Rotational Dynamics
Gen. motion] ← Today

Wed. March 27 2019

2D rigid objects

General facts:

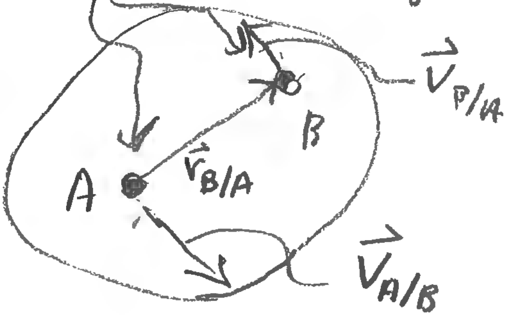
$$\boxed{\vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A}}$$

$\vec{V}_{B/A}$: $\vec{V}_B - \vec{V}_A$

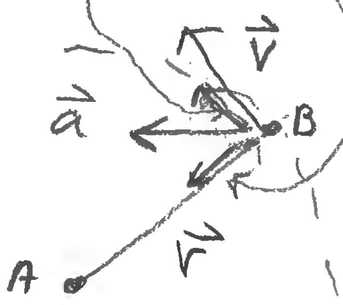
$\vec{\omega}$: $\dot{\theta}\hat{k}$

$\dot{\theta}$ ← rate of change of any angle on object

any 2 pts, fixed on object



$$\vec{a}_{B/A} = \underbrace{\dot{\vec{\omega}} \times \vec{r}_{B/A}} + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})}_{= -\omega^2 \vec{r}_{B/A}}$$



accel. towards center of circle due to change of directions

acc. tan to circle due to change of speed

Moment of Inertia

$$I^G = \int |\vec{r}_{/G}|^2 dm = \int r_{/G}^2 dm$$

polar mass moment of Inertia about G

CoM = G : $m_{tot}\vec{r}_G = \int \vec{r}\, dm$; $\sum \vec{r}_i m_i$

Center of Mass at G

$\vec{L}$ = Lin mom $\equiv \sum m_i \vec{v}_i$

Mechanics

LMB: $\boxed{\sum \vec{F}_j^{ext} = \underbrace{\sum m_i \vec{a}_i}_{\dot{\vec{L}}}}$

2D rigid object

$$\boxed{\sum \vec{F}^{ext} = m_{tot}\vec{a}_G}$$

and all other systems.

However, on rigid object, G is fixed w.r.t. object. A pt. "on" the object.

<u>AMB</u>: $\sum \vec{M}^{ext}_{/c} = \frac{d}{dt} \vec{H}_{/c}$

pt. C is fixed in space

$= \sum \vec{r}_{i/c} \times m_i \vec{a}_i$ ✓

C doesn't have to be fixed in space

$$\boxed{\vec{H}_{/c} = \sum \vec{r}_{i/c} \times m_i \vec{v}_{i/c}}$$ = Ang Mom. About C

