MAE 2030 Lecture #1:

* cameras & microphones on!

Today: ① Intro
② Harmonic oscillator

Course Topics: How things move in 1, 2, 3 dimensions
pos & rotations

particles & rigid objects

Newtons & Eulers, N-E laws, LMB, AMB

Applications: machines, planets, animal motion, vibrations, robotics

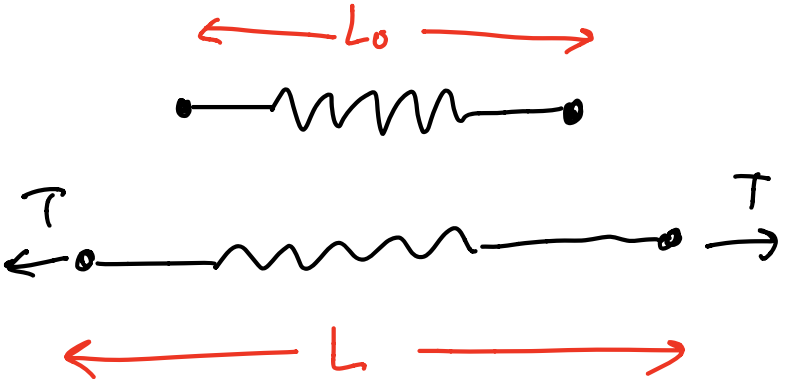
* build your skills

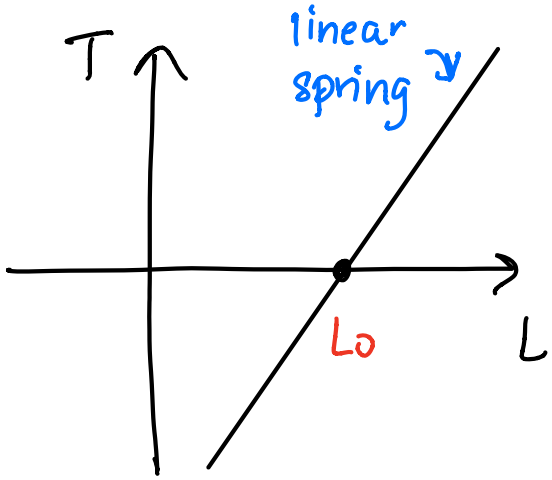
Skill Building: statics, ODEs, Matlab skills, modeling & problem solving skills

Organization: simple motion ⇒ more complex motions

Today: no motion / constant vel motion, 1D dynamics

Harmonic Oscillator : made of spring & mass



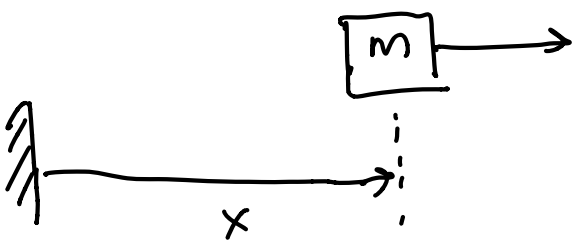


Constitutive law for spring

$$T = K(L - L_0)$$

"$F = Kx$"

mass



sign conventions

→ +

Kinematics

$$v = \frac{dx}{dt} = \dot{x}$$

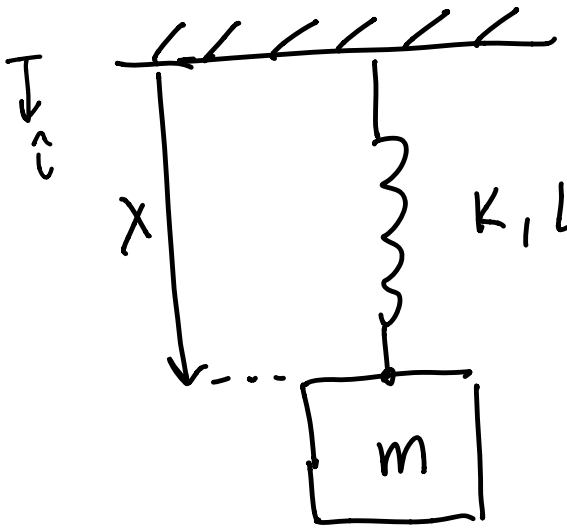
$$a = \frac{dv}{dt} = \dot{v} = \ddot{x}$$

