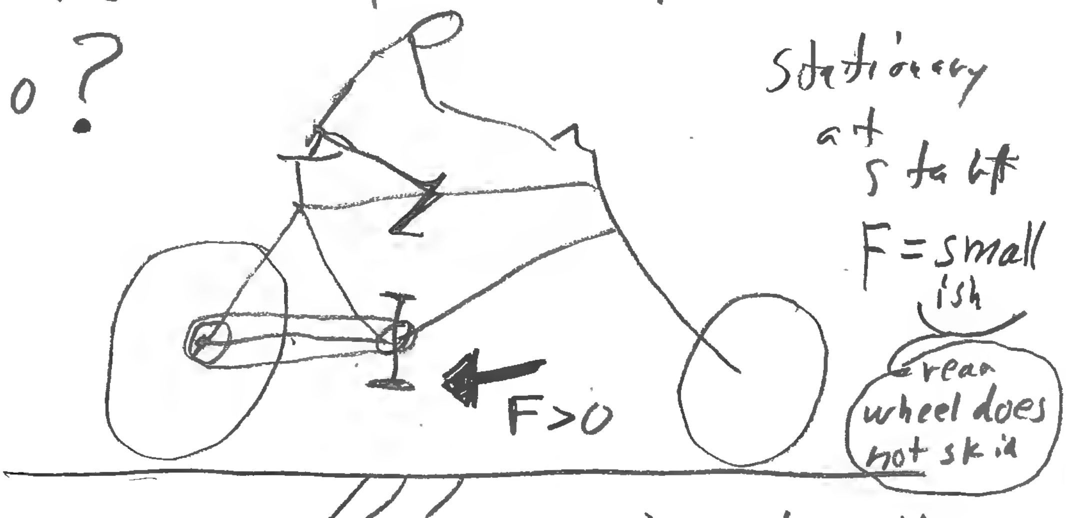
MONDAY MAY 6, 2019

LAST LECTURE

TODAY

1) Bike pedal puzzle
2) Falling ladder & instantaneous center
3) Intro. to 3D

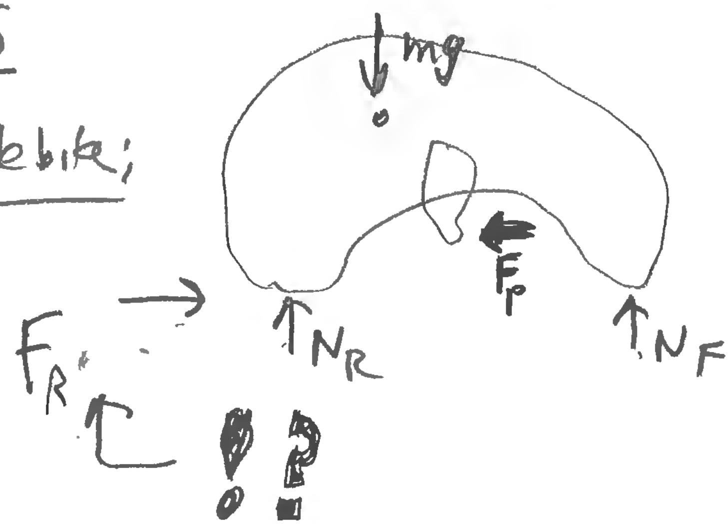
Q) Which way does the bike go?



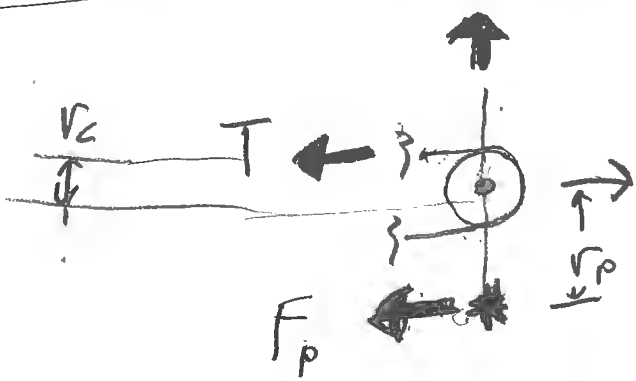
a) doesn't move
b) left
c) right
d) depends on info. not given
e) Not well posed.

