The kinetic energy of the system E_{K} is the sum over the kinetic energy of each particle,

$$E_{\mathrm{K}} = \frac{1}{2}\sum_i m_i v_{i/\mathrm{O}}^2,$$

where $v_{i/\mathrm{O}}^2 = \mathbf{v}_{i/\mathrm{O}} \cdot \mathbf{v}_{i/\mathrm{O}}$ and the point O is origin of some Newtonian reference frame. The position of the center of mass $\mathbf{r}_{\mathrm{G/O}}$ can be written as the mass-weighted average of the position of each particle $\mathbf{r}_{i/\mathrm{O}}$,

$$\mathbf{r}_{\mathrm{G/O}} = \frac{\sum_i m_i \mathbf{r}_{i/\mathrm{O}}}{\sum_i m_i},$$

and similarly, the velocity of the center of mass $\mathbf{v}_{\mathrm{G/O}}$ can be written as the mass-weighted average of the velocity of each particle $\mathbf{v}_{i/\mathrm{O}}$

$$\mathbf{v}_{\mathrm{G/O}} = \frac{\sum_i m_i \mathbf{v}_{i/\mathrm{O}}}{\sum_i m_i}.$$

We may write the velocity of each particle in terms of the velocity of the center of mass and the velocity of the particle relative to the center of mass $\mathbf{v}_{i/\mathrm{G}}$,

$$\mathbf{v}_{i/\mathrm{O}} - \mathbf{v}_{\mathrm{G/O}} = \mathbf{v}_{i/\mathrm{G}},$$

which then, after expanding

$$v_{i/\mathrm{O}}^2 = \mathbf{v}_{i/\mathrm{O}} \cdot \mathbf{v}_{i/\mathrm{O}} = v_{\mathrm{G/O}}^2 + v_{i/\mathrm{G}}^2 + 2\mathbf{v}_{\mathrm{G/O}} \cdot \mathbf{v}_{i/\mathrm{G}},$$

we retrieve the kinetic energy as

$$E_{\mathrm{K}} = \frac{1}{2}\sum_i m_i v_{\mathrm{G/O}}^2 + \frac{1}{2}\sum_i m_i v_{i/\mathrm{G}}^2 + \mathbf{v}_{\mathrm{G/O}} \cdot \sum_i m_i \mathbf{v}_{i/\mathrm{G}}.$$

We denote $m_{\mathrm{tot}} = \sum_i m_i$ as the total mass, so the first term is the kinetic energy of the center of mass

$$E_{\mathrm{KG}} = \frac{1}{2}\sum_i m_i v_{\mathrm{G/O}}^2 = \frac{1}{2} m_{\mathrm{tot}} v_{\mathrm{G/O}}^2.$$

The third term, using the previous relations, is seen to sum to zero by

$$\sum_i m_i \mathbf{v}_{i/\mathrm{G}} = \sum_i m_i \left(\mathbf{v}_{i/\mathrm{O}} - \mathbf{v}_{\mathrm{G/O}}\right) = m_{\mathrm{tot}} v_{\mathrm{G/O}} - m_{\mathrm{tot}} v_{\mathrm{G/O}} = 0,$$

so we retrieve $E_{\mathrm{K}} = E_{\mathrm{KG}} + E_{\mathrm{K/G}}$, where

$$E_{\mathrm{K/G}} = \frac{1}{2}\sum_i m_i v_{i/\mathrm{G}}^2.$$

Now, if we assume that all of the particles are part of a single rigid object, and since this problem is in two-dimensions, the velocity of each particle relative to the center of mass will be strictly due to planar rotation of the rigid object, i.e.

$$v_{i/\mathrm{G}} = \omega_\mathrm{G} r_{i/\mathrm{G}},$$

where ω_G is the angular frequency at which the rigid object rotates. Upon seeing the appearance of the moment of inertia of the rigid object about its center of mass,

$$I_\mathrm{G} = \sum_i m_i r_{i/\mathrm{G}}^2,$$

we now have the simple relation

$$E_{\mathrm{K}/\mathrm{G}} = \frac{1}{2}\, I_\mathrm{G}\omega_\mathrm{G}^2.$$