<u>Rigid Object</u> 2D

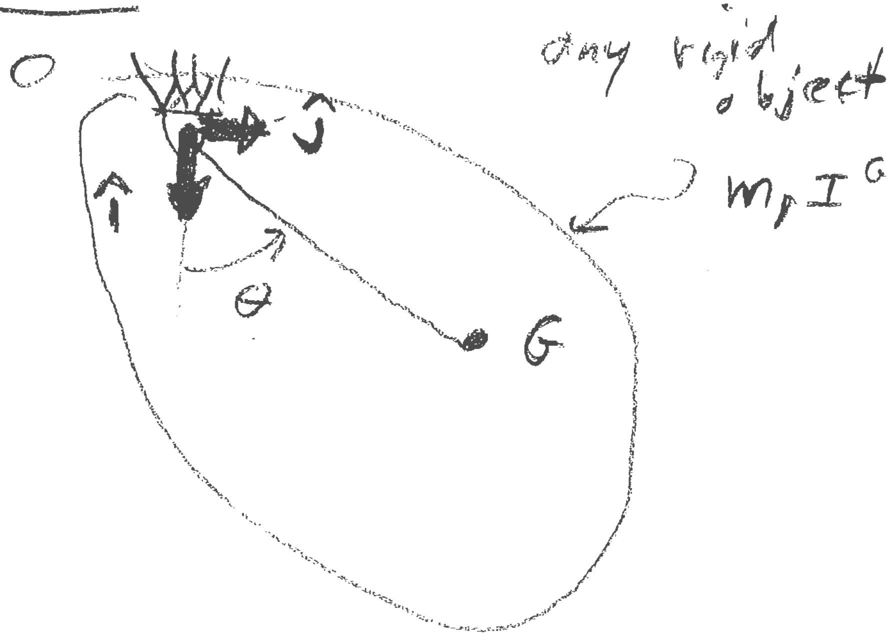
$$\boxed{\sum \vec{M}^{ext}_{c}} = \dot{\vec{H}}_{/c} = \boxed{\vec{r}_{G/c} \times m \vec{a}_G + I \alpha \hat{k}}$$

$\dot{\omega} = \ddot{\theta}$

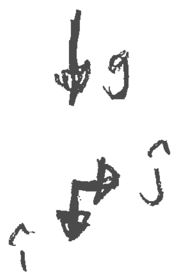
angle of line marked in object

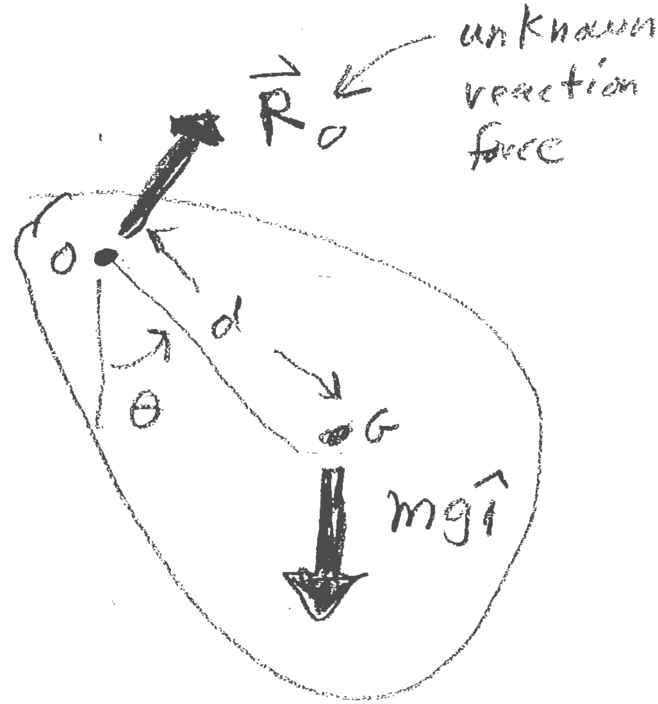
does <u>not</u> have to be fixed in space

Pendulum



FBD





EoM?

Ans: $\sum \vec{M}_{/O} = \overset{d\hat{e}_r}{\vec{r}_{G/O}} \times m\vec{a}_G + I\alpha\hat{k}$

$\vec{a}_G$: $(d\ddot{\theta}\hat{e}_\theta + d\dot{\theta}^2\hat{e}_r)$

$\vec{r}_{G/O} \times mg\hat{i}$

$\vec{r}_{G/O}$: $\underbrace{d(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})}_{\hat{e}_r}$

$$\{-mgd\sin\theta\hat{k} = d^2\ddot{\theta}m\hat{k} + I^G\ddot{\theta}\hat{k}\}$$

$$\{\}\cdot\hat{k} \Rightarrow -mgd\sin\theta = \underbrace{(I^G + md^2)}_{[I^O]}\ddot{\theta}$$

