TODAY March 18, 2019

Quizes (Review)
Circular Motion

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3 Quizzes were done first. Here they are at the end of this lectures notes

3 quizes

1. $\vec{F} = -c\,\vec{v}\,|\vec{v}|^{1/2}$

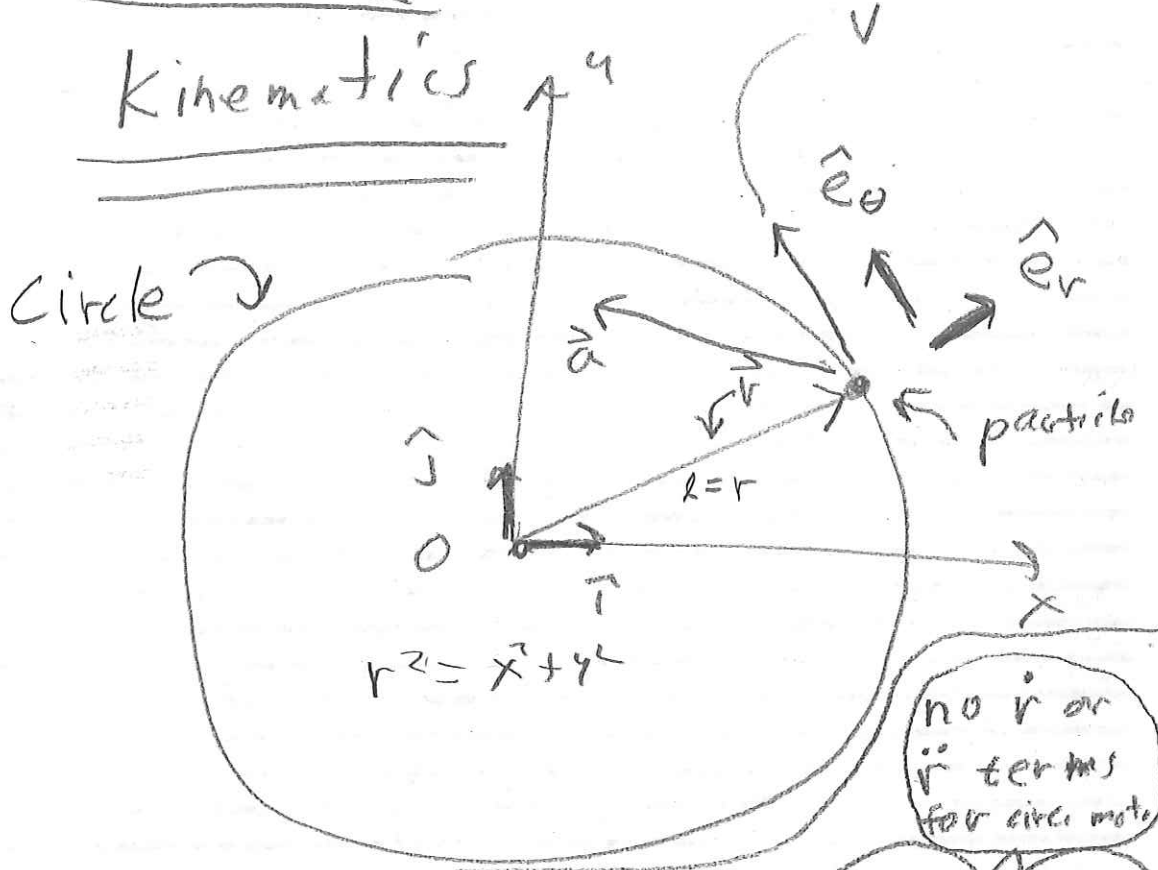
2. 

3) 

