TODAY: Monday March 11

Pulleys (cont'd)
2D forces in 1D motion.

PULLEYS

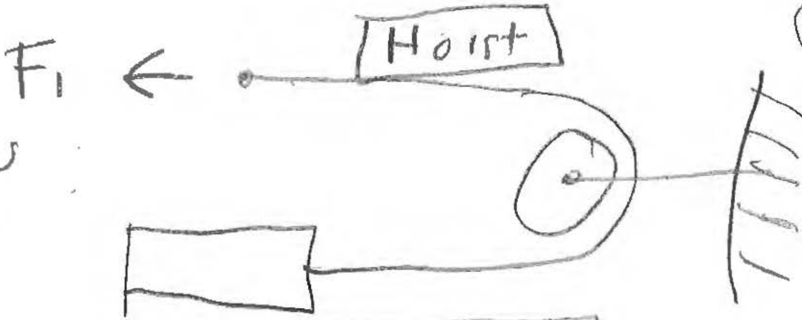
Use pulley to increase/decrease force on/motion of object. (4 uses). Also reverse direction.

2x2

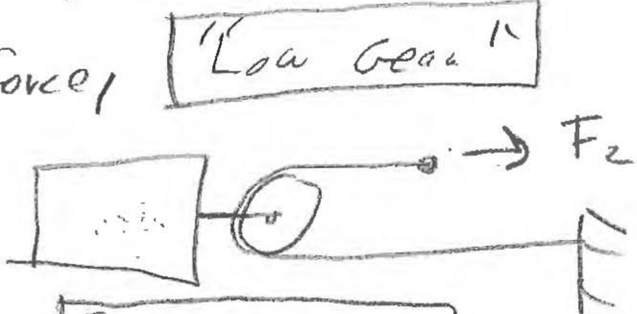
Puzzle

Draw 3 pulley systems that catch all uses above.
(Soln. on next page)

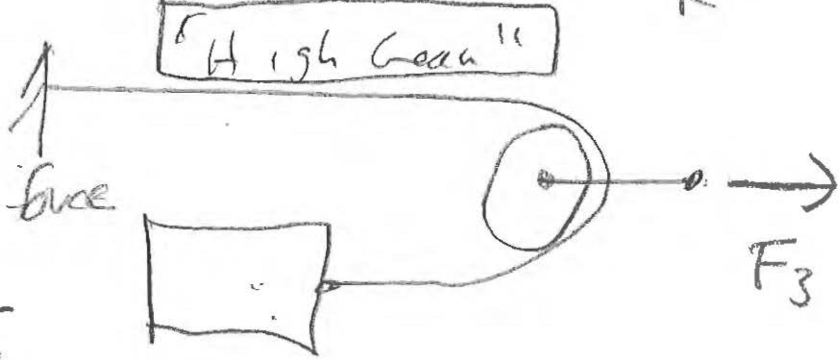
reverses motion



doubles force, halves motion



halves force doubles motion



Power balance (too)

(Power in) = (Power Out)