aside

$$\boxed{\ddot{\theta} = \frac{-mgd}{I^G + md^2}\sin\theta}$$

$$\boxed{\ddot{\theta} = -c\sin\theta}$$

Recall: simple pend

$c = \frac{g}{\ell} = g/d$

New idea:

mean square distance of mass from G

$$r_{gyr}^2 \equiv (\text{"radius of gyration"})^2$$

$$= I^G/m$$

$$\Rightarrow \quad I^G = m\, r_{gyr}^2$$

$$\ddot{\theta} = \frac{-\cancel{m} g d}{\cancel{m} r_{gyr}^2 + \cancel{m} d^2} \sin\theta$$

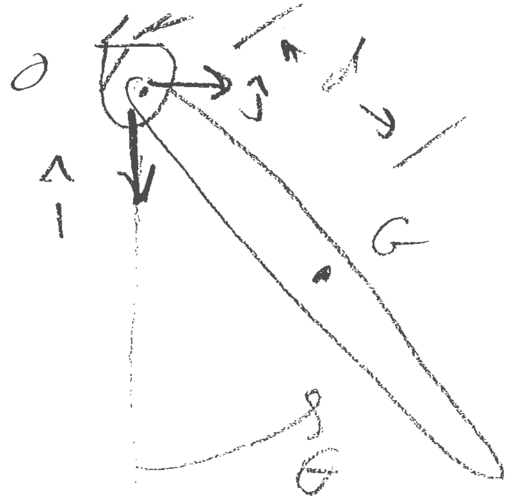
$$= \frac{-g}{d} \underbrace{\left(\frac{1}{1 + \left(r_{gyr}/d \right)^2} \right)}_{\text{shape factor}} \sin\theta$$

ex) pt. mass: $r_{gyr} = 0$

shape factor = 1

ex)

