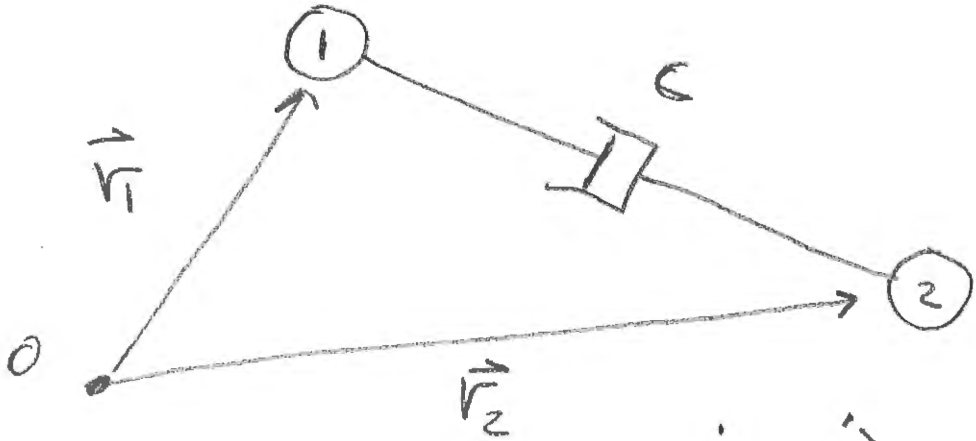
Wed. March 13, 2019

TODAY

Quiz particle in space
Pulley reviews
Car braking example

(Notes here complete the algebra that was not done in lecture for car braking problem)

Quiz



Given $\vec{r}_1, \vec{r}_2, \dot{\vec{r}}_1, \dot{\vec{r}}_2, c$

Find force on 1. ?

$\boxed{\text{3 minutes}}$

Answers on next page, you will have 30 s to pick one,

[Don't write components explicitly.]

force on 1 = $\vec{F}$ = ?

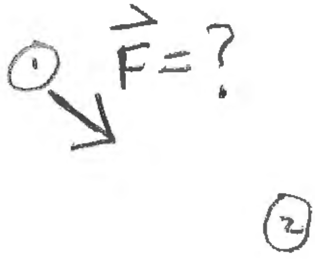
Define

$$\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\dot{\vec{r}} = \dot{\vec{r}}_2 - \dot{\vec{r}}_1$$

$$r = |\vec{r}|$$

$$\hat{\lambda} = \vec{r} / |\vec{r}|$$



OPTIONS

a) $\vec{F} = c(\vec{r} \cdot \dot{\vec{r}})\vec{r} / r^2$

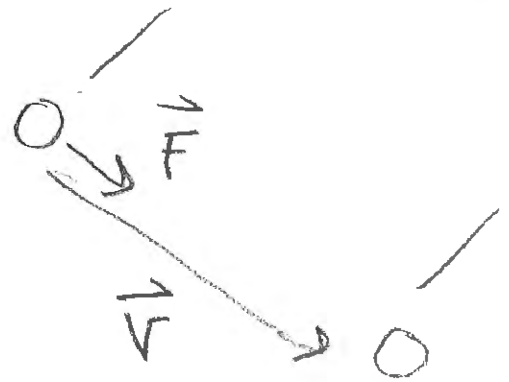
b) $\vec{F} = c(\hat{\lambda} \cdot \dot{\vec{r}})\hat{\lambda}$

c) $\vec{F} = \dfrac{c(\vec{r}_2 - \vec{r}_1) \cdot (\dot{\vec{r}}_2 - \dot{\vec{r}}_1)(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^2}$

d) 2 of above

e) all 3 of above ✓

Soln 1



$\vec{r} = \vec{r}_2 - \vec{r}_1$

$\vec{r}_{12} = \vec{r}_{2/1}$

$\dot{\vec{r}} = \dot{\vec{r}}_2 - \dot{\vec{r}}_1$

$$\ell = |\vec{r}| = \sqrt{\vec{r} \cdot \vec{r}} = r$$

$$\boxed{\vec{F} = \underbrace{c\,\dot{\ell}}_{\text{scalar part}}\ \underbrace{\hat{\lambda}}_{\vec{r}/|\vec{r}| = \frac{\vec{r}}{r}}} \quad (1)$$

