Wed. March 6, 2019

TODAY

Normal Mode Quiz (last pages)

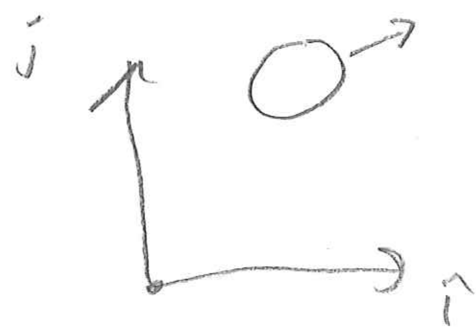
Lots of particles

1 D constrained motion

Lots of particles

See Matlab

ex)



Gen. case

- $\vec{F}_i = m_i \vec{a}_i$ for each particle
- can calc. $\vec{F}_i$ from all $\vec{r}_i$ & $\vec{v}_i$ $\Rightarrow$ $\dot{z} = f(t, z)$

$\Rightarrow$ 100s, 1000s & billions of particles

Apply to atoms

"predicts the future" - Laplace

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ex) Solid Chrystal

$\Rightarrow \dot{z} = f(t, z)$

1D constrained motion

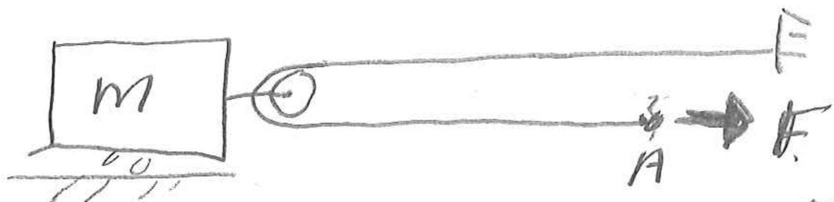
a) Quiz, next pages

b) Quiz, Soln

c) 2 masses

QUIZ

1 D, no friction
no gravity



$m = m$ $\qquad$ $F = F$

$$\frac{a_A}{a_B} = ?$$

a) 1/2

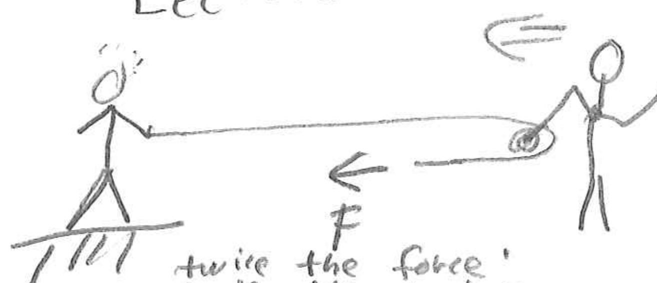
b) 1

c) 2

d) 4

e) 16

Lecture demo

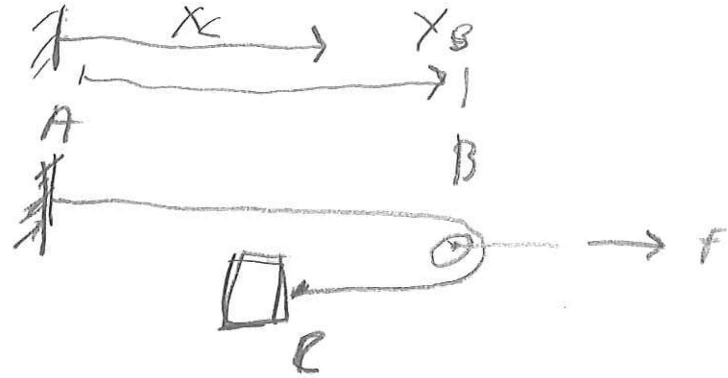


twice the force
half the motion

VS

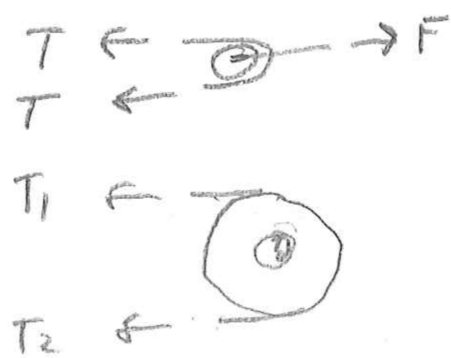
half the force
twice the motion

ex)



FBDs

$\Rightarrow T = F/2$



$\Rightarrow T_1 = T_2$

LMB $\boxed{a_c = \frac{T}{m} = \frac{F}{2m}}$

Kinematics; Length of rope = const.

$$L = x_B + (x_B - x_C) + \pi R_{pulley}$$

$$\frac{d}{dt} L = 0 = \dot{x}_B + \dot{x}_B - \dot{x}_C = 0$$

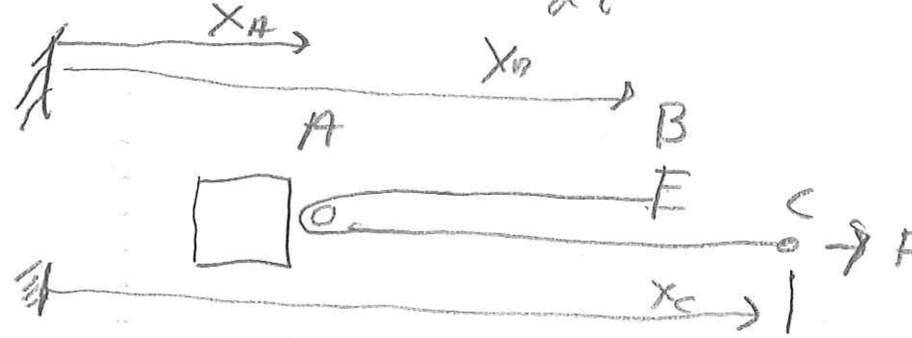
$$\Rightarrow \boxed{\ddot{x}_B = \ddot{x}_C/2 = \frac{F}{4m}}$$