FBDs

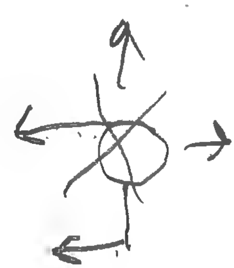
Whole bike:



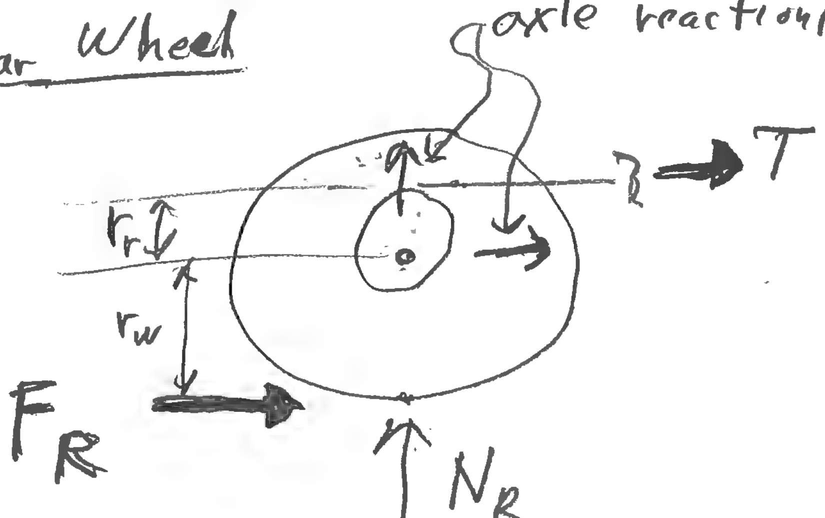
Front Sprocket & Crank



bad FBD



Rear Wheel



4 unknown reactions

& $\vec{a}_b = a_b \hat{\imath}$

$\llcorner$ unknown

LMB Whole bike

$$\boxed{F_R - F_p = m a_B}$$

Is F_R bigger or smaller than F_p?

use statics for internal parts.

AMB of crank / hinge,

$$\boxed{T = \frac{r_p}{r_c} F_p}$$

Rear Wheel

$$F_R = \frac{r_r}{r_w} T$$

sabutitute

$$F_R = \underbrace{\frac{r_r}{r_w} \frac{r_p}{r_c}}_{\text{usually } < 1} F_p$$

Force increase because crank > front sprocket

force decrease because rear wheel > rear Sprocket

Energy approach

Work of $F_p > 0$

for bike to gain Energy

$\Rightarrow$ pedal goes backwards

Kinematics analysis

$\Rightarrow$ pedal moves same dir. of bike. Happens if

A few calcs. w/ $\vec{\omega} \times \vec{r}$ $\Rightarrow$ $\Rightarrow$ $\frac{r_r}{r_w} \frac{r_p}{r_c} < 1$

