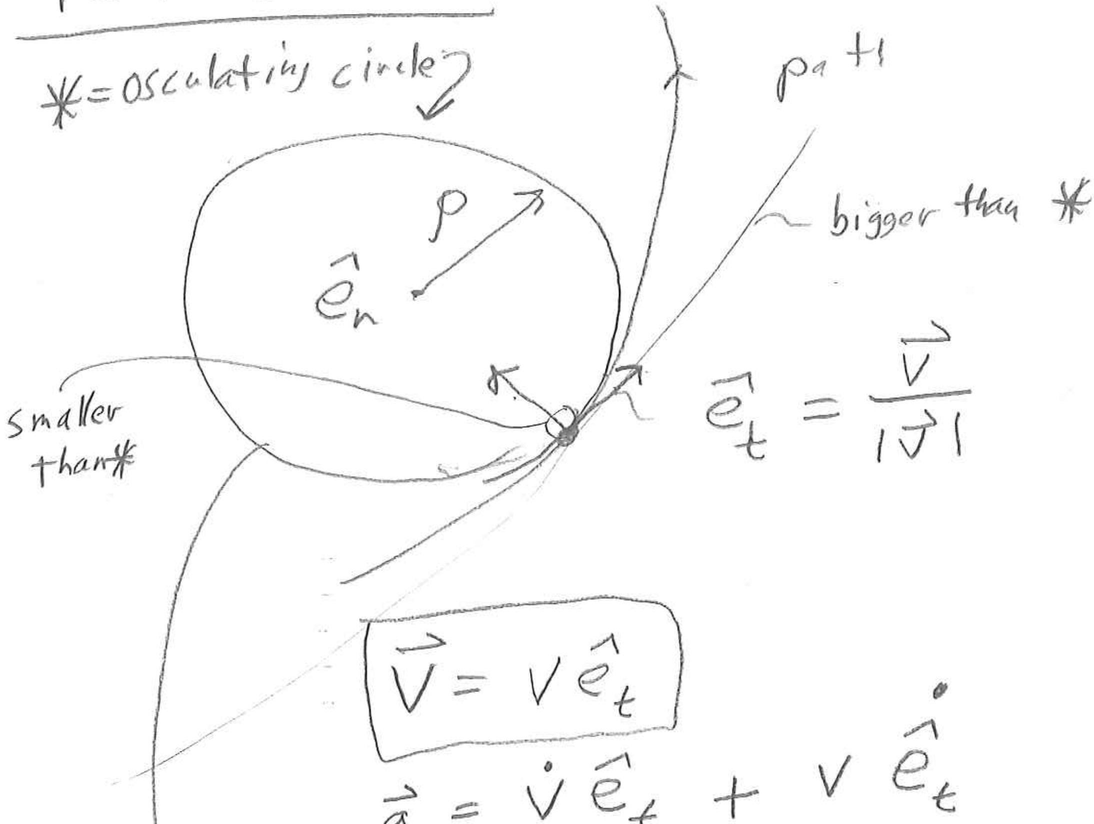
$$\Rightarrow \boxed{\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta}$$

skip steps

(see text sect 14.1)

Path coord:

*= osculating circle



$$\hat{e}_t = \frac{\vec{v}}{|\vec{v}|}$$

$$\boxed{\vec{v} = v\hat{e}_t}$$

$$\vec{a} = \dot{v}\hat{e}_t + v\dot{\hat{e}}_t$$

$$\boxed{\vec{a} = \dot{v}\hat{e}_t + v^2 \kappa\, \hat{e}_n}$$

Curvature $= 1/\rho$

Centrip. Accel.

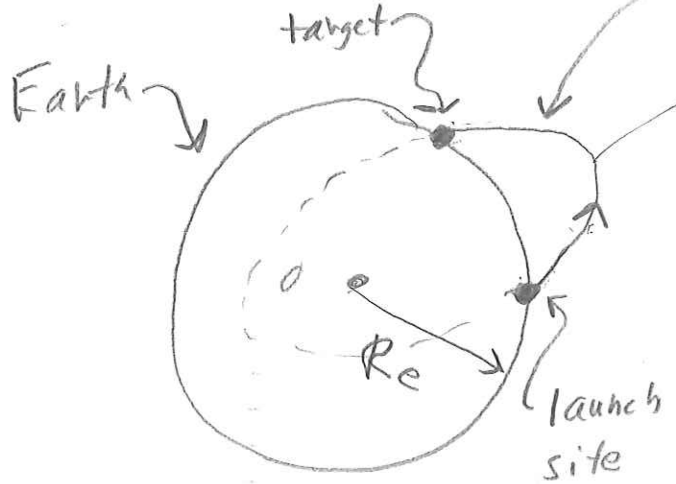
radius of curvature

"Kissing" osculating circle

$\vec{a}$ = 1 part for speeding up;
+1 part for going around corners

Ex)

Nuclear War: ballistic missile



a) parab.

b) circle

c) elipse

d) hyperbola

e) other

$$\vec{F} = F\,\hat{\lambda}$$

$$\hat{\lambda} = -\vec{r}\,/\,|\vec{r}| = -\hat{e}_r$$

$$F = \frac{m g R_e^2}{r^2} = \frac{GMm}{r^2}$$

G = Cavindish Const

M = mass of Earth

$$\vec{F} = -\frac{mgR_e^2}{r^2}\,\frac{\vec{r}}{r}$$

$$= -\frac{mgR_e^2}{r^3}\,\vec{r}$$

LMB $\qquad \vec{F} = m\vec{a}$

$$\boxed{\begin{aligned} \dot{\vec{V}} &= \vec{a} = \frac{1}{m}\left(-\frac{mgR_e^2}{r^3}\vec{r}\right) \\ \dot{\vec{r}} &= \vec{V} \end{aligned}}$$

$$\dot{Z} = \begin{bmatrix} [\dot{\vec{r}}] \\ [\dot{\vec{V}}] \end{bmatrix} = \begin{bmatrix} [V] \\ [\dot{\vec{V}}] \end{bmatrix}$$

See Computer Solution of this problem, from lecture

Thms & Facts

$\vec{F} = m\vec{a}$ has solns

w/ various properties:

Lin. Mom

$$\int_{t_1}^{t_2} \vec{F}\,dt = m(\vec{v}_2 - \vec{v}_1)$$

Power

$$\text{Power} = \dot{E}_K$$

$E_K = \frac{1}{2}m\,\vec{v}\cdot\vec{v}$

$\vec{F}\cdot\vec{v}$

$$\int P\,dt = \Delta E_K$$

$$\int \vec{F}\cdot d\vec{r} = \Delta E_K$$

If Conservative Forces

$$-\Delta E_p = \Delta E_K$$

$$\Delta(E_p + E_K) = 0$$

Angular Momentum

$$\underbrace{\vec{r}_{/0} \times \vec{F}}_{\vec{M}_{/0}} = \frac{d}{dt}\Big(\underbrace{\vec{H}_{/0}}_{\vec{r}_{/0} \times (m\vec{v})}\Big)$$

USES of THMS.

$\Rightarrow$ * Checks

* help solve

* intuition

Thms: $\vec{L}$, $\vec{H}$, P, W, E_{TOT}