$$\dot{\ell} = \frac{d}{dt}(\ell) = \frac{d}{dt}\sqrt{\vec{r} \cdot \vec{r}}$$

$$= \frac{1}{2\sqrt{\vec{r} \cdot \vec{r}}} \frac{d}{dt}(\vec{r} \cdot \vec{r})$$

$$= \frac{1}{2r}(\vec{r} \cdot \dot{\vec{r}} + \dot{\vec{r}} \cdot \vec{r}) =$$

$$= \frac{\dot{\vec{r}} \cdot \vec{r}}{r} \quad (2)$$

(1) & (2) $\Rightarrow$

$$\vec{F} = c\,\dot{\ell}\,\hat{\lambda} = c\,\frac{\dot{\vec{r}} \cdot \vec{r}}{|\vec{r}|}\,\frac{\vec{r}}{|\vec{r}|} = c(\dot{\vec{r}} \cdot \hat{\lambda})\hat{\lambda}$$

Soln 2 (Quiz cont'd)

hard part is $\dot{\ell} = ?$

Note: $\{ \ell^2 = \vec{r} \cdot \vec{r} \}$

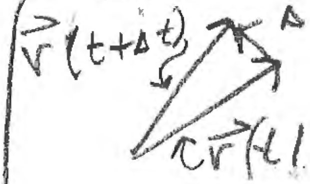
$$\frac{d}{dt}\{\ \} \Rightarrow 2\ell\dot{\ell} = \vec{r} \cdot \dot{\vec{r}} + \dot{\vec{r}} \cdot \vec{r}$$

$$\dot{\ell} = \frac{\vec{r} \cdot \dot{\vec{r}}}{\ell}$$

$\ell = r = |\vec{r}|$

$$\boxed{\dot{\ell} = \dot{\vec{r}} \cdot \hat{\lambda}}$$

example



$\Longleftrightarrow$ $\boxed{\dot{\ell} = |\dot{\vec{r}}|}$ ✗

wrong

$\dot{\vec{r}} \neq \vec{0}$, yet

$\dot{\ell} = 0$!

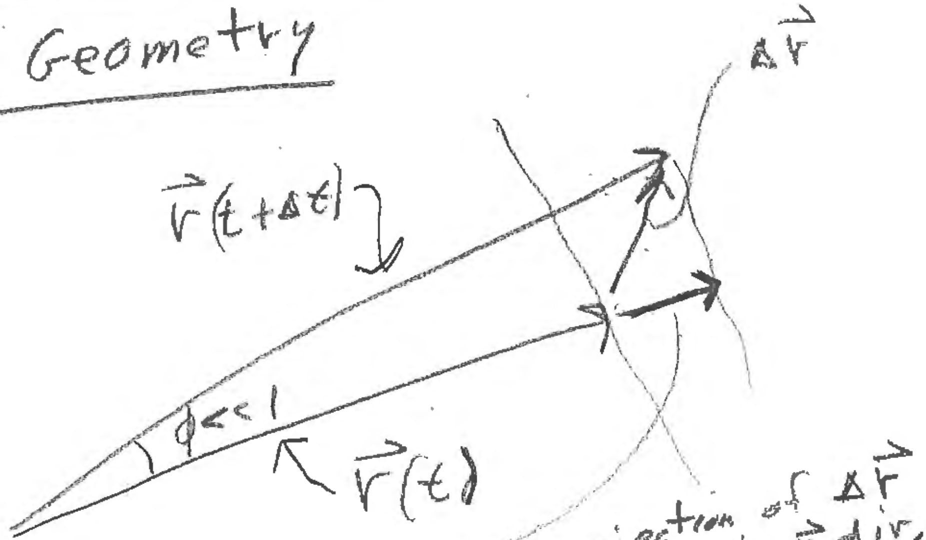
$$\frac{\dot{\vec{r}}}{|\dot{\vec{r}}|}|\dot{\vec{r}}|$$

Ignore this: talking to student

WATCH OUT FOR THIS MISTAKE.

