Wed. April 25, 2019

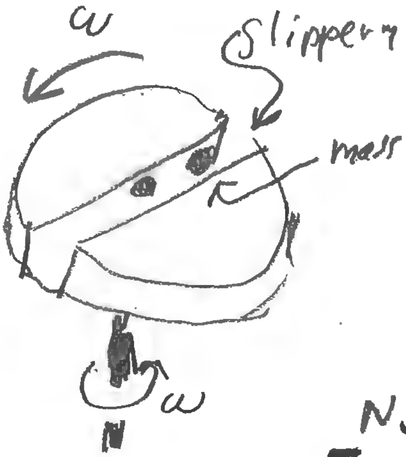
[The missing 2 pages by (great) student are posted online]

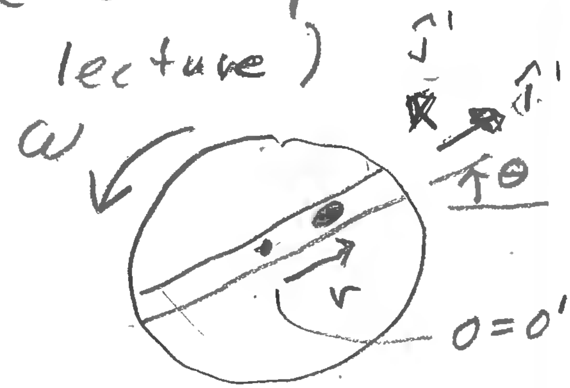
TODAY

1) bead in slot (cont'd)
2) bead on wire
Vibrating base pendulum
~~(slider crank mechanism)~~

NO MORE NEW TOOLS!

Bead in slot (see also prev. lecture)





$\omega = \text{const.}$

FBD

To find $\vec{a}$, see prev. lecture

LMB: $\sum \vec{F} = m\vec{a}$

$$N\hat{j}' = (\ddot{r} - r\omega^2)\hat{\imath}' + 2\dot{r}\omega\,\hat{j}'$$

$\{\cdot\hat{\imath}' \Rightarrow$ $\boxed{\ddot{r} = \omega^2 r}$ $\Rightarrow r \sim \underline{e^{\omega t}}$

Look at path

chain rule, twice

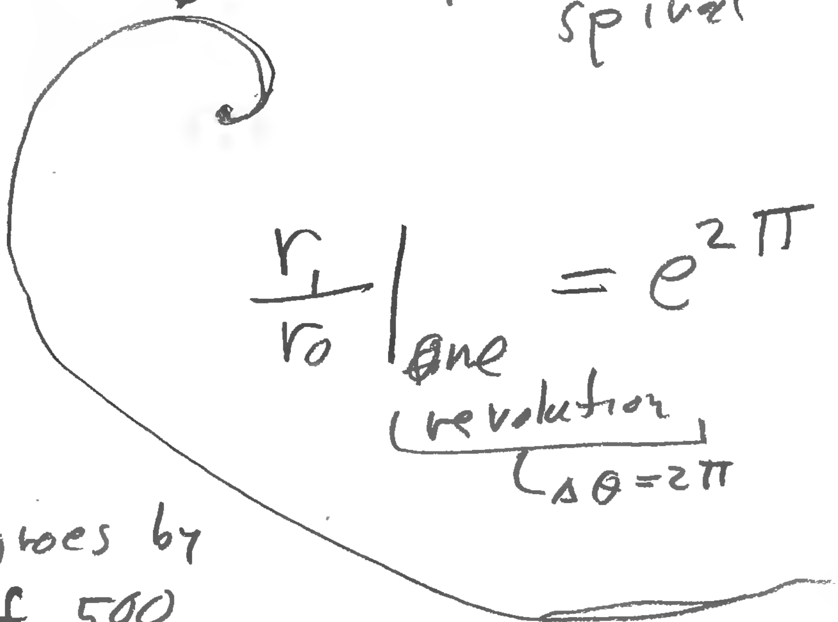
$$\frac{d^2 r}{d\theta^2} = \frac{d^2 r}{dt^2}\left(\frac{dt}{d\theta}\right)^2 = \frac{1}{\omega^2}\frac{d^2 r}{dt^2}$$

no. coeff.

