MAE 4730/5730 Fall 2020 Prelim 2 Problem 3 Summary

Juan Berrio jgb249

November 2020

Contents

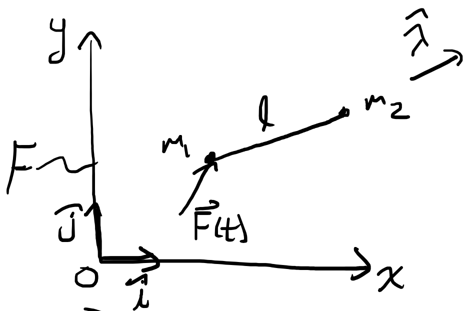
1 Solution

(On next two pages)

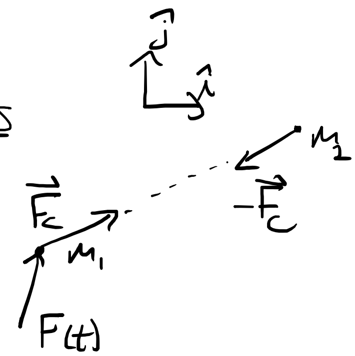
Fall 2020 Prelim 2 Problem 3

Monday, November 9, 2020 5:40 PM

3) Two particles. 2D. Two particles m_1 and m_2 are connected by a rigid massless rod. A known and given external force $\vec{F}(t)$ acts on m_1. Write the equations of motion using the DAE approach, clearly defining any matrices or column vectors you define.



FBDs



$\vec{r}_1 = (x_1\hat{\imath} + y_1\hat{\jmath})$ $\quad \vec{r}_2 = x_2\hat{\imath} + y_2\hat{\jmath}$ $\quad \ell = |\vec{r}_2 - \vec{r}_1|$

$\vec{F}_c = F_c\hat{\lambda}$ $\quad \hat{\lambda} = \dfrac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|} = \dfrac{\vec{r}_2 - \vec{r}_1}{\ell}$ $\quad \vec{F}(t) = F_x\hat{\imath} + F_y\hat{\jmath}$

unknowns: $\begin{bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{x}_2 \\ \ddot{y}_2 \\ F_c \end{bmatrix}$

5 unknowns $\longrightarrow$ 5 equations

LMB1

$$\sum \vec{F}_1 = m_1\vec{a}_{1/F}$$

$$\left\{ \vec{F}(t) + \vec{F}_c = m_1(\ddot{x}_1\hat{\imath} + \ddot{y}_1\hat{\jmath}) \right\} \text{LMB}_1$$

LMB2

$$\sum \vec{F}_2 = m_2\vec{a}_{2/F}$$

$$\left\{ -\vec{F}_c = m_2(\ddot{x}_2\hat{\imath} + \ddot{y}_2\hat{\jmath}) \right\} \text{LMB}_2$$

LMB1 $\cdot\, \hat{\imath}$

$$\vec{F}(t)\cdot\hat{\imath} + \vec{F}_c\cdot\hat{\imath} = m_1\ddot{x}_1$$

$$① \quad \boxed{F_x + F_c\left(\frac{x_2 - x_1}{\ell}\right) = m_1\ddot{x}_1}$$

LMB2 $\cdot\, \hat{\imath}$

$$-\vec{F}_c\cdot\hat{\imath} = m_2\ddot{x}_2$$

$$② \quad \boxed{-F_c\left(\frac{x_2 - x_1}{\ell}\right) = m_2\ddot{x}_2}$$

LMB1 $\cdot\, \hat{\jmath}$

$$\vec{F}(t)\cdot\hat{\jmath} + \vec{F}_c\cdot\hat{\jmath} = m_1\ddot{y}_1$$

LMB2 $\cdot\, \hat{\jmath}$

$$-\vec{F}_c\cdot\hat{\jmath} = m_2\ddot{y}_2$$

$$\vec{F}(t)\cdot\hat{j} + \vec{F}_c\cdot\hat{j} = m_1\ddot{y}_1$$

(3) $$\boxed{F_y + F_c\left(\frac{y_2-y_1}{l}\right) = m_2\ddot{y}_2}$$

$$-\vec{F}_c\cdot\hat{j} = m_2\ddot{y}_2$$

(4) $$\boxed{-F_c\left(\frac{y_2-y_1}{l}\right) = m_2\ddot{y}_2}$$

Constraint Equation

$$l = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$l^2 = (x_2-x_1)^2 + (y_2-y_1)^2$$