Soln 3 to quiz

USE Geometry



$$\Delta l \cong (\Delta \vec{r}) \cdot \frac{\vec{r}}{|\vec{r}|}$$

$$\dot{l} = \frac{\Delta \vec{r}}{\Delta t} \cdot \frac{\vec{r}}{|\vec{r}|}$$

$$\boxed{\dot{l} = \dot{\vec{r}} \cdot \hat{\lambda}}$$

Student asked: "What is $\Delta \vec{r}$?"

Answer $\Delta \vec{r} = \vec{r}(t+\Delta t) - \vec{r}(t)$

= change in $\vec{r}$ in time Δt

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%Particles in space Example (lecture Mar 13, 2019
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```
for x = 1:10
    disp(x)
end
```

```
%Given r1, r1dot, r2, r2dot
c= rand(1);
r1    = rand(3,1); r2    = rand(3,1);
r1dot = rand(3,1); r2dot = rand(3,1);

%Find a vector in the direction r12
%with magnitude   c * d/dt (|r12|)

r    = r2    - r1;       % rel position
rdot = r2dot - r1dot;    % rel velocity

lambda = r/norm(r);      % uinit vector

Fscalar = c * dot(rdot,r)/norm(r); %see lecture notes

Fvector = Fscalar * lambda  % the answer, a vector
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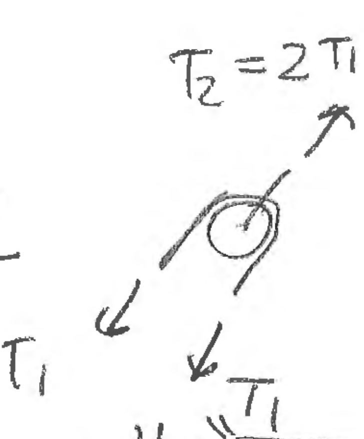
Pulleys (review)

FBD of each pulley



Assumptions: "ideal pulley" round, good bearings, & negligible inertia

Inextensible rope

each rope: $\ell = \text{const}$

$\Rightarrow$ write length(s) in terms of key positions of masses, connection points

If concerned with V & a & not position

$\Rightarrow$ can neglect πR

& other constant terms,

Another "trick"

power balance to solve problems,

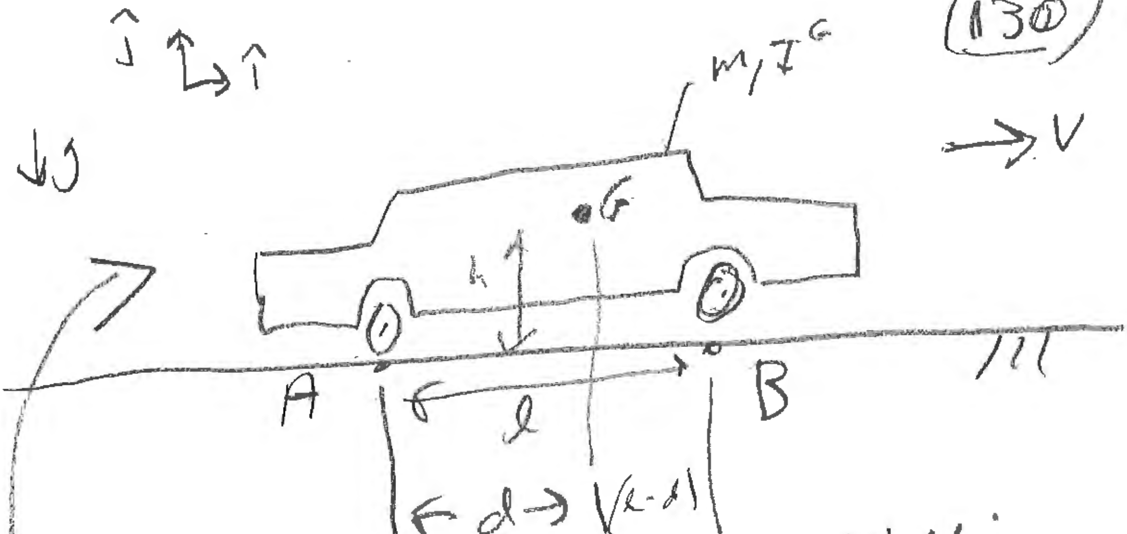
$$\boxed{P_{in} = \dot{E}_K}$$

for system, includg masses

And to check problems.