(Applied Force) · (Applied vel.) =

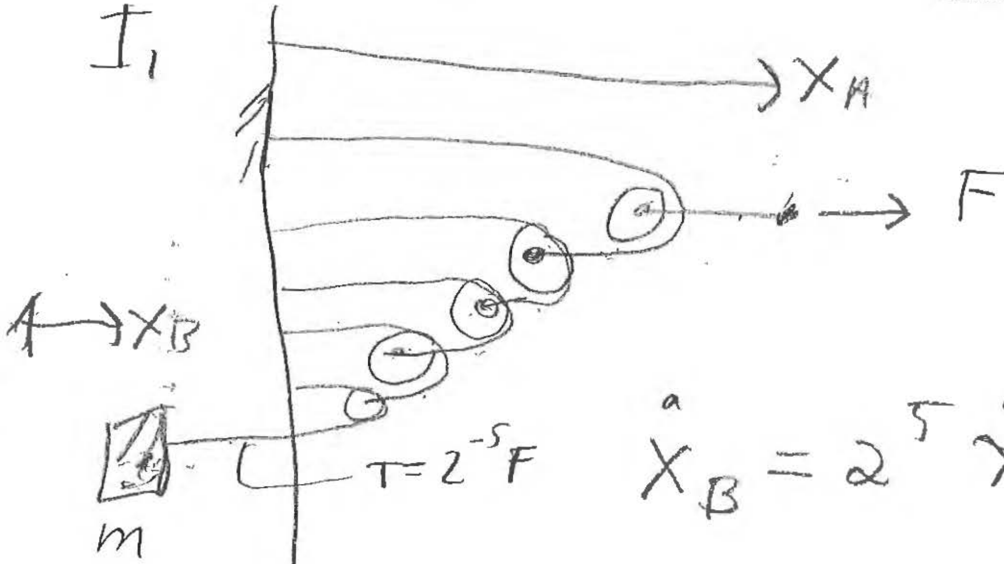
$\underbrace{\text{(Force on mass)}} \cdot \underbrace{\text{(mass vel.)}}$