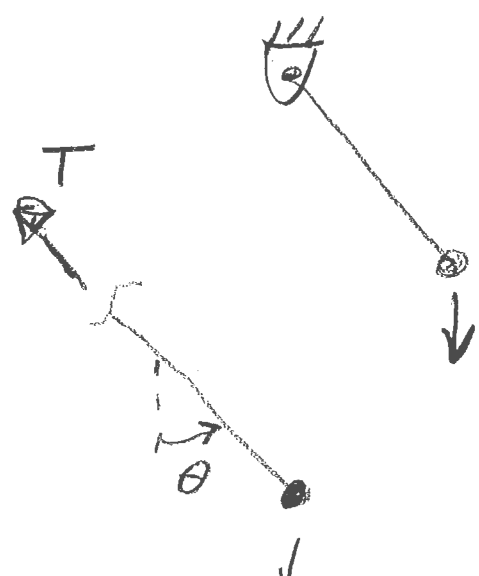
Stick as pendulum



line mass

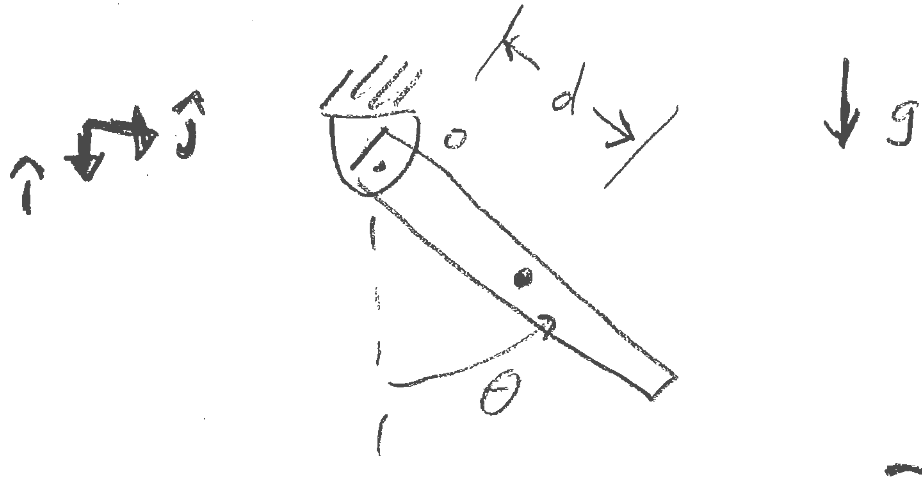
uniform

Recall: pt. mass pendum



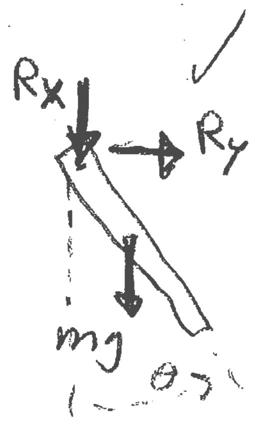
FBD

Stick-pendulum

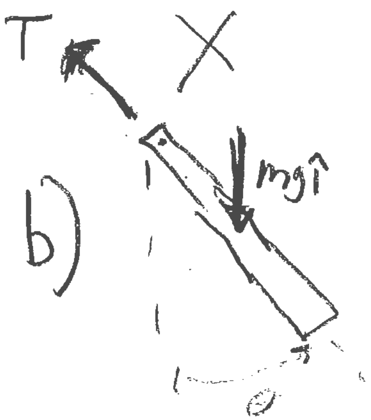


Which FBD is wrong?

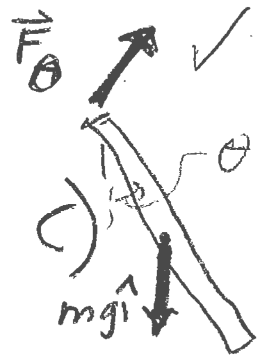
a)



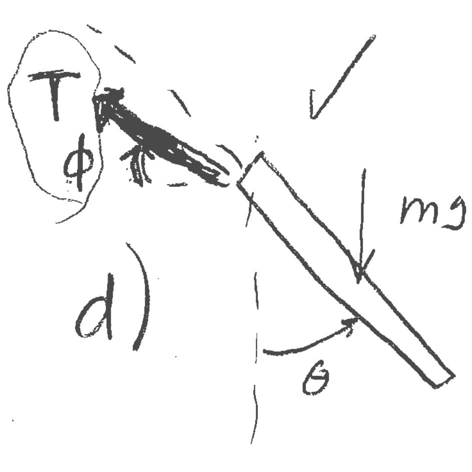
b)



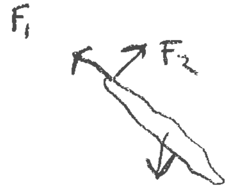
c)