$$\boxed{P_{in\ total} = 0}$$

for massless pulley system w/out masses



skidding braking

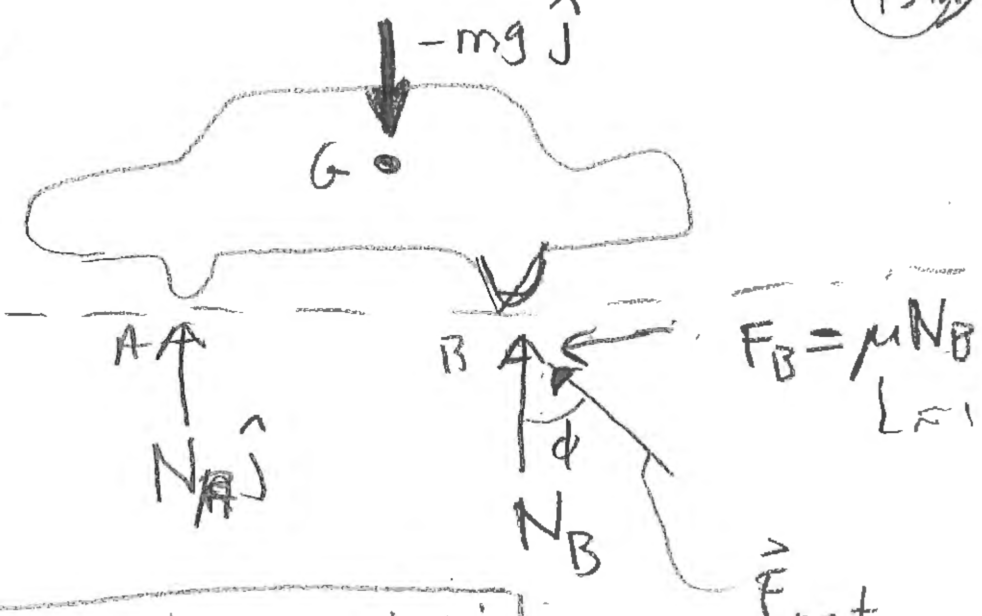
ideal rolling

round
bearings good
negligible inertia

CAR BRAKING PROBLEM

$a_G = ?$

FBD



ASIDE about Friction

$$\tan\phi = \mu$$

ϕ ← friction angle, equiliv. way to express μ

$$\mu = 1 \iff \phi = \pi/4$$

Kinematics:

$$\omega = 0, \quad \dot{\omega} = 0$$

$$\vec{a}_G = a_G\,\hat{\imath}$$

ie. No rotation & G moves horizontally

13 lb

LMB

$$\sum \vec{F}_i^{ext} = m_{tot} \vec{a}_G$$

$$\left\{ \boxed{N_A \hat{j} + N_B \hat{j} - mg \hat{j} - \underbrace{F_B}_{(\mu N_B)} \hat{i} = m a_G \hat{i}} \right\} \circledast$$

$$\{\ast\} \cdot \hat{i} \Rightarrow -F_B = m a_G \quad (1)$$

$$\{\ \} \hat{j} \Rightarrow N_A + N_B = mg \quad (2)$$

(4) unknowns N_A, N_B, F_B, a_G

$$F_B = \mu N_B \quad (3)$$

Need another eqn

AMB /G ; $\sum \vec{M}_{/G} = I^G \overset{0}{\cancel{\alpha}} \hat{k}$

$$\underbrace{\vec{r}_{A/G}}_{-d\hat{i}-h\hat{j}} \times N_A \hat{j} + \overbrace{\vec{r}_{B/G}}^{(l-d)\hat{i}-h\hat{j}} \times (N_B \hat{j} - F_B \hat{i})$$

$$= \vec{0} \quad (4)$$

AMB/$_G$ (cont'd)

$$\{(4)\}\cdot\hat{k} \Rightarrow \quad -dN_A + (\ell-d)N_B - F_B h = 0 \qquad (4)$$

Need to Solve (1)-(4) for a_G.