CIRCULAR MOTION

Particles

Kinematics



position $\vec{r}$ $= x\hat{i} + y\hat{j} = r\hat{e}_r$

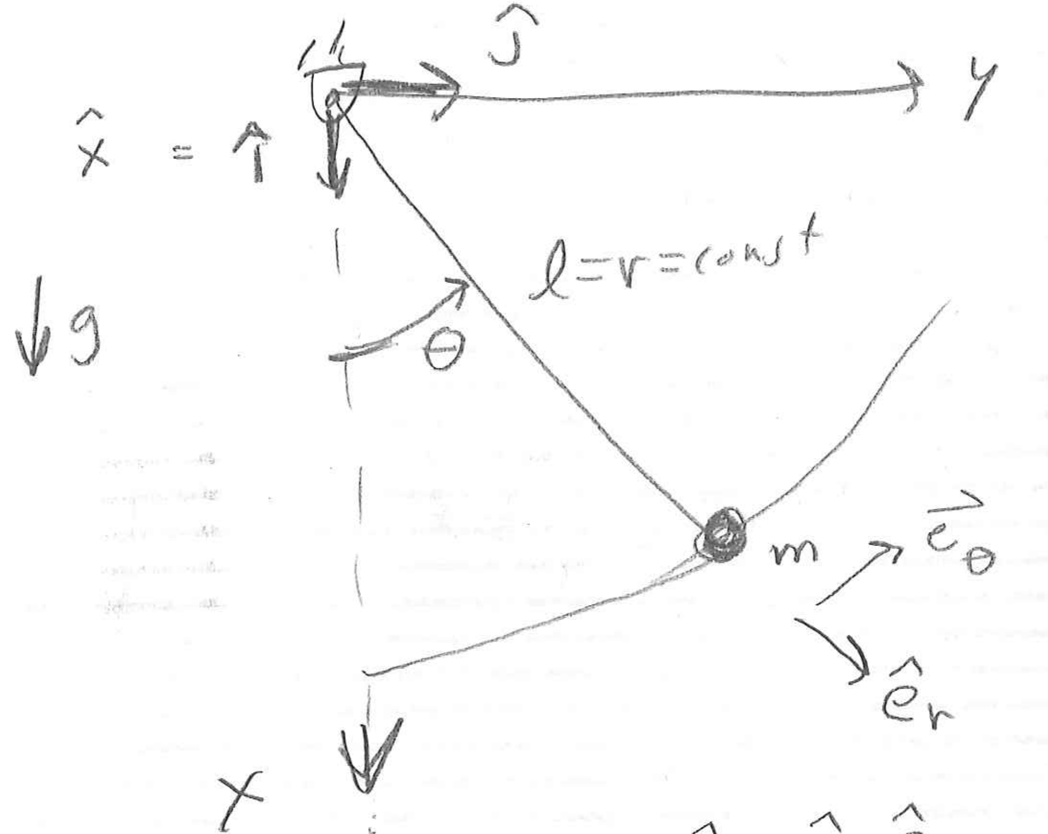
vel. $\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} = r\dot{\theta}\hat{e}_\theta = v_\theta \hat{e}_\theta = v\hat{e}_\theta$

accel $\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} = a_\theta \hat{e}_\theta + \frac{-v_\theta^2}{r}\hat{e}_r$

$= r\ddot{\theta}\hat{e}_\theta + -r\dot{\theta}^2\hat{e}_r$

Pendulum: 1037 ways

↑ slight exaguration



$\hat{e}_\theta = \hat{k} \times \hat{e}_r$

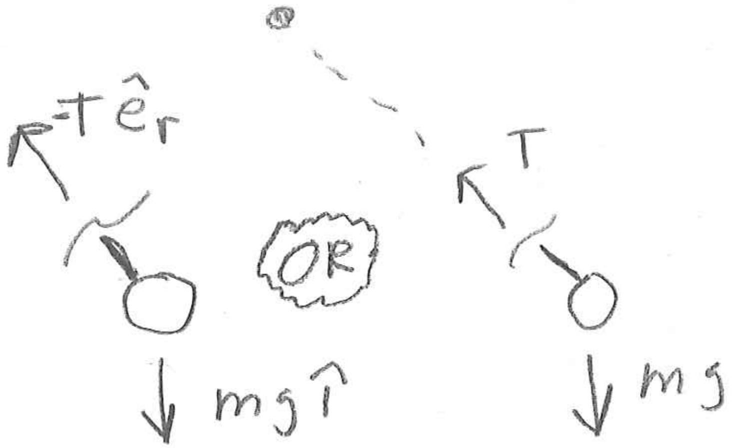
$\hat{e}_r = \vec{r}/|\vec{r}|$

$\hat{e}_r = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$

$\hat{e}_\theta = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

FBDs



Method 1

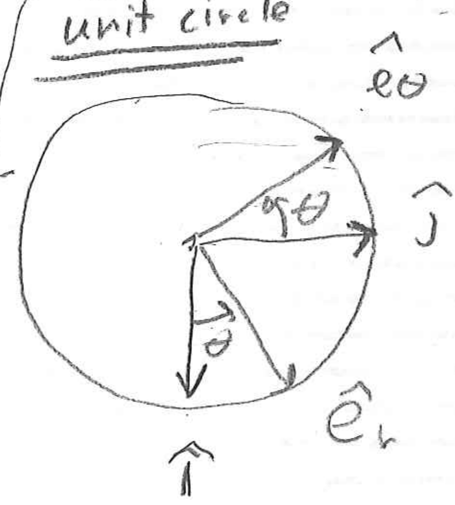
LMB:

