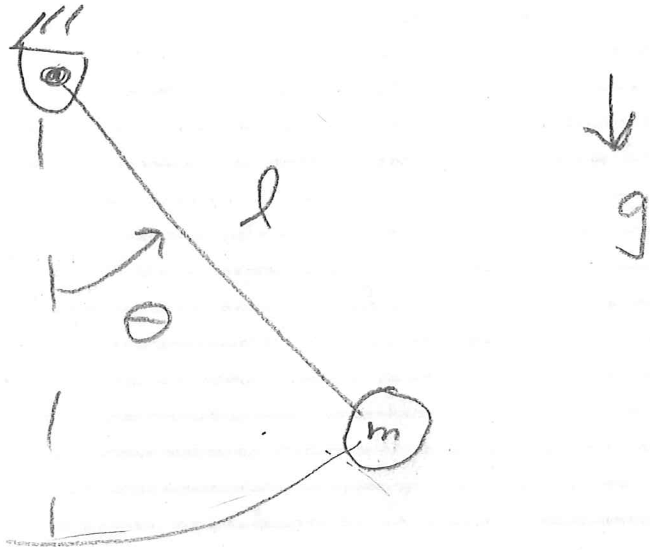
Lecture 16, March 20, 2019

TODAY

QUIZ, Rotations

See also computer demo on Rotations.

QUIZ



Assuming small $\theta \ll 1$

$\sin(\underbrace{\lambda t}_{2\pi})$ one cycle

a) $\lambda = g/\ell$

b) $\lambda = \sqrt{g/\ell}$ ✓

c) $\lambda = 1$

d) $\lambda = \sqrt{\ell/g}$

e) $\lambda = \ell/g$

$T = 2\pi/\lambda$

$\lambda T = 2\pi$

$\lambda = 2\pi/T$

Why "λ"?

omega (ω)

overloading

ω = ang. vel

α = freq.

ODE;

$$\boxed{\ddot{\theta} = -\frac{g}{l}\sin\theta}$$

small angle approximation

$\theta << 1$

$$\Rightarrow \quad \sin\theta \approx \theta - \cancel{\frac{\theta^3}{3!}} + \cancel{\frac{\theta^5}{5!}} - \cdots$$

$$\Rightarrow \quad \boxed{\ddot{\theta} = -\frac{g}{l}\theta}$$

$$\boxed{\ddot{\theta} + \frac{g}{l}\theta = 0}$$

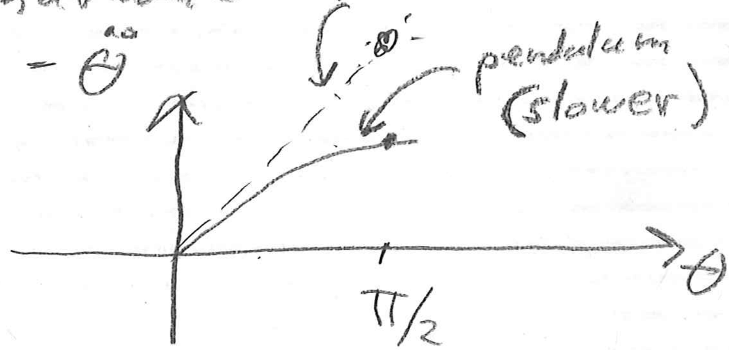
$A\sin(\sqrt{\frac{g}{l}}t)$

ω

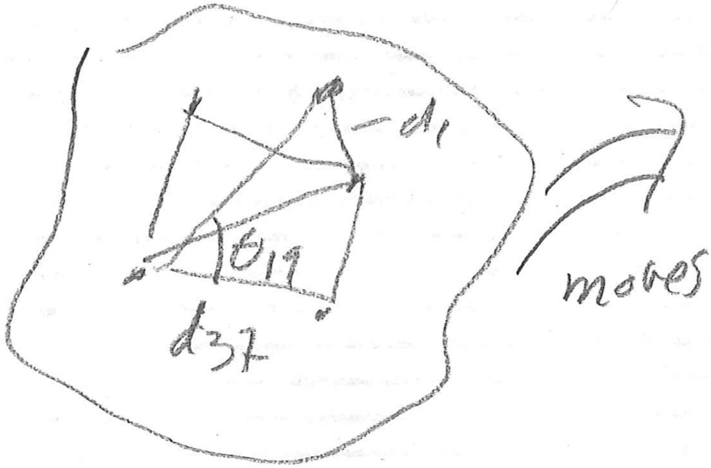
whatever

" $\ddot{X}$ + [illegible] $X = 0$ "