$$(2) \Rightarrow \qquad N_A = mg - N_B \qquad (5)$$

Subst. (5) into (4)

$$-d\underbrace{(mg - N_B)}_{N_A} + (\ell-d)N_B - \overbrace{\mu N_B}^{F_B} h = 0 \qquad (6)$$

Solve (6) for N_B:

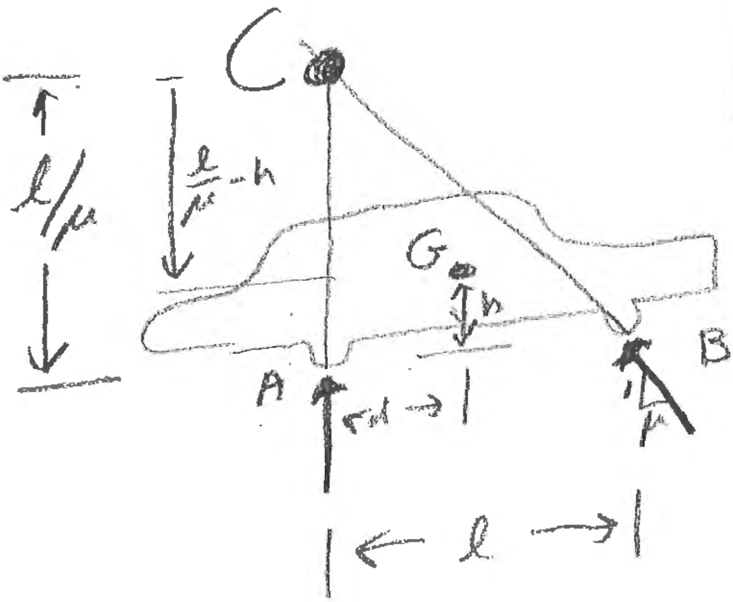
$$N_B = \frac{dmg}{d + (\ell-d) - \mu h} \qquad (7)$$

$$(7)\ \&\ (3) \Rightarrow \boxed{a_G = \frac{-dg}{\ell/\mu - h}}$$

Alternative approach to car

Wheel forces have no moment about C.

So, AMB/C is one eqn. in one unknown, a_G.



$a_G \hat{\imath}$

AMB/C $\quad \sum \vec{M}_{/C}^{ext} = \vec{r}_{G/C} \times m\vec{a}_G + I^G \cancel{\alpha}^{0} \hat{k}$

$\vec{r}_{G/C} \times -mg\hat{\jmath}$

$-\left(\frac{\ell}{\mu}-h\right)\hat{\jmath} + d\hat{\imath}$

$$-d\,mg = \left(\frac{\ell}{\mu}-h\right) m a_G$$

$$\boxed{a_G = \frac{-d \cdot g}{\ell/\mu - h}}$$

Comment on Soln.

We had soln:

$$a_G = \frac{-dg}{\ell/\mu - h}$$

$$N_B = \frac{dmg}{\ell - \mu h}$$

$$N_A = mg - N_B$$

$$= \left(\frac{(\ell-\mu h) - d}{\ell - \mu h}\right) mg$$

Note: $N_A = 0$ if $\ell - \mu h - d = 0$

$\Rightarrow \mu = (\ell - d)/h$

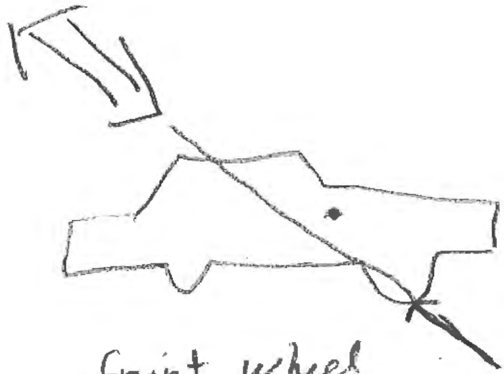
friction force goes through $G = $ CoM $\Rightarrow$

Then, also

$$N_B = mg$$

$$a_G = -\mu g$$

for $\mu > \frac{\ell - d}{h}$



then $N_A < 0 \Rightarrow$ NONSENSE

Car doesn't tip is
invalid assumption in
this case