$$\sum \vec{F} = m\vec{a}$$

$$\left\{ +mg\hat{\imath} - T\hat{e}_r = m\left(r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r\right) \right\}$$

$$\{\}\cdot\hat{e}_\theta \Rightarrow$$

$$\cancel{m}g\underbrace{\hat{\imath}\cdot\hat{e}_\theta}_{= -\sin\theta} = \cancel{m}r\ddot{\theta}$$



$$\Rightarrow -g\sin\theta = r\ddot{\theta}$$

$$\boxed{\ddot{\theta} = \frac{-g}{r}\sin\theta}$$

EoM

Method 2

LMB

$$\left\{ mg\hat{i} - T\hat{e}_r = m(r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r) \right\}$$

$$\boxed{\ddot{\theta} = \frac{-g}{r}\sin\theta}$$

$\hat{e}_r \times \{ \} \Longrightarrow$ (same thing)

Method 3

AMB/$_O$

$$\underbrace{\sum \vec{M}_{/O}}_{r\hat{e}_r \times (mg\hat{i} - T\hat{e}_r)} = \underbrace{\vec{r}_{G/O}}_{r\hat{e}_r} \times m\underbrace{\vec{a}_G}_{(r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r)} + \cancel{I^G}\ddot{\theta}\hat{k}$$

Mom. of Inertia of pt. mass is zero 0

Circular motion

$$\Longrightarrow \boxed{\ddot{\theta} = \frac{-g}{r}\sin\theta \quad \text{again}} \text{ (again)}$$

Method 4

Cons. of Energy

$$E_{TOT} = \text{const.}$$

$$\text{const.} = \underbrace{E_p}_{mgh} + \underbrace{E_K}_{\frac{1}{2}mv^2}$$

$$\left\{ \text{const} = \overbrace{-mgr\cos\theta}^{mgh} + \frac{1}{2}(r\dot{\theta})^2 m \right\}$$

$$\frac{d}{dt}\{\ \} \Rightarrow \boxed{\ddot{\theta} = \frac{-g}{r}\sin\theta}$$

(again)

Method 5

Power Balance

$$P = \dot{E}_K \leftarrow \frac{d}{dt}\left(\frac{(r\dot{\theta})^2 m}{2}\right)$$

$$P = \vec{V} \cdot (mg\hat{\imath}) \qquad \vec{V} = r\dot{\theta}\hat{e}_\theta$$

$$\Rightarrow \boxed{\ddot{\theta} = \frac{-g}{r}\sin\theta}$$

(again)

Method 6

use cart coord

LMB: $\vec{F} = m\vec{a}$

$$mg\hat{\imath} - T\hat{e}_r = m(\ddot{x}\,\hat{\imath} + \ddot{y}\,\hat{\jmath})$$

$$\hat{e}_r = \left(\frac{x}{r}\,\hat{\imath} + \frac{y}{r}\right)\hat{\jmath}$$

T, $\ddot{x}$, $\ddot{y}$ as unknowns

Have 2 eqs for 3 unknowns

another eqn: $l = \text{const}$

$$\Rightarrow \quad l^2 = \text{const}$$

$$\Rightarrow \quad \left\{ x^2 + y^2 = \text{const} \right\}$$

$\frac{d^2}{dt^2}\{\ \} \Rightarrow$ rel. bet. $x, \dot{x}, \ddot{x}$; $y, \dot{y}, \ddot{y}$

3rd eqn. $\longrightarrow$

[TO BE CONT'D IN ANOTHER LECTURE]

Quiz 1

Drag force on particle is prop. to $v^{3/2}$. Write $\vec{F}_D$ as a vector: $\vec{F} = ?$