$$\Rightarrow \boxed{\frac{d^2 r}{d\theta^2} = r}$$

$$\Rightarrow \quad r = \underbrace{c_1 e^{\theta}}_{\text{dominates}} + c_2 e^{-\theta}$$

exponential spiral



$$\left.\frac{r_1}{r_0}\right|_{\substack{\text{one revolution} \\ \Delta\theta = 2\pi}} = e^{2\pi} \approx 500$$

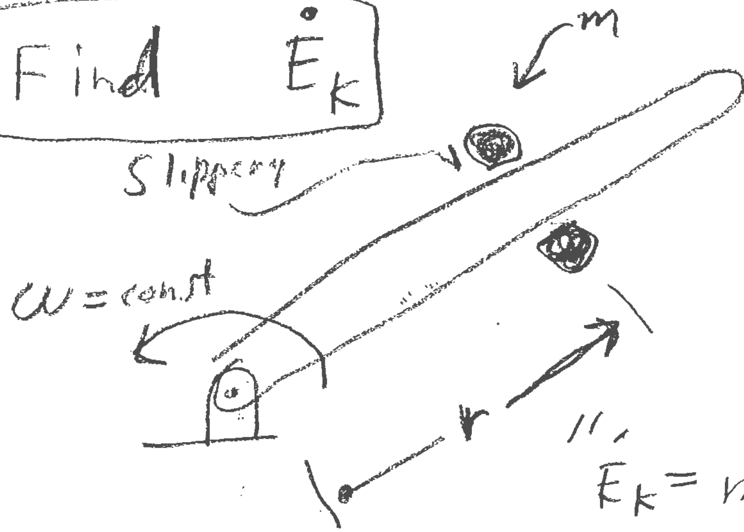
(actually 535...)

radius grows by factor of 500 each revolution

QUIZ

Given $\omega = \text{const}$, r, $\dot{r}$, m

$\boxed{\text{Find } \dot{E}_K}$



No gravity

Student guesses

"$\dot{E}_K = m r \omega^2 \dot{r}$" ✗

$E_K = \frac{1}{2} m r^2 \omega^2$ ✗

$I^O \omega^2/2$ ✗

"$\dot{E}_K = 2 m \omega \dot{r}^2$
$= P$
$= \vec{F} \cdot \vec{V}$
$= \dots$" ✗

a) ? got it

b) ? maybe

c) ? not yet

d) ? having trouble

e) ? No way

$\vec{\text{vel.}} = \vec{V} = \dot{r}\,\hat{\imath}' + \omega r\,\hat{\jmath}'$

$$\vec{V} = \dot{r}\,\hat{\imath}' + r\omega\,\hat{\jmath}'$$

$$\vec{F} = m\vec{a}$$

$$\Rightarrow \left\{ N\hat{\jmath}' = \left[(\ddot{r} - r\omega^2)\,\hat{\imath}' + 2\dot{r}\omega\,\hat{\jmath}'\right] m \right\}$$

$$\{\ \}\cdot\hat{\jmath}' = \quad N = 2\dot{r}\omega m$$

$$\vec{F} = 2\dot{r}\omega\,\hat{\jmath}' m$$

$$\dot{E}_K = \vec{F}\cdot\vec{V}$$

$$= (2\dot{r}\omega\,\hat{\jmath}')\cdot(\dot{r}\,\hat{\imath}' + r\omega\,\hat{\jmath}')$$

