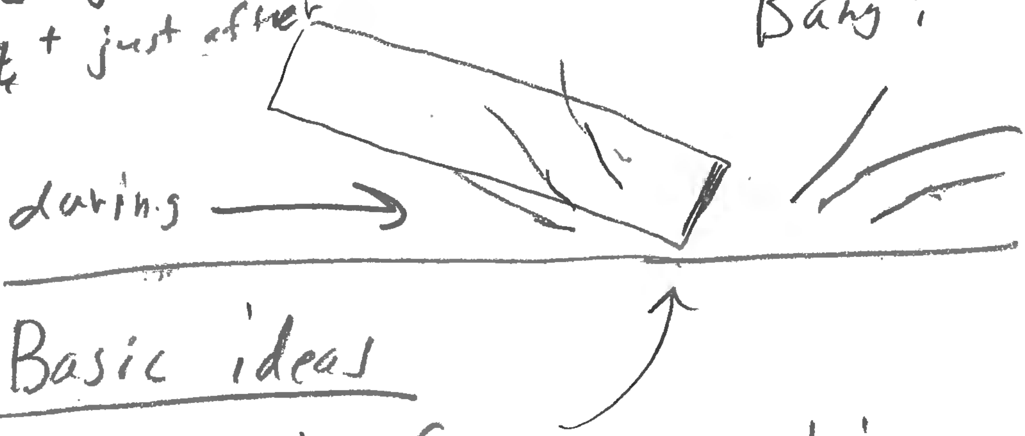
TODAY

Collisions of rigid objects

t^- just before coll.

t^+ just after



Basic ideas

* Collision forces are big

⇒ neglect non-collision forces

⇒ don't show non-coll. forces on collision FBD

* Short time ⇒ config. doesn't change BUT vels. do change.

from t^- to t^+
$\vec{v}^+ = \vec{v}^-$
$\theta^+ = \theta^-$

* Integrate eqs in time

"$F = ma$" ⇒ "Impulse $= m(\Delta v)$"

ex) uniform stick

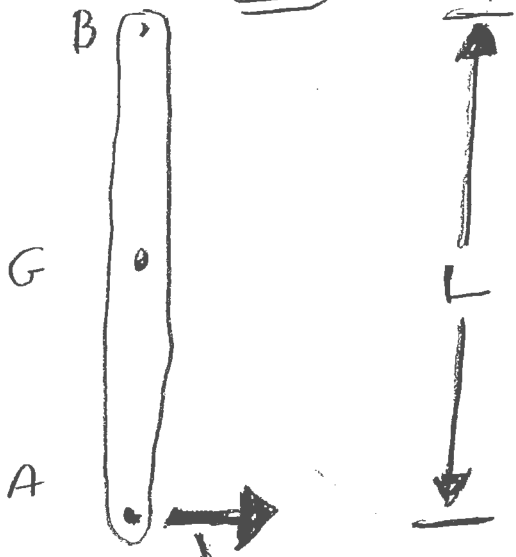
$\vec{V}_G(0) = \vec{0}$

$\dot{\theta}(0) = \vec{0}$

$\hat{j}$ $\hat{i}$

FBD

m, I^G



given

$\underbrace{\int F\,dt}_{\text{"I", "P"}} = \text{Impulse}$

$\vec{V}_B^{\,+} = ?$ $\qquad \omega^+ = ?$

LMB $\qquad \int \sum \vec{F}\,dt = m\Delta\vec{V}_G$

$$\int F\,dt\,\hat{i} = m\Delta\vec{V}_G$$

$$= m\Big(\underbrace{\vec{V}_G(t^+)}_{\vec{V}_G^{\,+}} - \underbrace{\cancel{\vec{V}_G(t^-)}}_{\vec{V}_G^{\,-}}\Big) \quad \vec{0}$$

$$\int F\,dt\,\hat{i} = m\vec{V}_G^{\,+}$$

2 pts

$$\boxed{\vec{V}_G^{\,+} = \int F\,dt\,\hat{\imath}\,/m}$$

AMB $/G$

note

$$\int \sum \vec{M}_{/G}\,dt = \Delta \vec{H}_{/G}$$

$$\vec{r}_{A/G} \times \left(\int F\,dt\,\hat{\imath}\right) = \vec{H}_G^{\,+} - \cancel{\vec{H}_G^{\,-}}$$

$\cancel{\vec{H}_G^{\,-}} \rightarrow \vec{0}$

$\vec{r}_{A/G} = -\frac{L}{2}\hat{\jmath}$

$$\frac{L}{2}\int F\,dt\;\hat{k} = \dot{\theta}^{+} I^{G}\,\hat{k}$$

$$\boxed{\dot{\theta}^{+} = \frac{\left(\int F\,dt\right)\frac{L}{2}}{I^{G}}}$$

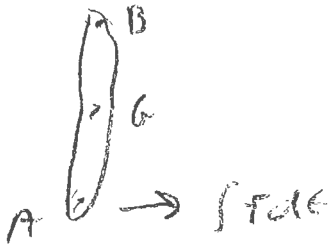
$$\vec{V}_B^{\,+} = V_B^{+}\,\hat{\imath}$$