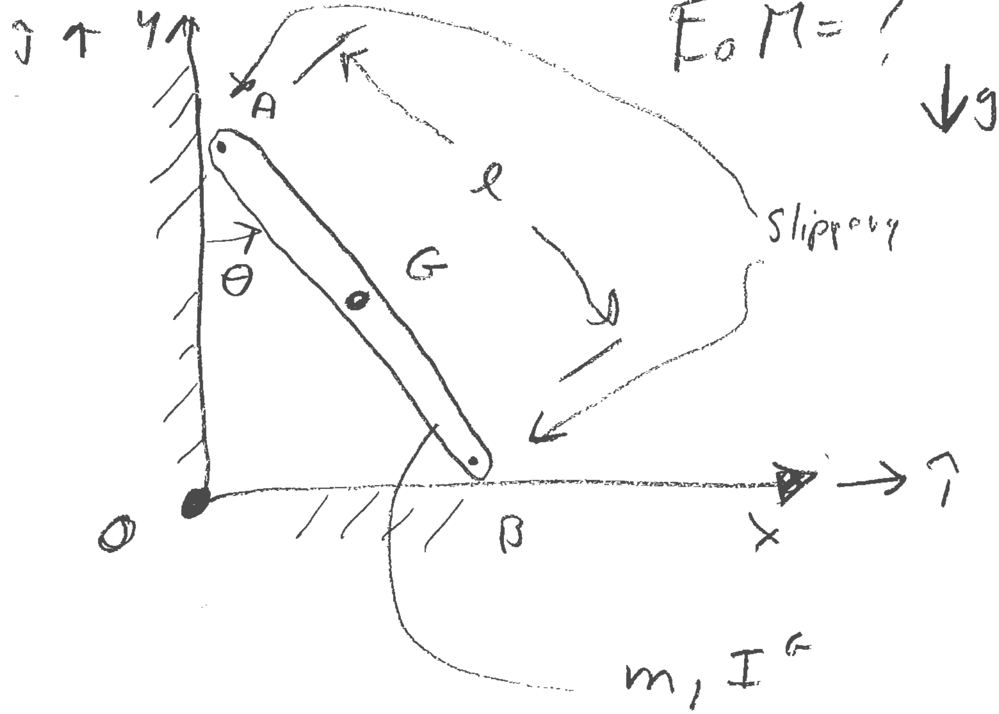
$\Rightarrow$ bike goes backwards

See veritasium video

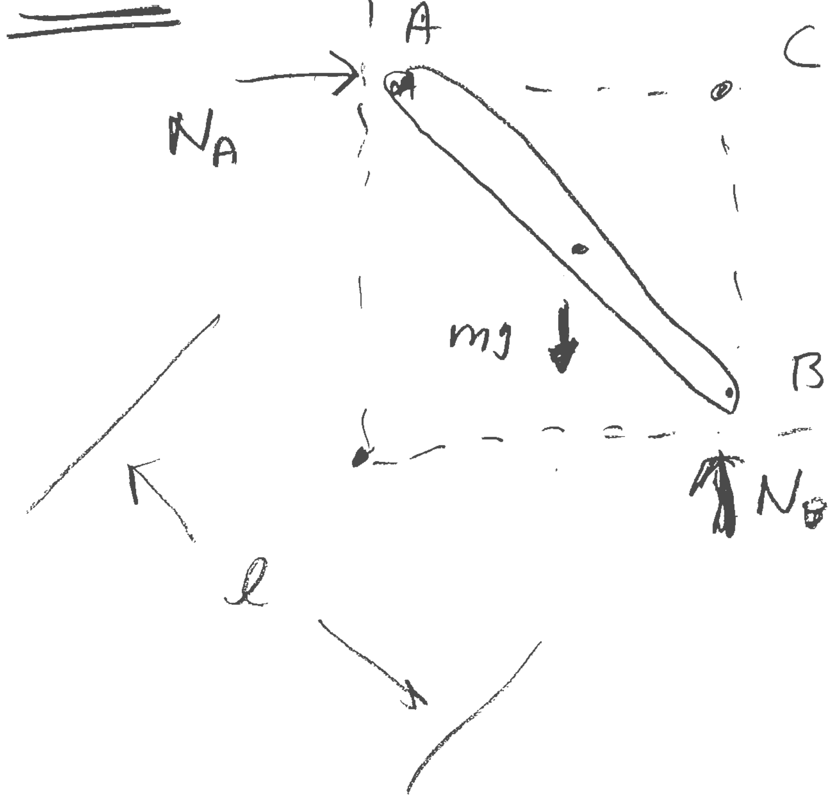
google it

ex) Falling ladder

$E_oM = ?$



FBD



Kinematics

DoF ?

ANS: 1 : θ

↑ good one

Need: $\vec{a}_G$, $\dot{\vec{\omega}}$ from θ, $\dot{\theta}$, $\ddot{\theta}$

Approach 2:

$$\vec{r}_G = \vec{r}_G$$

$$x_G \hat{i} + y_G \hat{j} = y_A \hat{j} + \underbrace{\vec{r}_{G/A}}_{\frac{\ell}{2}(\sin\theta \hat{i} - \cos\theta \hat{j})}$$

also

$$= x_B \hat{i} + \vec{r}_{G/B}$$

etc.

$\Rightarrow$ x_G, y_G in terms of θ

vel: $\vec{V}_G = \vec{V}_G$ two ways

acc: $\vec{a}_G = \vec{a}_G$ two ways

$\vec{\omega} = \ddot{\theta}\hat{k}$

$\Rightarrow$ $\vec{a}_G$, $\dot{\vec{\omega}}$ in terms of $\theta, \dot{\theta}, \ddot{\theta}$

this works (1).

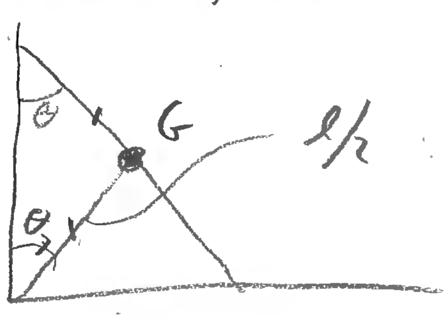
Generally easier for hard problems

Approach 1

Generally easier for easy problems

$$\boxed{\vec{a}_{G/f} = \frac{d^2}{dt^2}\left(\vec{r}_{G/O}\right)}$$

Always, for all probs



G moves in circle w/ radius $l/2$

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$$\vec{r}_{G/O} = \frac{l}{2}\left(\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}\right)$$

$$\Rightarrow \vec{v}_{G/O} = \text{-----}$$

$$\Rightarrow \vec{a}_{G/O} = -\dot{\theta}^2 \frac{l}{2}\left(\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}\right)$$

$$+ \ddot{\theta}\frac{l}{2}\left(\cos\theta\,\hat{\imath} + -\sin\theta\,\hat{\jmath}\right)$$

