MAE 4730/5730 Fall 2020 Prelim 1 Problem 2 Summary

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Contents

1 Solution

(On next two pages)

Fall 2020 Prelim 1 Problem 2

Monday, October 19, 2020 11:18 AM

2) Particle with four forces on it. 2D. A particle m is connected to the origin with a (1) linear spring (k, ℓ_0) and (2) a linear dashpot c. Also, (3) gravity g acts in the $-y$ direction. Finally, (4) there is also a drag force opposing the direction of motion, with magnitude dv^2, where d is a drag constant. Assume that MATLAB code has been written where, at the time of interest, all of the following quantities have already been defined, or found:

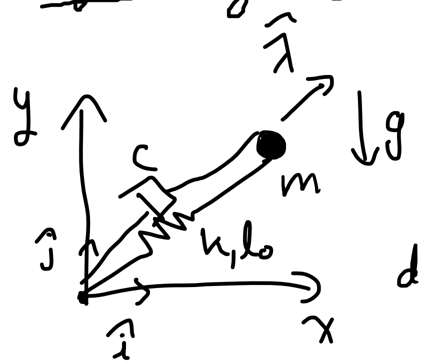
`m, g, k, ell_o, c, d`, and
`r = [x,y]'`, and `v = [v_x,v_y]'`.

Write MATLAB code, the last line of which should be

`a = ________________,`

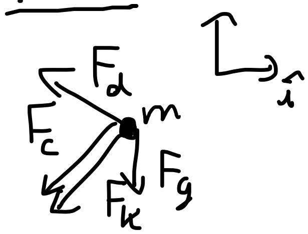
where `a` is a two component vector giving the x and y components of the acceleration. Short and clear lines of code are far preferred to long lines of code. Comments need not follow MATLAB notation (*i.e.*, circles and arrows are ok). Informal notation is ok (*e.g.*, ℓ_0 and v_x instead of `ell_0` and `v_x`).

System Diagram



d : drag constant

FBD



Define quantities

$\vec{r} = x i + y\hat{j}$ $\quad \vec{V} = \dot{\vec{r}} = \dot{x}\hat{i} + \dot{y}\hat{j}$

$\vec{a} = \dot{\vec{V}} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$ $\quad \hat{\lambda} = \vec{r}/\|\vec{r}\|$

$\vec{F}_k = -k(\|\vec{r}\| - l_0)\hat{\lambda}$

$\vec{F}_c = -c(\vec{V}\cdot\hat{\lambda})\hat{\lambda}$

$\vec{F}_d = -d\|\vec{V}\|^{\cancel{2}}\left(\frac{\vec{V}}{\cancel{\|\vec{V}\|}}\right) = -d\|\vec{V}\|\vec{V}$

$\vec{F}_g = mg\hat{j}$

LMB

$$\sum \vec{F} = m\vec{a}$$

$$\sum \vec{F} = \vec{F}_k + \vec{F}_c + \vec{F}_d + \vec{F}_g = m\vec{a}$$

$$\vec{a} = \frac{1}{m}\left(\vec{F}_k + \vec{F}_c + \vec{F}_d + \vec{F}_g\right)$$

Parameters, r, and v at time of interest have already been defined/found.

Code

```
lambda = r/norm(r);

Fk = -k*(norm(r)-ell_0)*lambda,

Fc = -c*dot(v, lambda)*lambda;

Fd = -d*norm(v)*v;

Fg = [0;-m*g];

a = (1/m)*(Fk+Fc+Fd+Fg);
```

2 Class Performance: The Good, the Bad, the Interesting...

2.1 The Good

Overall most students knew what this problem was asking for and how to get there. The statistics below demonstrate this. The mistakes were not related to understanding how to solve the problem as a whole, but specific parts of it.

2.2 The Bad (Common Errors)

1. **Dashpot**: The most glaring error which happened to be the most common error had to do with the dashpot force. Almost all students correctly remembered that it was proportional to the rate of change of the length of the dashpot and pointing in the direction opposite to the velocity. Unfortunately, more than half the students incorrectly assumed that the scalar component of this force was simply proportional to the velocity of the mass and wrote down the following or some variation of the following (Rate of change of length $\neq$ velocity).

$$\vec{F}_c = -c|\vec{v}|\frac{\vec{v}}{|\vec{v}|} = -c\vec{v} \tag{1}$$

   A few students remembered that the rate of change of the dashpot is the same as the component of velocity that is in the direction of the position vector of the mass ($\vec{v} \cdot \frac{\vec{r}}{|\vec{r}|}$) and were able to write down the correct expression.

$$\boxed{\vec{F}_c = -c\Big(\vec{v} \cdot \frac{\vec{r}}{|\vec{r}|}\Big)\frac{\vec{r}}{|\vec{r}|}} \tag{2}$$

   To clarify the difference between the two, imagine the mass moving on a perfectly circular path of some radius about the origin. The velocity would be nonzero, but the rate of change of the length of the dashpot would be 0 and so the dashpot would not exert any force on the mass.

2. **Diagrams (System defining ones and FBDs alike)**: This one is second only because it was not nearly as common as the first. Diagrams defining the system and FBDs in general need to show what's happening in the system. Mistakes ranged from forgetting to include coordinate systems to omitting the system parameters in the diagram to forgetting the FBDs entirely (that last was worth 5 points). When using LMB/AMB, the system needs to be well defined beforehand. Please look to Professor Ruina's textbook for a guide on FBDs as I will not explain here what is required in one.