$$\frac{d}{dt}\quad 0 = \cancel{2}(x_2-x_1)(\dot{x}_2-\dot{x}_1) + \cancel{2}(y_2-y_1)(\dot{y}_2-\dot{y}_1)$$

$$\frac{d}{dt}\quad \boxed{0 = (\dot{x}_2-\dot{x}_1)^2 + (x_2-x_1)(\ddot{x}_2-\ddot{x}_1) + (\dot{y}_2-\dot{y}_1)^2 + (y_2-y_1)(\ddot{y}_2-\ddot{y}_1)}$$ (5)

rewrite as . . .

$$\boxed{\begin{gathered} 0 = V_{2/1}^2 + (x_2-x_1)(\ddot{x}_2-\ddot{x}_1) + (y_2-y_1)(\ddot{y}_2-\ddot{y}_1) \\ V_{2/1}^2 = (\dot{x}_2-\dot{x}_1)^2 + (\dot{y}_2-\dot{y}_1)^2 \end{gathered}}$$ (5)

$$Ax = b \qquad x = A^{-1}b$$

$$\boxed{\begin{bmatrix} m_1 & 0 & 0 & 0 & -\frac{(x_2-x_1)}{l} \\ 0 & m_1 & 0 & 0 & -\frac{(y_2-y_1)}{l} \\ 0 & 0 & m_2 & 0 & \frac{(x_2-x_1)}{l} \\ 0 & 0 & 0 & m_2 & \frac{(y_2-y_1)}{l} \\ -(x_2-x_1) & -(y_2-y_1) & (x_2-x_1) & (y_2-y_1) & 0 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{x}_2 \\ \ddot{y}_2 \\ F_c \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ 0 \\ 0 \\ -V_{2/1}^2 \end{bmatrix}}$$

2 Class Performance: Confusion

I think the name of the section speaks for itself.

The DAE method is used for solving for the accelerations of each body (This is why maximal coordinates must be used). Simplifying the system to reduce the number of acceleration quantities to solve for will result in solving the system using minimal coordinates which is nothing more than just rewriting the equations of motion using a matrix. Constraints are also omitted this way. This alternative is a fundamentally different method that is not DAE.

A more common issue I saw was the use of extra variables. This was done by either splitting up the tension in the rod (Commonly written as T_x and T_y) or by adding an additional variable θ and searching for equations to solve for $\ddot{\theta}$. The former is fine, although most students who attempted this were unable to find an additional correct equation to solve for two constraint forces instead of just one. The latter is problematic. There is no need to solve for the second derivative of θ because θ can be solved for simply using the position of the two masses which you are already solving for. Including θ and $\ddot{\theta}$ is doing something akin to "super maximal coordinates" and it is over-determining the system. In short, it's wrong.

I should point out that for every DAE solution that was not obviously correct or obviously incorrect (e.g. ones that solved for extra variables), I tested them out using matlab and compared the output to the output produced by my solution. Very few of them worked. Many of them had matrices that were singular or close to it.

Aside from that, other common errors included the following (in no particular order):

- FBD errors such as forgetting to include a coordinate system
- Having scalars and vectors in the same equation
- Incorrectly doing AMB (furthermore AMB was not required for this problem)
- Incorrectly writing the constraint equation (this one surprised me, many people forgot how to solve for the length of the rod using the position of the masses)
- Over-simplifying the system to a 1 degree of freedom system (where motion and/or forces are constrained to just the $\hat{\imath}$ direction or the rod is free only to rotate about its center.
- Thinking the applied force was another quantity to be solved for.