$$\boxed{\ddot{x}_B = \frac{F}{4m}}$$

$$f(t) = g(t)$$

$$\frac{d^{37} f(t)}{dt^{37}} = \frac{d^{37} g(t)}{dt^{37}}$$

ex1



<u>FBDs</u>

$$\boxed{m} \rightrightarrows \begin{matrix} F \\ F \end{matrix} \Rightarrow a_A = \frac{2F}{m}$$

<u>LMB</u>

<u>Kinematics</u>

$L = \text{const}$

$$\left\{ \text{const} = (x_B - x_A) + (x_C - x_A) \right.$$

$$\frac{d}{dt}\{\ \} \Rightarrow 0 - \dot{x}_A + \dot{x}_C - \dot{x}_A$$

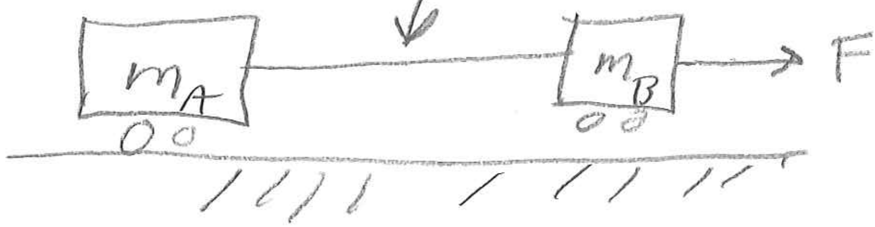
$$\Rightarrow \ddot{x}_C = 2\ddot{x}_A \Rightarrow \boxed{a_C = \frac{4F}{m}}$$

(compare to prev. example)

Ex)

1D constraint

inextensible
$\Rightarrow$ constraint



$a_A = ?$ $\quad a_B = ?$

FBDs

LMB $\Rightarrow$ $T = m_A \ddot{x}_A$

LMB $\Rightarrow$ $F - T = m_B \ddot{x}_B$

Instead of Const. Law $\quad \ddot{x}_A = \ddot{x}_B$

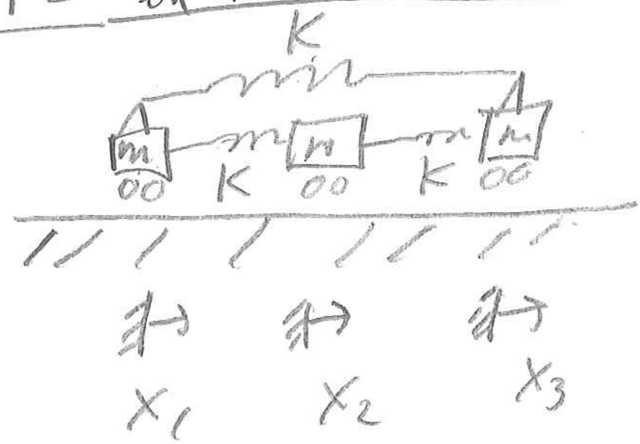
$\boxed{\text{Constraint Eqn.}}$

Solve Simult.

$\Rightarrow \ddot{x}_A = \ddot{x}_B = \dfrac{F}{m_A + m_B}$

& $T = \dfrac{F m_A}{m_A + m_B}$

QUIZ on Normal Modes



$$M = m\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \; ; \; K = K\begin{bmatrix}2&-1&-1\\-1&2&-1\\-1&-1&2\end{bmatrix}$$

$$A = M^{-1}K = \begin{bmatrix}2&-1&-1\\-1&2&-1\\-1&-1&2\end{bmatrix}$$

Normal Modes?

Frequencies

See Matlab

$$\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix}, \begin{bmatrix}0\\1\\-1\end{bmatrix}$$

$$\begin{bmatrix}5\\7\\-12\end{bmatrix}$$

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* How many ind. normal, ~~[illegible]~~ modes? a) 1 b) 2, c) 3 d) 4 e) 5

* How many of above are normal modes?

a) 1
b) 2
c) 3
d) 4
e) 5

guess

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Math

check:

$$\underbrace{\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}}_{A} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= 0 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\omega^2 = 0$

$\omega = 0$

Physical $K_{eff} = 0$ ✓

guess

$$A \cdot \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$\omega^2 = 3$

Physical $K_{eff} = 3$ ✓

guess $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ ✓

$$A \cdot V_1 = \lambda V_1$$

$$A V_2 = \lambda V_2$$

$$\Rightarrow A(6V_1 - 37V_2) = \lambda(6V_1 - 37V_2)$$ ✓

For M = diagonal, K = symmetric

$\Rightarrow$ A = symmetric

$\Rightarrow$ e-vectors mutually orthogonal

Orthogonal to $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ are vectors whose coeff. add to 0. 2D space of e-vectors w/ $K_{eff} = 3$ is vectors w/ $\sum v_i = 0$

e.g. $[5 \;\; 7 \;\; -12]'$