Write Matlab commands for

$F = ?$
$\leftarrow 2\times 1$

Given $\vec{V}$, c, $V = \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$

Q1: $\vec{F}$

F, Matlab

a) $\vec{F} = -cV^{3/2}$ ✗	MATLAB error: `F = -c*V^(3/2)` ✗
b) $\vec{F} = -c\lvert\vec{V}\rvert^{1/2}\,\vec{V}$ ✓	$\cancel{\texttt{F = -c*V*norm(V)}}$ $\underbrace{\texttt{V*norm(V)}}_{\propto V^2}$ ✗
c) $\vec{F} = -c\vec{V}^{3/2}$ ✗	`F = -c*norm(V)^...` `(3/2)*V` ✗
d) $\vec{F} = -c\lvert\vec{V}\rvert^{3/2}*\vec{V}$ ✗	`F = -c* V/norm(V)...` `^(3/2)` ✗

e) NONE OF ABOVE ✓

Correct Matlab

```
F=-c*V*...
  (norm(V))^(1/2)
```

`V.^(3/2)`

$$= \begin{bmatrix} V_x^{3/2} \\ V_y^{3/2} \end{bmatrix} \quad ✗$$

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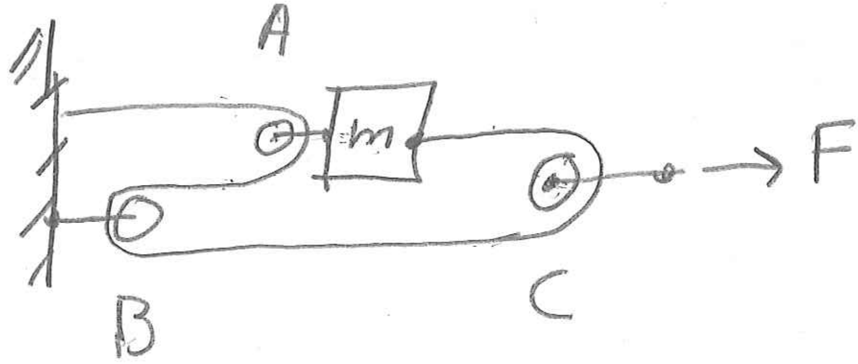
QUIZ 2

$\ddot{x}_c = ?$



a) $F/(4m)$ ✓

b) $F/(2m)$

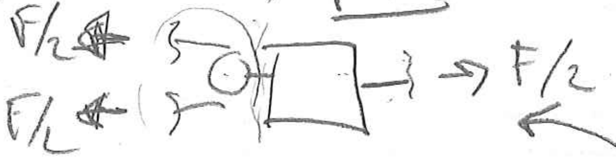
c) F/m

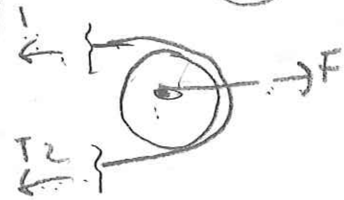
d) $-F/m$

e) $-F/(2m)$

Q2:

FBDS

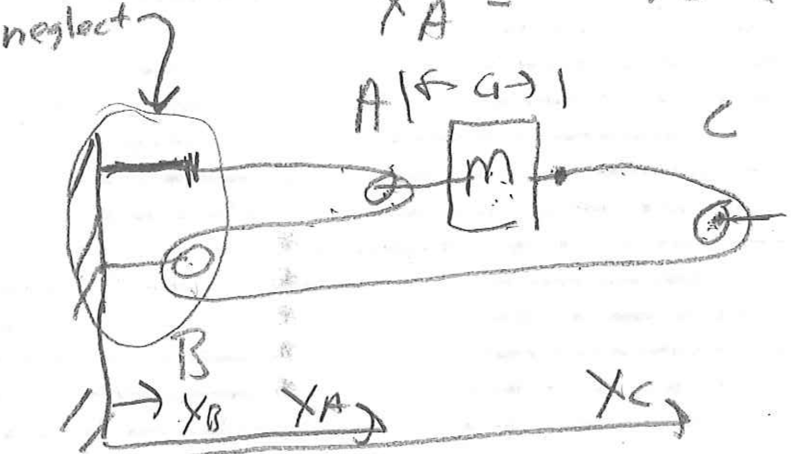




OR

LMB: $m\ddot{x}_A = -F/2$

$\ddot{x}_A = -F/2m$



$$\left\{ \begin{aligned} \text{Const} &= \text{length} \\ &= x_A + (x_A - x_B) + x_B \\ &\quad + (x_C - x_B) + x_C - (x_A + c_1) \end{aligned} \right\}$$