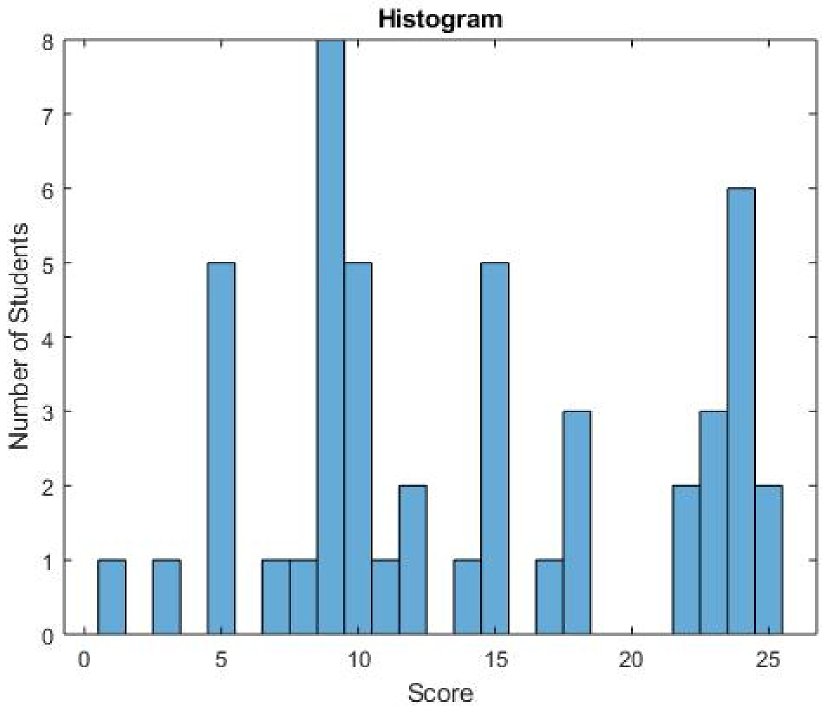
The diversity in errors/mistakes students made I think contributed to unusual distribution. I am quite surprised by it.

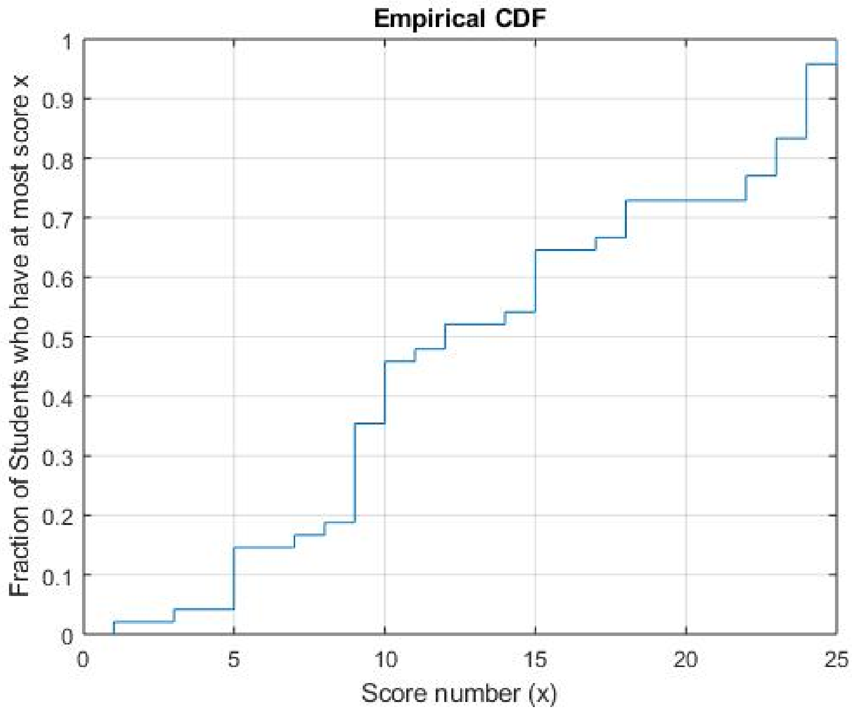
3 Problem Statistics

Like last time, here is probably all you could ever want to know about the performance of the class on this problem in terms of numbers and graphs.

Mean	13.9167
Median	12
Standard Deviation	7.07

Table 1: Distrubtion info




Empirical CDF
Fraction of Students who have at most score x
Score number (x)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20
25