$\Rightarrow$ Force amplification $\Leftrightarrow$ motion attenuation

Very large numbers

5 pulleys how big a number can an answer be? $\boxed{10^6}$

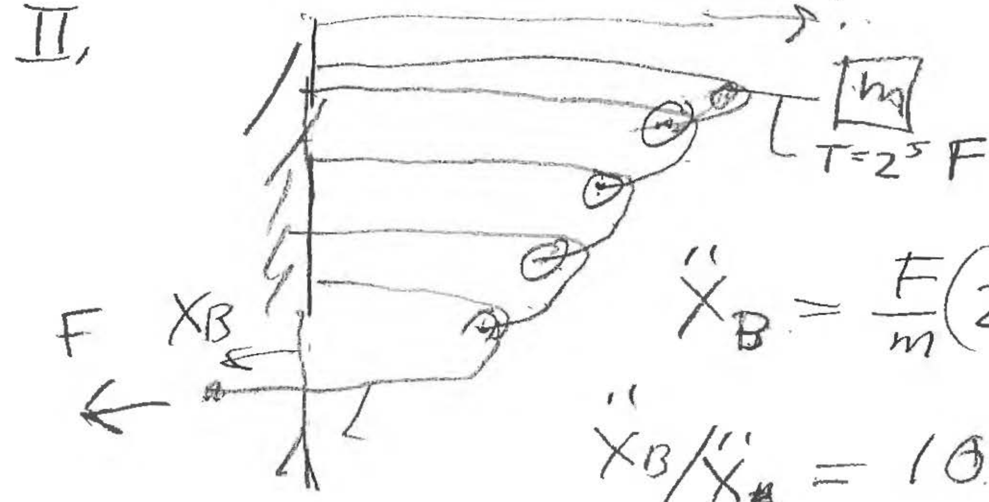
I,



$$\dot{X}_B = 2^5 \dot{X}_A$$

$$\ddot{X}_A = \frac{F}{m}\left(2^{-5}\right)^2$$

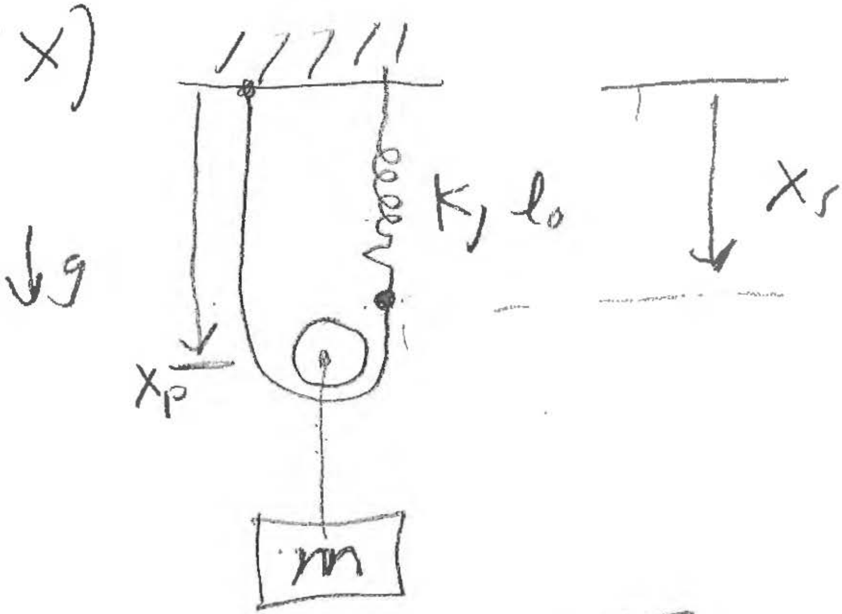
II,



$$\ddot{X}_B = \frac{F}{m}\left(2^5\right)^2$$

$$\ddot{X}_B / \ddot{X}_A = 10^6$$

ex)



$$\omega = c\sqrt{K/m}$$

a) $c = 1$

b) $c = 2$

c) $c = 1/2$

d) $c = 1/4$

e) $c = 4$

Soln on next pages,

ex) (cont'd)

Kinematics

$$a_m = \ddot{X}_p$$

$$l_r = const_1 = X_p + (X_p - X_s)$$

$$2X_p = X_s + l_r$$

length of rope

$$X_s = 2X_p - l_r$$

$$\Delta l_s = X_s - l_0$$

rest length of spring

$$\boxed{\Delta l_s = 2X_p - l_r - l_0}$$

one factor of 2

FBD:

LMB

$$\sum \vec{F} = m\vec{a}$$

(115)

$\{\}\cdot\hat{T}$

└ LMB

$\downarrow\hat{\imath}$

!

$$mg - 2T_s = m\ddot{x}$$

└ $K(\Delta \ell)$

!

└ $2x_p - \ell_p - \ell_s$

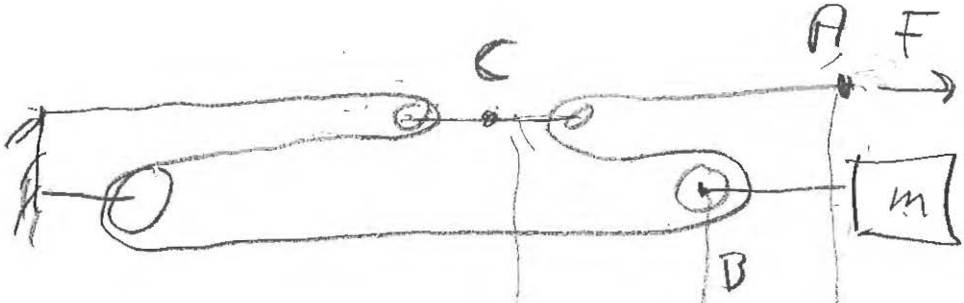
$$m\ddot{x} + 4kx = \begin{pmatrix}\text{const}\\ \text{stuff}\end{pmatrix}$$