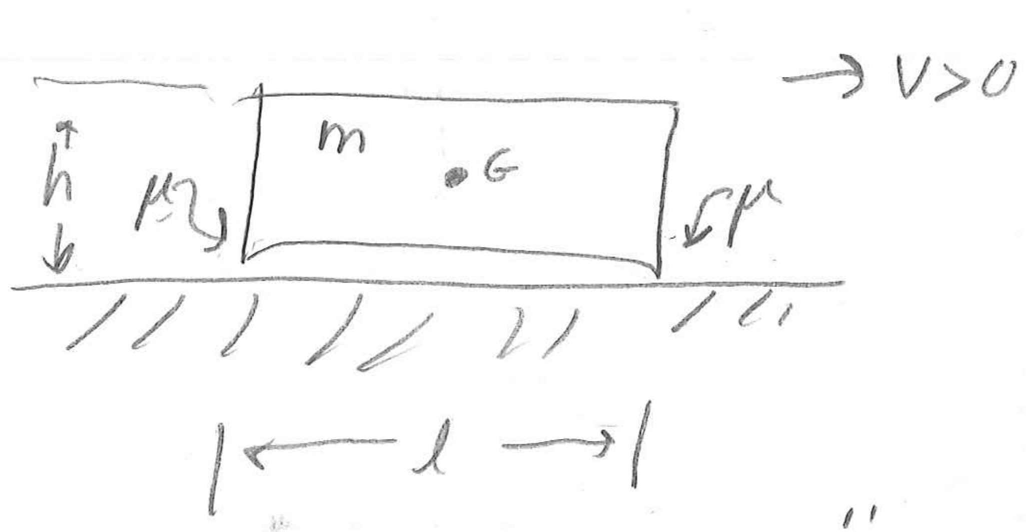
$\frac{d^2}{dt^2}\{\}$

$$\Rightarrow \quad 0 = \ddot{x}_A + 2\ddot{x}_C$$

$$\ddot{x}_C = -\ddot{x}_A/2 = \boxed{F/4m}$$

QUIZ 3

$\downarrow g$ $\quad \hat{j}$ $\hat{i}$



$a_G = ?$ $\qquad a_G = \ddot{x}_G$

a) $-\mu g$ ✓

b) $-\mu/(2g)$

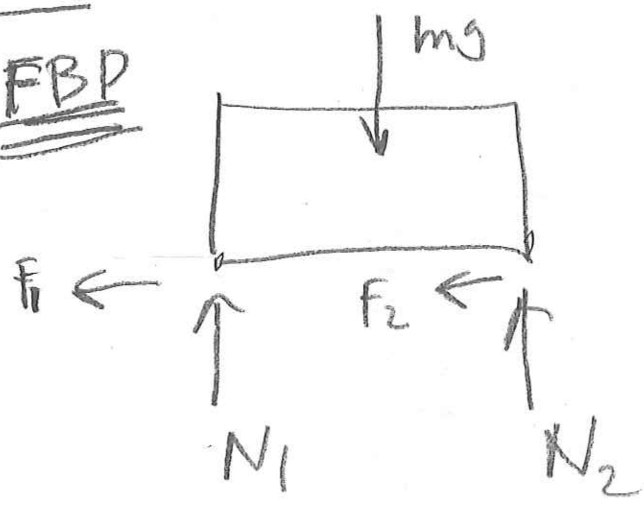
c) $-\mu(h/l)\,g$

d) $-\mu g(h/2 - l/2)/l$

e) $-\mu g/3$

Q3 (148)

FBP



Friction: $F_1 = \mu N_1$

$F_2 = \mu N_2$

$\{LMB\} \cdot \hat{j}$

$$\Rightarrow (N_1 + N_2) - mg = m\cancelto{0}{\ddot{y}_G}$$

$$N_1 + N_2 = mg$$

$$\Rightarrow F_1 + F_2 = \mu mg$$

$$\{LMB\} \cdot \hat{i} \Rightarrow -F_1 - F_2 = m\ddot{x} \quad ✓$$

$$\boxed{\ddot{x} = -\mu \cancel{m} g / \cancel{m}}$$