$$\boxed{\dot{E}_K = 2\dot{r}r\omega^2 m}$$

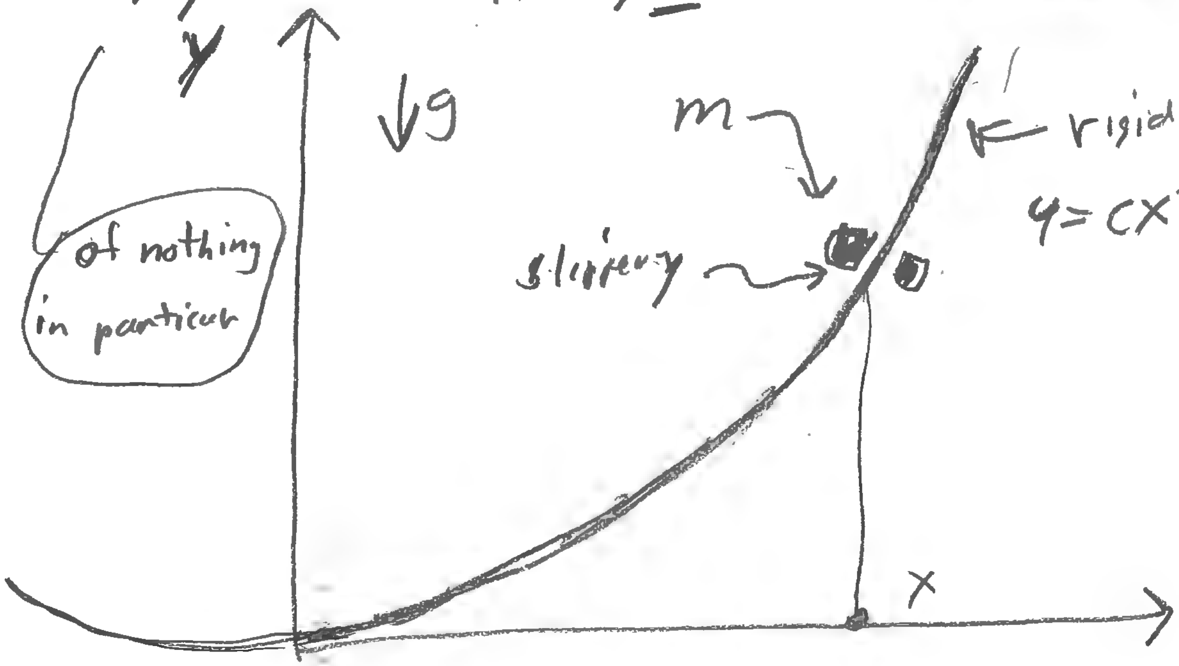
Alternative way to find $\dot{E}_K$: $E_K = \frac{1}{2} m\, \vec{V}\cdot\vec{V}$

$$= \frac{1}{2} m (\dot{r}^2 + r^2\omega^2)$$

then differentiate and use

$$\ddot{r} = 2\omega\dot{r}$$

ex) Bead on wire
slippery, in gravity



1 DoF want 1 EoM $\ddot{x}$

Given State: $x, \dot{x} \Rightarrow y, \dot{y}$

Parameters : m, c, g

Find $\ddot{x}, \ddot{y}$, Reaction Force

FBD!

LMB:

$$\sum \vec{F} = m\vec{a}$$

$$\underbrace{-mg\hat{J}}\ N\hat{e}_n = m\left[\ddot{x}\hat{\imath} + \ddot{y}\hat{J}\right] \quad *$$

$\hat{n}$

Want to get rid of $y, \dot{y}, \ddot{y}$

Kinematics & geometry

curve $\Rightarrow$ $y = cx^2$

$$\Rightarrow \dot{y} = 2cx\dot{x}$$

$$\Rightarrow \ddot{y} = 2c\left[\dot{x}^2 + x\ddot{x}\right] \quad \ddot{y}$$

$$\vec{a} = \ddot{x}\hat{\imath} + 2c(\dot{x}^2 + x\ddot{x})\hat{J}$$

find this

$$\vec{V} = \dot{x}\hat{\imath} + 2cx\dot{x}\hat{J}$$

$$\hat{e}_t = \vec{V}/|\vec{V}|, \quad \hat{e}_n = \hat{k}\times\hat{e}_t$$

Back to LMB

$\checkmark$ = known

$$\left\{ -mg\hat{J} + N\hat{e}_n = m\left[\ddot{x}\hat{\imath} + 2c(\dot{x}^2 + x\ddot{x})\hat{J}\right]\right\}$$

unknown

2 eqs in
2 unknowns

1 2D vec. eqn.