3. **Bad Code**: Good code has succinct lines. For this problem only a few lines were required to define the forces, sum them, and divide by the mass to get acceleration. Some students chose to do all that in one/two lines (two if they decided to write out the components of all forces to define a_x or a_y). While the following is technically correct, it is not good code (this is not a specific student's solution but a general representation of what I saw several times).

```
ax = (-k*(1-ell_0/norm(r))*r(1)-c*dot(v,r/norm(r))*v(1)/norm(v)-d*norm(
```

```
ay = (-k*(1-ell_0/norm(r))*r(2)-c*dot(v,r/norm(r))*v(2)/norm(v)-d*norm(
```

Notice it falls off the page. Please don't make your grader read that :).

2.3 The Interesting (An interesting common error)

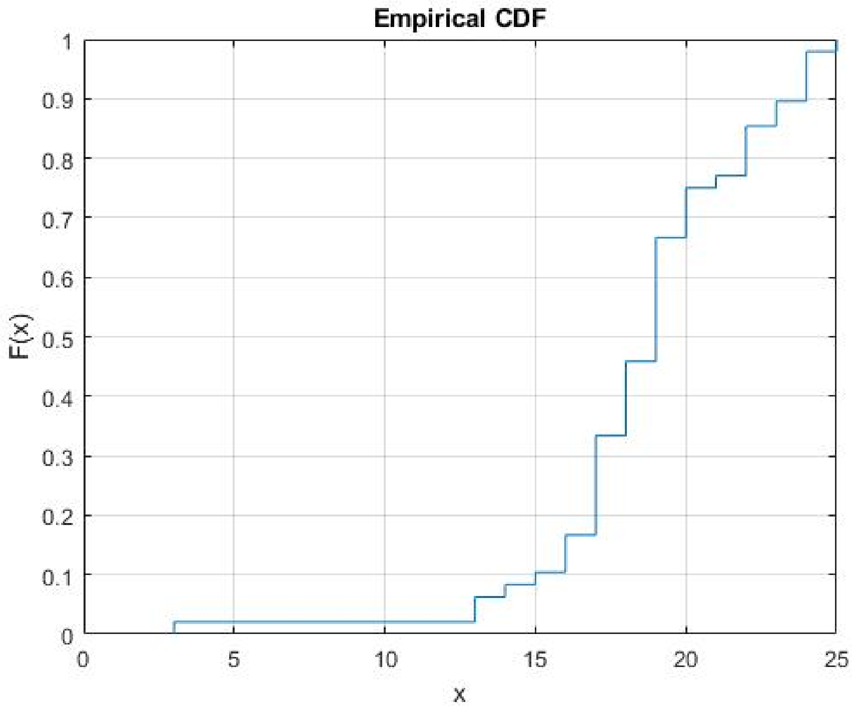
The problem statement makes it very clear that you are looking for the acceleration at a time of interest. Furthermore it explained that position and velocity and the parameters at the time of interest had already been defined or found. All that was left to do was to define the forces, add them up, and divide by mass and done! For some reason, many students chose to write fully fleshed out code using ode45 and a myrhs function as well. Students were not deducted points for failing to follow the instructions in this way (unless there were problems with their code). I am genuinely surprised by the number of students who chose to double the required amount of work to successfully answer this problem. My only guess as to why this is the case is that many students did not read the instructions carefully.

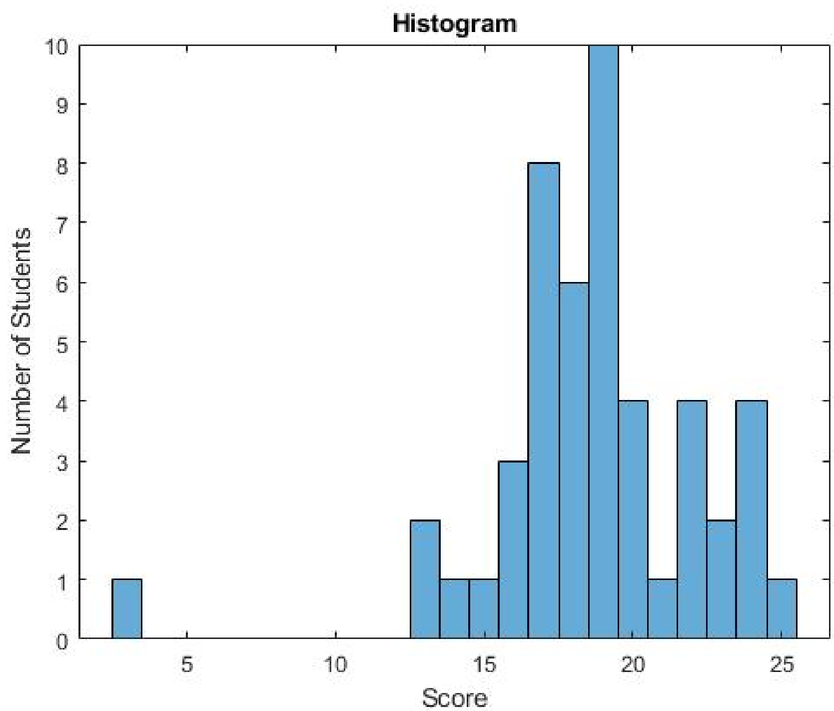
3 Problem Statistics

Here is probably all you could ever want to know about the performance of the class on this problem in terms of numbers and graphs.

Mean	$18.\bar{6}$
Median	19
Standard Deviation	3.692

Table 1: Distrubtion info


Empirical CDF
F(x)
x
0
5
10
15
20
25
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Histogram
Number of Students
Score
5
10
15
20
25
0
1
2
3
4
5
6
7
8
9
10