LMB

↑ linear momentum balance

$\dot{L}$; $L = mv$

$F = ma$



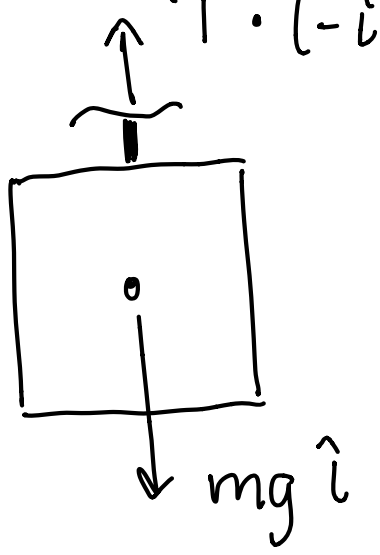
Q: Given m, L_0, g, K, t, x_0

Given initial conditions (ICs): $x(0) = x_0$ @ $t=0$

released from rest: $v(0) = 0$

Find x

FBD of mass



LMB $\quad \sum \vec{F} = m\vec{a}$

$$\{ mg\hat{i} - T\hat{i} = m\ddot{x}\hat{i} \}$$

$T = k(L - L_0) = k(x - L_0)$

$\{\ \} \cdot \hat{i} \Rightarrow \quad mg - k(x - L_0) = m\ddot{x}$

$\Rightarrow$

Standard 2^{nd} order form

all x stuff $\longleftarrow | \longrightarrow$ no x's

$$\boxed{m\ddot{x} + kx = kL_0 + mg} \quad (1)$$

Standard prob. ODE : (1) with

ICs $\begin{cases} x(0) = x_0 \\ \dot{x}(0) = 0 \end{cases}$

sol'n : $x(t) = x_h(t) + x_p(t)$

$x_h(t)$: homogeneous sol'n

$x_p(t)$: particular sol'n

sol'n to (1) w/ R.H.S. set to 0

$\Rightarrow \boxed{m\ddot{x}_h + kx = 0}$

any old solution, the simplest one

$m\ddot{x} + kx = kL_0 + mg$

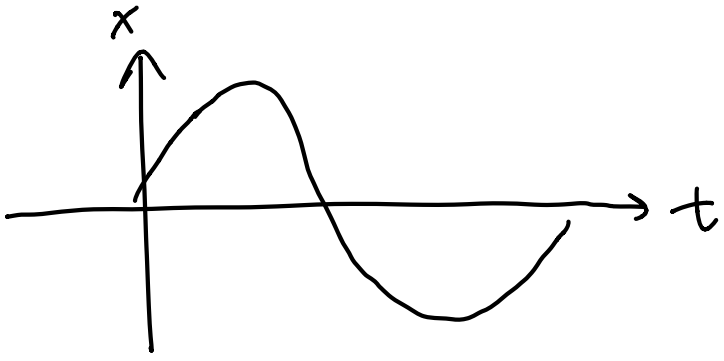
$$\boxed{x_p = L_0 + \frac{mg}{k}}$$

aside

Think about $\ddot{x} = -x$

$x > 0 \Rightarrow$ curved down

$x < 0 \Rightarrow$ curved up



$$\Rightarrow \boxed{x_h = B\sin(\omega_0 t) + A\cos(\omega_0 t)}$$

"omega zero"

$$\omega_0 = \sqrt{\frac{k}{m}}$$

General sol'n: $x(t) = x_h + x_p$

$$= A\cos(\omega_0 t) + B\sin(\omega_0 t) + L_0 + \frac{mg}{k}$$

→ plug in ICs to find A & B

$$\dot{x}(0) = 0 \Rightarrow -A\omega_0 \sin(\omega_0 t) + B\omega_0 \cos(\omega_0 t) + 0 = 0$$

@ $t = 0$

$$\Rightarrow \boxed{B = 0}$$

$$x(0) = x_0 \Rightarrow A\cos(\omega_0 t) + L_0 + \frac{mg}{k} = x_0$$

($\cos(\omega_0 t) \to 1$ at $t = 0$)

$$\Rightarrow \boxed{A = x_0 - \left(L_0 + \frac{mg}{k}\right)}$$

$$\boxed{x = \left[x_0 - \left(L_0 + \frac{mg}{k}\right)\right]\cos(\omega_0 t) + L_0 + \frac{mg}{k}}$$

($\omega_0 \to \sqrt{\frac{k}{m}}$)