$\{\ \}\cdot\hat{e}_t$ or $\{\ \}\cdot\vec{v}$

$\vec{v}$ is $\perp$ to $\hat{e}_n$

$\hat{e}_n\cdot\hat{e}_t=0$

$\hat{e}_t=\vec{v}/|\vec{v}|$

$$\left\{-\cancel{m}g\hat{j}+\cancel{N\hat{e}_n}=\cancel{m}\left[\ddot{x}\hat{i}+2c(\dot{x}^2+x\ddot{x})\right]\right\}\cdot \qquad *$$

$$\underbrace{(\dot{x}\hat{i}+2cx\dot{x}\hat{j})}_{\vec{v}}$$

Some algebra

EoM

$$** \Rightarrow \boxed{\ddot{x} = \text{a mess}}$$

involving $x, \dot{x}, c, \cancel{m}, g$ (goes away)

Now, How to find N?

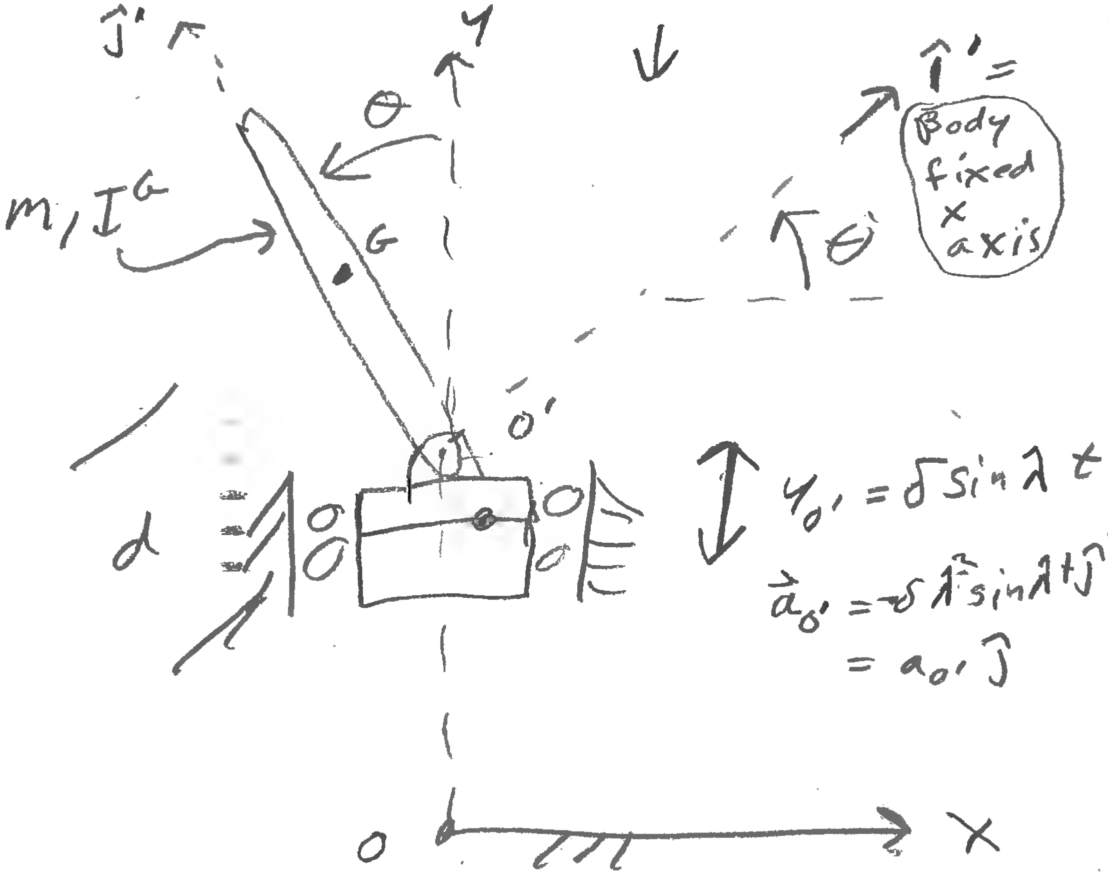
$\{$use $*$, sub. in $**$, $\}\cdot\hat{e}_n$