$2\times 2 = 4$

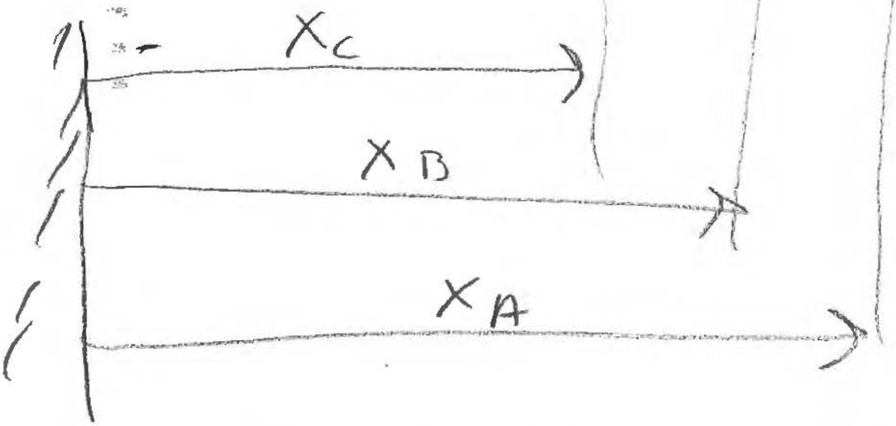
$$\Rightarrow \omega = \sqrt{4K/m} = \boxed{2\sqrt{K/m}}$$

soln. to puzzle on page 113.

ex) A partially indeter-minate problem



$\ell = \text{const}$



B moves w/ mass

$$\ell = \text{const} = 2x_C + (x_B - x_C) + x_B + x_A - x_C$$

1 eqn for 3 unknowns

Odd problem (cont'd) (117)

Can't solve for motion of

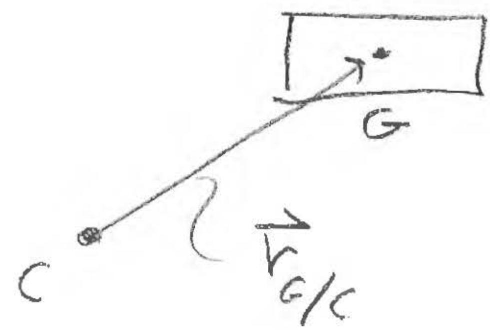
x_C. Not determinable using simple methods.

Rigid objects in 2D w/ 1D motion

((Car braking problems))

Key eqn:

For rigid object & any pt. C



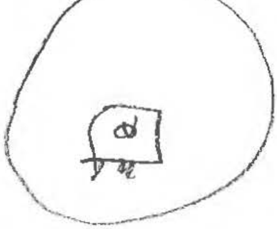
$$\sum \underline{M}^{ext}_{/c} = \vec{r}_{G/c} \times m\vec{a}_{G/F} + I^{G} \alpha \hat{K} \quad (*)$$

$\alpha = 0$ for no-rotation problems

F: fixed frame

recall

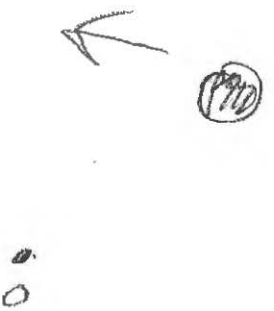
A)



" $M = I\alpha$ "

" $T = J\dot{\omega}$ "

B)