AMB $/C$ $\quad \sum \vec{M}_{/C} = \dot{\vec{H}}_{/C}$

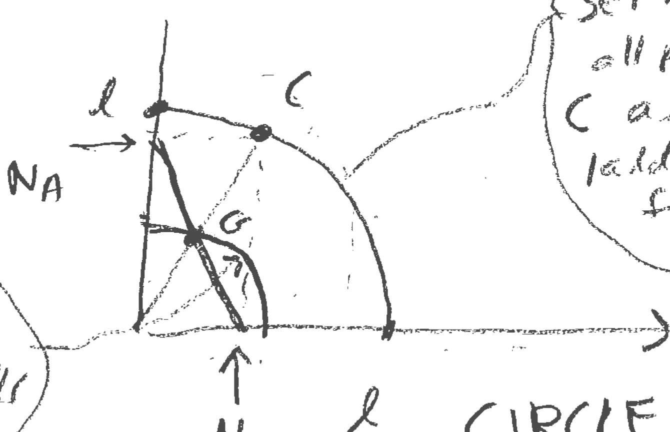
"Kills N_A & N_B"

$$\left\{ \vec{r}_{G/C} \times (-mg\hat{\jmath}) = \vec{r}_{G/C} \times m\vec{a}_G + I^G\ddot{\theta}\hat{k} \right\}$$

$$\{\}\cdot\hat{k} \Rightarrow \boxed{\text{EoM}}$$

Curious facts

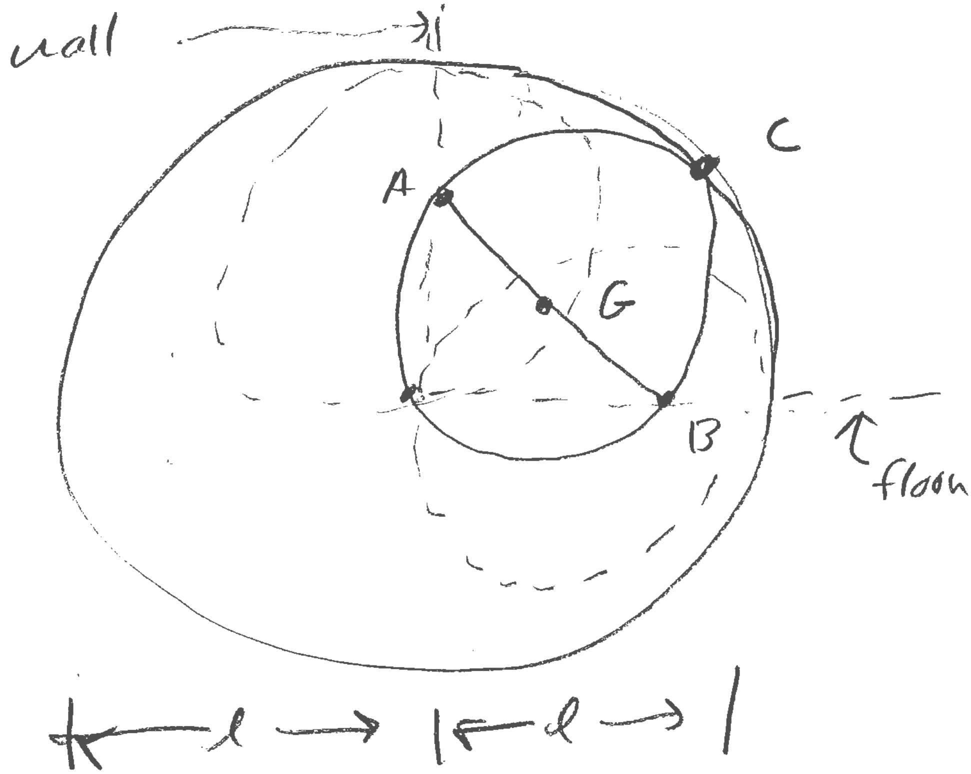
Set of all pts G as ladder falls



Set of all pts C as ladder falls

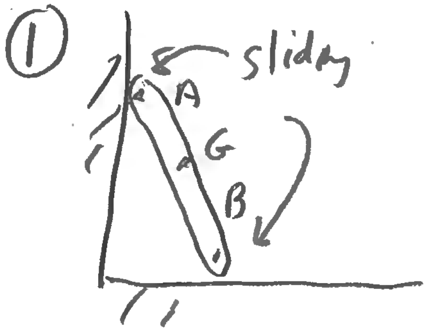
CIRCLES!