$$\Rightarrow \boxed{N = \text{another mess}}$$

ex) Shaken inverted pendulum

(Recall demo: or google or old Ruina lectures

$\lambda \neq \omega = \dot{\theta}$

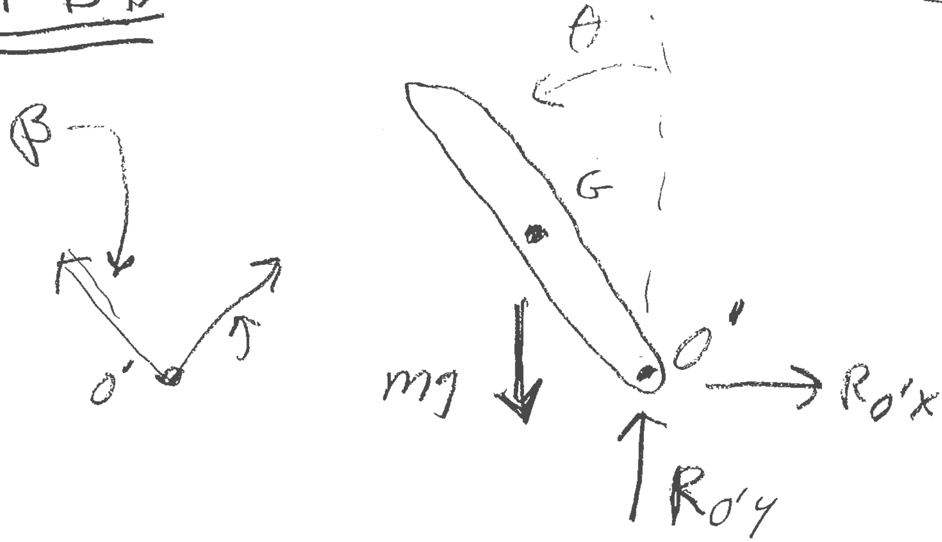


Given: $\delta, \lambda, g, m, I^G,$

Find EoM $\Rightarrow$ find $\ddot{\theta}$, given parameters & θ & $\dot{\theta}$

That is,

FBD



AMB $/O'$:

$$\sum \vec{M}_{/O'} = \text{"}\dot{\vec{H}}_{/O'}\text{"}$$

challenge

$$\underbrace{\vec{r}_{G/O'}}_{d\hat{J}'} \times (-mg\hat{J}) = \vec{r}_{G/O'} \times m\vec{a}_{G/\mathcal{F}} + I^G \underbrace{\alpha}_{\ddot{\theta}} \hat{k}$$

$$\vec{a}_{G/\mathcal{F}} = \underbrace{\overset{\checkmark}{\vec{a}_{O'/\mathcal{F}}}}_{-\delta\lambda^2 \sin(\lambda t)\hat{J}} + \cancel{\vec{a}_{G/\mathcal{B}}}^{\vec{0}} + -\overset{\dot{\theta}\hat{k}}{\omega^2}\,\underbrace{\vec{r}_{G/O'}}_{d\hat{J}'}$$

$$+ \underbrace{\dot{\vec{\omega}}_{\mathcal{B}/\mathcal{F}}}_{\ddot{\theta}\hat{k}} \times \vec{r}_{G/O'} + 2\vec{\omega}_{\mathcal{B}/\mathcal{F}} \times \cancel{\vec{v}_{P/\mathcal{B}}}^{\vec{0}}$$

TO BE CONTINUED