$V_B^{+} > 0$ (a)

< 0 ✓ (b)

$= 0$ (c)

$$\vec{V}_B^{\,+} = \vec{V}_G^{\,+} + \vec{V}_{B/G}^{\,+}$$



$$= \frac{\int F\,dt}{m}\,\hat{\imath}$$

$$\frac{L}{2}\,\hat{J}$$

$$+ \;\vec{\omega}^{\,+} \times \vec{r}_{B/G}$$

$$L\;\dot{\theta}^{+}\,\hat{k}$$

$$\frac{\int F\,dt}{I^G}$$

$$\dot{\hat{e}}_r = \vec{\omega} \times \hat{e}_r$$

$$\dot{\hat{\imath}}' = \vec{\omega} \times \hat{\imath}'$$

$$\dot{\vec{Q}} = \vec{\omega} \times \vec{Q}$$

$$=$$

$$= \int F\,dt \left[\frac{1}{m} - \frac{L^2/4}{I^G} \right] \hat{\imath}$$

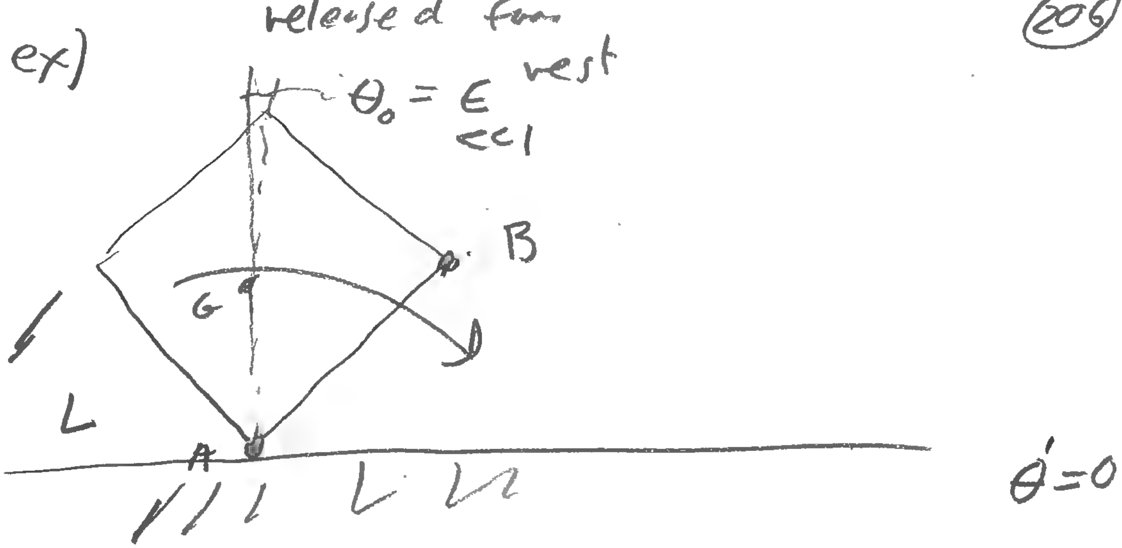
always: $I^G \leq m\,L^2/4$

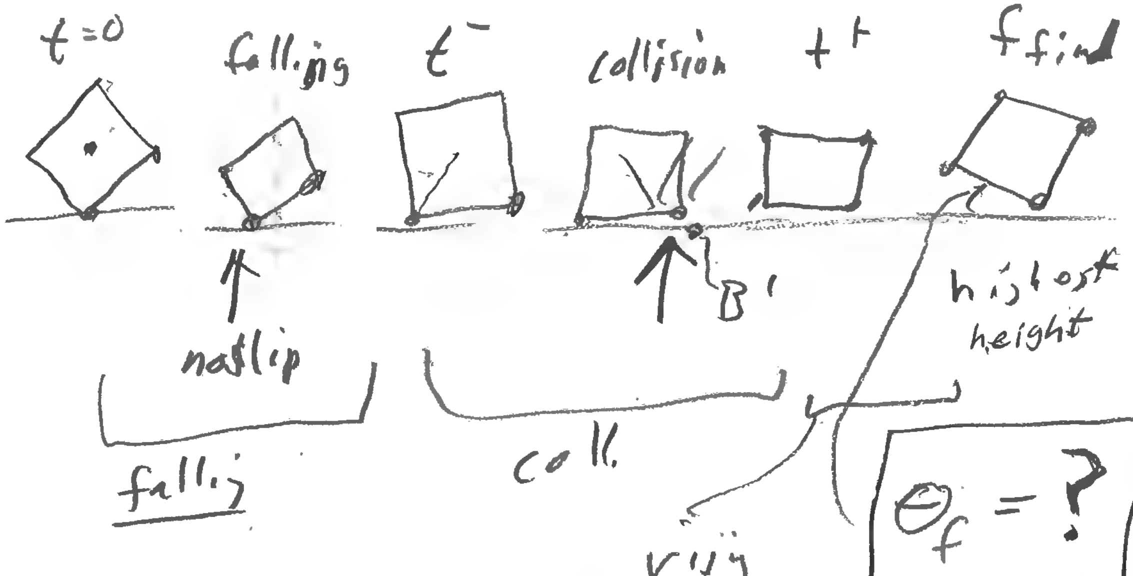
ex) $I^G = mL^2/4 \Rightarrow V_B^+ = 0$