$\vec{r} \times \vec{F} = \vec{r} \times (m\vec{a})$

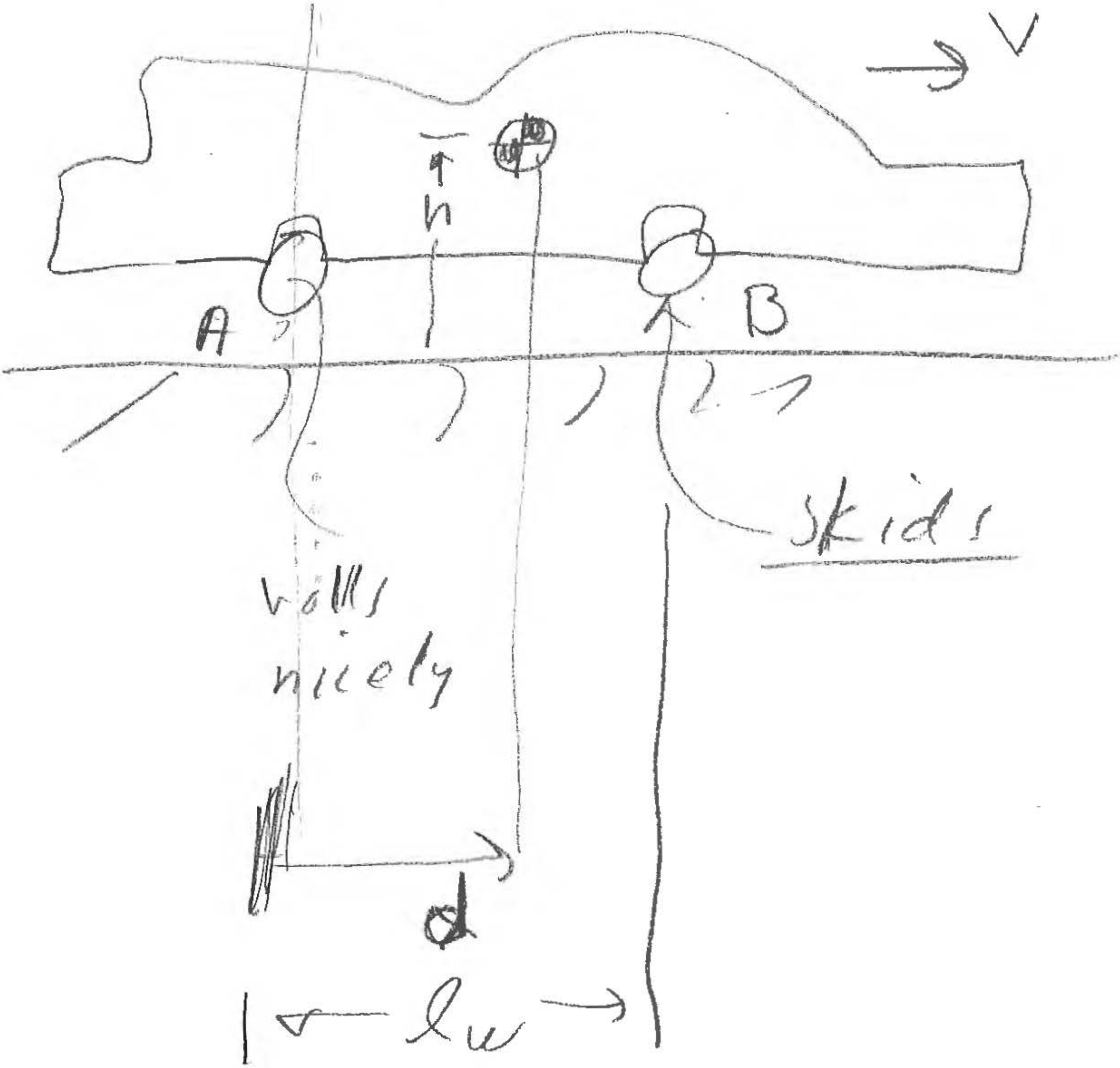
particle motion

$(A) + (B) \Rightarrow *$

intuitive aid for eqn *

$\boxed{\text{Eqn * is key for rest of semester}}$

ex) Front Wheel Braking



assume no rotation

$\alpha = 0$

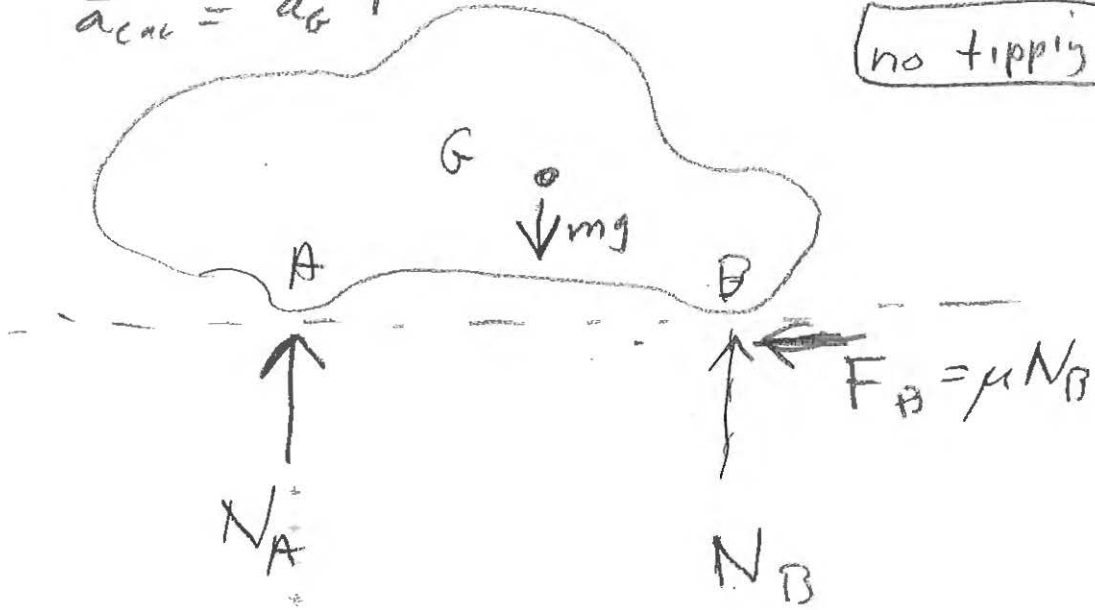
(12c)

$a_{car} = ?$ will be $a_G < 0$

$\vec{a}_{car} = a_G \hat{\imath}$

$\hat{\jmath}$, $\hat{\imath}$

$\boxed{\text{no tipping}}$



Unknowns $\quad a_G$ = scalar

$F_f, \ N_A, \ N_B$

4 Eqs

Eqs: $\quad F_B = \mu N_B \quad (1)$

LMB: $(N_A + N_B)\hat{\jmath}$ $\quad (2)$