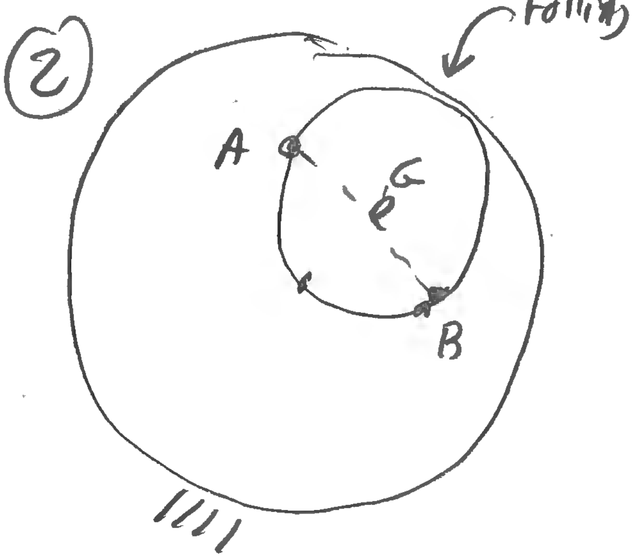
Consider disk w/ diameter l rolling inside circle w/ radius l

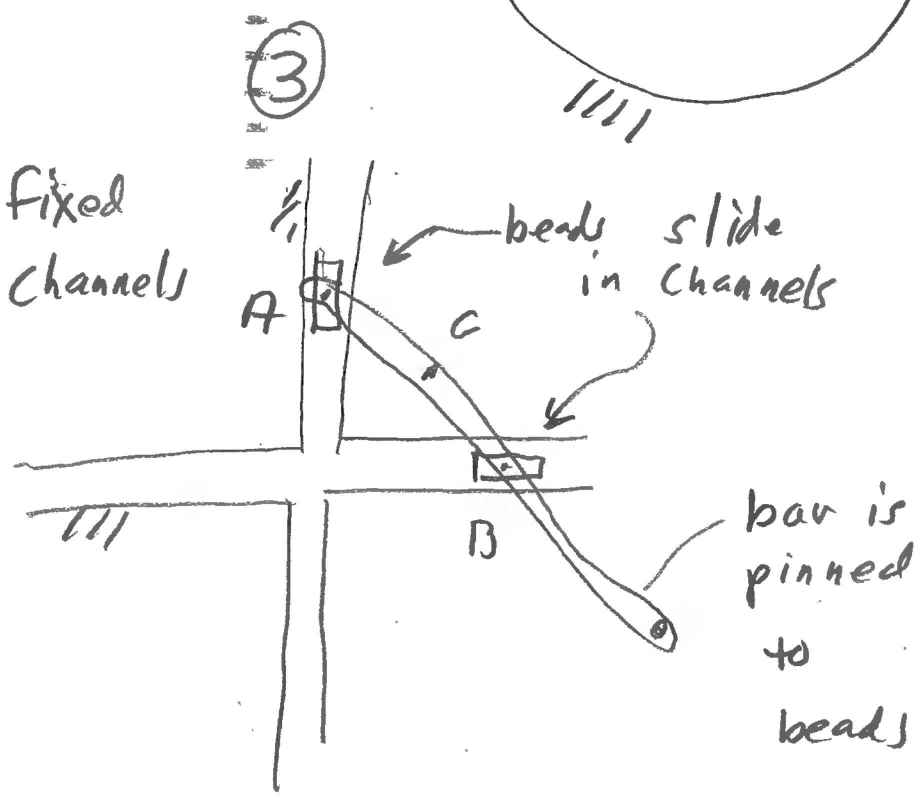


Motion of ladder is motion of diameter of inner disk.

These 3 problems are kind of the same







A & B & G have same motions for all 3

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*3260 is not a pre-req.

Whats in it?

I. harder 2D problems (many moving parts) ← Diff. Alg. Eqs & Lagrange Eqs

II. 3D

Main thing:

$$\vec{H}_{/G} = \begin{cases} \dot{\theta} I^G \hat{k} & \text{2D} \\ \underline{\underline{I}}^G \cdot \dot{\vec{\omega}} + \vec{\omega} \times (\underline{\underline{I}}^G \cdot \vec{\omega}) & \text{3D} \end{cases}$$

3D vector

moment of inertia tensor