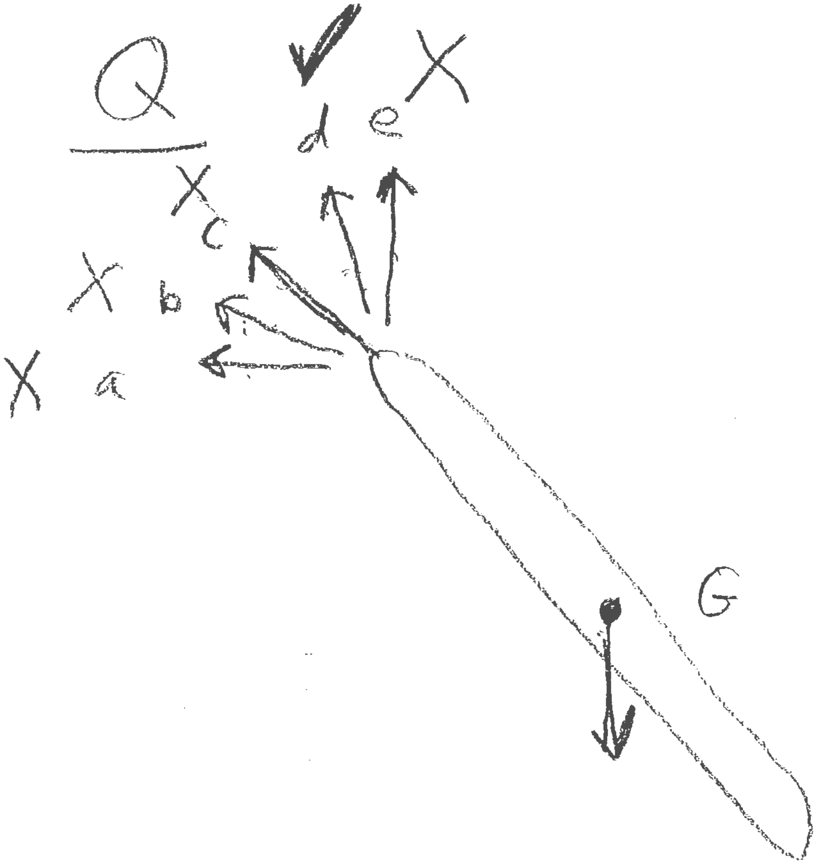


d)



e) None, all are O.K.





a) stick has vert. accel $\neq g$
$\Rightarrow$ needs vert. comp. (a) has none
$\Rightarrow$ X

$\ddot{\theta} > 0$

b) $AMB_{/G} \Rightarrow$ CCW ang. accel, but
$\ddot{\theta} < 0$ in this config w. $\theta > 0$
$\Rightarrow$ X

c) $AMB_{/G} = 0$ if no torques. But
$\ddot{\theta} \neq 0 \Rightarrow$ torque-free C is wrong!

d) all O.K

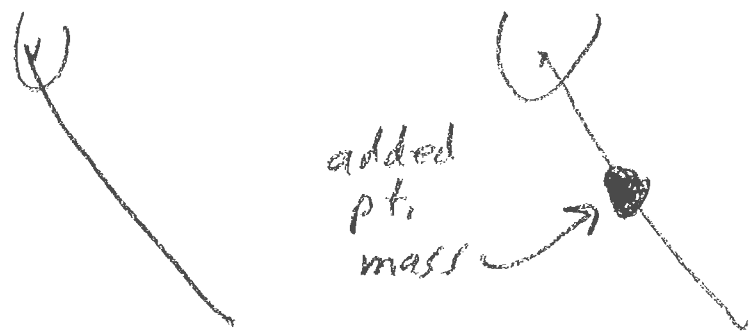
e) Doesn't give known-to-exist horiz. accel.

Q

$\downarrow g$

Uniform Stick

Uniform Stick w/ pt. mass added 1/2 way down



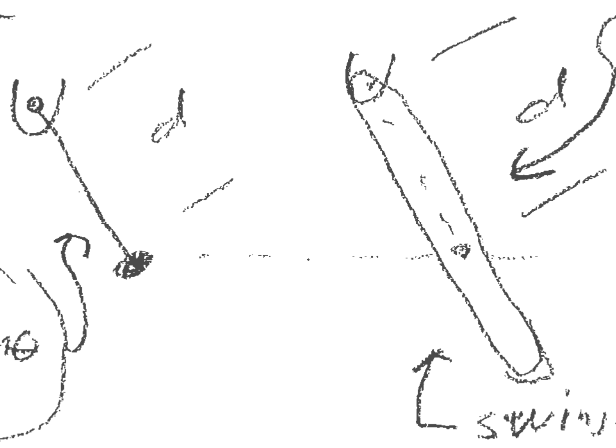
frequency of oscillation changed?

a) faster?

b) Same?

c) Slower?

Note



$$\ddot{\theta} = \frac{-g}{\ell}\sin\theta$$

$$\ddot{\theta} = -\frac{g}{\ell}\underbrace{\left(\frac{1}{1+([illegible])^2}\right)}_{<1}\sin\theta$$

swings slower than pt. mass

Adding pt. mass slows the pendulum.