$-mg\hat{\jmath} - F_B\hat{\imath} = m a_G \hat{\imath}$

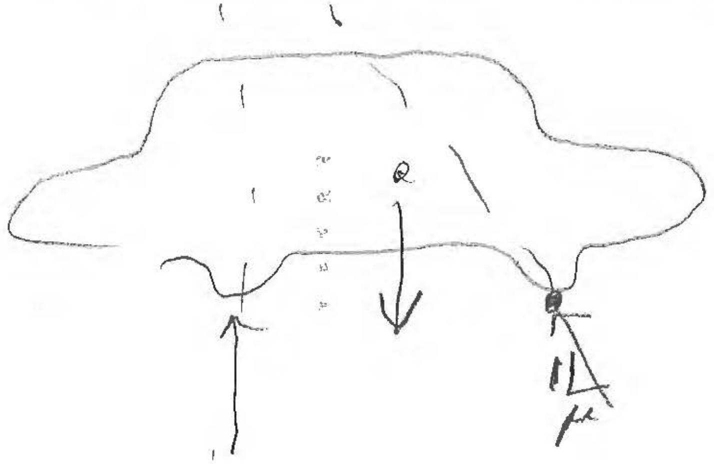
AMB/G: $\vec{r}_{B/G} \times N_B \hat{\jmath} + \vec{r}_{A/G} \times N_A \hat{\jmath}$ (1)

$+ \vec{r}_{B/G} \times (F_f \hat{\imath}) = I^G \alpha \hat{K}$

$\Rightarrow$ 4 eqs & 4 unknowns

Short Cut

H = pt. at intersection of lines of action of forces at A & B.



$$\sum \vec{M}_{/H} = \vec{r}_{G/H} \times m(a_G \hat{i}) + \cancel{I\alpha}^{\,0}\,\hat{k}$$

N_A, N_B, F_B drop out.

$\Rightarrow$ 1 eqn in

1 unknown a_G

(note $-mg\hat{j}$ is in eqn. but it is known)