most extreme

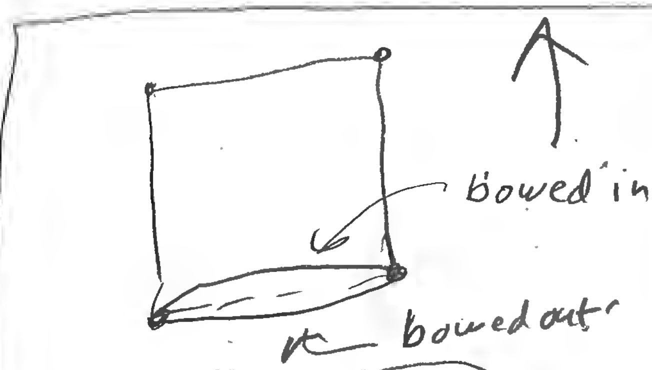
$$\boxed{V_B^{+} \leq 0}$$

ex)





$\boxed{\theta_f = ?}$



very different!

Solve 3 sub-problems

1) Falling; energy cons.

$\Rightarrow \dot{\theta}^-$

2) Collision;

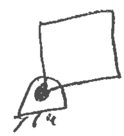
$\underbrace{AMB_{/B};}_{\text{no net moment impulses}} \Rightarrow \dot{\theta}^+$

3) rising; energy cons. again

Steps, to solve

1) Falling: En. Cons.

$$E^{0}_{tot} = E^{t^-}_{tot}$$



$$\cancel{\vec{E}^{0}_{K}} + E^{0}_{p} = E^{-}_{K} + \underbrace{E^{-}_{p}}_{\frac{L}{2} mg}$$

$$\underbrace{\frac{L}{2}\sqrt{2}\, mg = \frac{1}{2} m |\vec{V}_c^-|^2 + I^G \dot{\theta}^{-2}/2}_{}$$

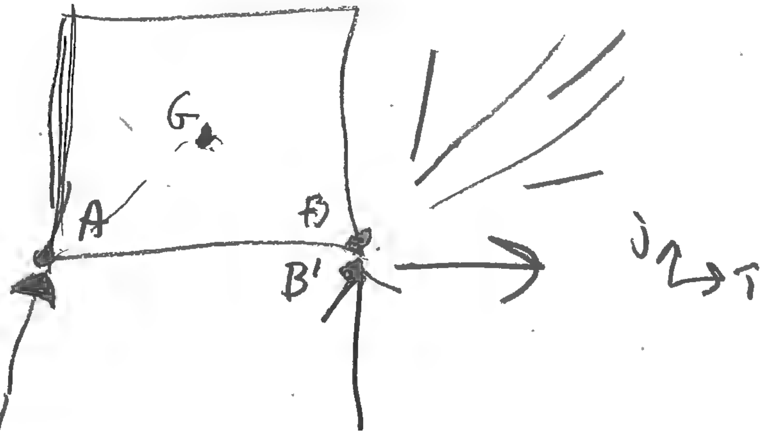
1 eqn for $\dot{\theta}^-$ $\quad |V_c^-| = \dot{\theta}\sqrt{2}\,\frac{L}{2}$

$\dot\theta^-$ known ✓

$\dot\theta^- = \sqrt{\text{mess}}$

2) Collision

Collision FBD



+

$\vec\omega = -\dot\theta \hat k$

rot. about A before coll.

assume no force here

<u>AMB</u> $/B'$

$$\int \cancel{\sum \vec M_{/B'}}\, dt = \Delta \vec H_{B'}$$

$\vec 0$

$$= \vec H^+_{/B'} - \vec H^-_{/B'}$$

$$\vec H^-_{/B} = \underbrace{\vec r_{G/B'} \times \vec V_G^- m}_{=\vec 0} + I^G \omega^- \hat K$$

$\omega^- = -\dot\theta^-$

$\vec r_{G/B'} = \frac{L}{2}(-\hat\imath + \hat\jmath)$

$\vec V_G^- = \vec\omega^- \times \vec r_{G/A}$

$\vec\omega^- = -\dot\theta^- \hat K$

$\vec r_{G/A} = \frac{L}{2}(\hat\imath + \hat\jmath)$

$$\vec{H}_B^{+} = \vec{r}_{G/B} \times \vec{v}_G^{+} m + I^G \omega^+ \hat{k}$$

$-\dot{\theta}^{+}$

$\vec{\omega}^{+} \times \vec{r}_{G/B}$

$\frac{L}{2}(-i + \hat{j})$

$\frac{L}{2}(-\hat{i} + \hat{j})$

$-\dot{\theta}^{+}$

$$\vec{H}_0^{-} = \vec{H}_B^{+} \Rightarrow \boxed{\dot{\theta}^{+}}$$

3) En. Cons. (again)

$$\Rightarrow \theta^{find} \simeq 1.2^\circ$$