$\Rightarrow$ harmonic oscillator



Rigid objects

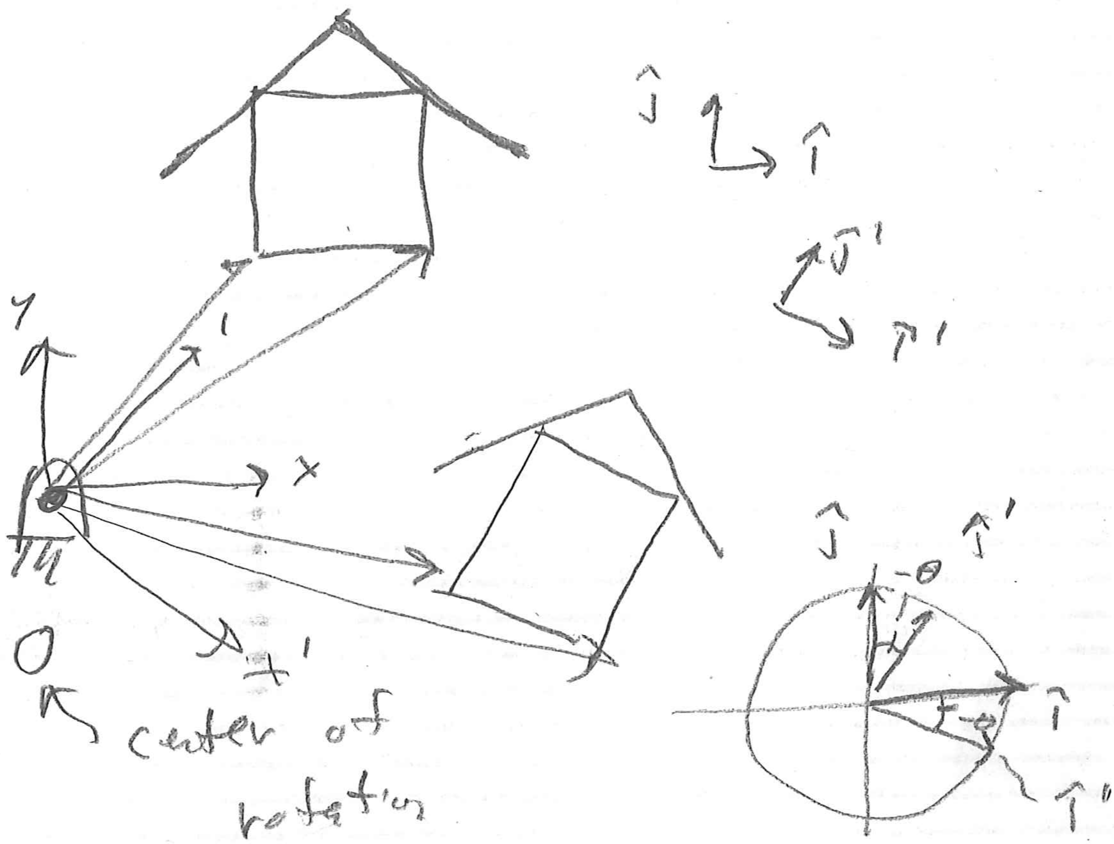


All ~~length~~ lengths &
all angles marked in
object are const.

rigid object

Review

no rotation
(Working car problem)



$\vec{r}_i$ = pos. vector of some pt.

$$Rot(\vec{r}_i) = \vec{r}_i^{\,*} = \vec{r}_i \text{, } \underline{\text{rotated}}$$

typical pt.

$$\underset{\text{orig. coords}}{\vec{r}_i} \xrightarrow{\text{rotation}} \underset{\text{new coords}}{\vec{r}_i^{\,*}}$$

$$\vec{r}_i = x_i \hat{\imath} + y_i \hat{\jmath}$$

TWO WAYS TO LOOK AT SAME THING

$$\xrightarrow{\text{rotate}} \Rightarrow \underset{\text{new}}{\vec{r}_i^{\,*}} = x_i^* \hat{\imath} + y_i^* \hat{\jmath}$$

$$= x_i \hat{\imath}' + y_i \hat{\jmath}'$$

$$x_i^* = ? \qquad y_i^* = ?$$

$$\boxed{\vec{r}_i^{\,*} = \vec{r}_i^{\,*}}$$

Laurie Anderson "Let X=X"

$$\left\{ x_i^* \hat{\imath} + y_i^* \hat{\jmath} = x_i \hat{\imath}' + y_i \hat{\jmath}' \right\}$$

$$\{\ \}\cdot \hat{\imath} \Rightarrow x_i^* = x_i \underbrace{\hat{\imath}' \cdot \hat{\imath}}_{\cos\theta} + y_i \underbrace{\hat{\jmath}' \cdot \hat{\imath}}_{-\sin\theta}$$

$\{\ \}\cdot\hat{j} \Rightarrow$

$$y_i^* = \sin\theta\, x_i + \cos\theta\, y_i$$

$$\begin{bmatrix} x_i^* \\ y_i^* \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} x_i \\ y_i \end{bmatrix}}_{\text{one pt}}$$

$$\begin{bmatrix} \text{New house coords} \end{bmatrix} = \begin{bmatrix} R \end{bmatrix} \underbrace{\left[\begin{array}{c|c|c|c} x_1 & x_2 & \cdots & \cdots \\ y_1 & y_2 & & \end{array}\right]}_{\text{diff. pts. on picture: 1, 2, 3...}}$$

$$\left[\begin{array}{c|c|c} x_1^* & x_2^* & \cdots \\ y_1^* & y_2^* & \end{array}\right]$$

Eqs. of Motion

for circular motion

Physics classes

$$\sum \vec{M}_{/O} = \begin{cases} I^{O} \dot{\omega} \hat{k} \\ \boxed{\vec{r}_{G/O} \times m \vec{a}_G + I^{G} \dot{\omega} \hat{k}} \end{cases}$$

our main eqn.

origin of rotation

$$I^{G} = \begin{cases} \sum r_{i/G}^2 \, m_i \\ \int r_{/G}^2 \, dm \end{cases}$$

Polar moment of inertia about G

$$\sum \vec{M}_{/C} = \underbrace{\begin{cases} \sum_i \vec{r}_{i/C} \times m_i \vec{a}_C \\ \int \vec{r}_{/C} \times \vec{a} \, dm \end{cases}}_{\dot{\vec{H}}_{/C}}$$

always

all dyn.

this is all if no rotation

$$\dot{\vec{H}}_{/C} = \vec{r}_{G/C} \times m \vec{a}_G + I^G \dot{\omega} \hat{k}$$

2 D rigid objects

α = angular accel.

~~Rotation~~