MAE 2030 Lecture #3: 2/12/21

Today: ① Num sol'n of ODEs

What is an ODE? An eqn with derivatives

ex) $\dot{x} = x$

What is a sol'n? A function $x(t)$ that satisfies ODE

ex) $x(t) = t^2$

→ plug in $\dot{x} \overset{?}{=} x$

$2t = t^2$ ✗

$x(t) = t^2$ does not solve $\dot{x} = x$

ex) $x(t) = e^t$

→ check $\dot{x} \overset{?}{=} x$

$e^t = e^t$ ✓

$x(t) = e^t$ [satisfies / solves] ODE $\dot{x} = x$

Can we solve "all" ODEs?

ex) $\dot{x} = f(t)$ ⟹ sol'n : $x(t) = \int_0^t f(t')dt'$

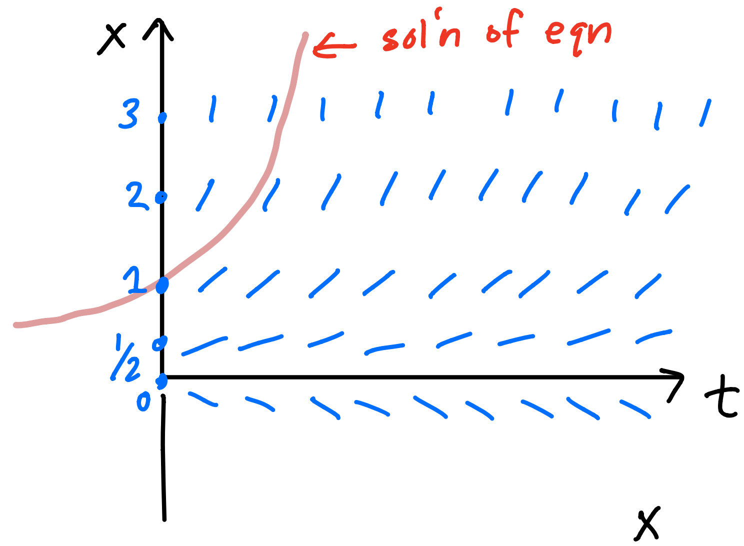
↗ given

ex) $f(t) = \dfrac{e^t}{t^2 + \ln(t)}$ ⟹ no formula for $x(t)$

Most ODEs have no analytical sol'n

→ need numerical sol'n

ex) $\dot{x} = x$, IC: $x(0) = 1$

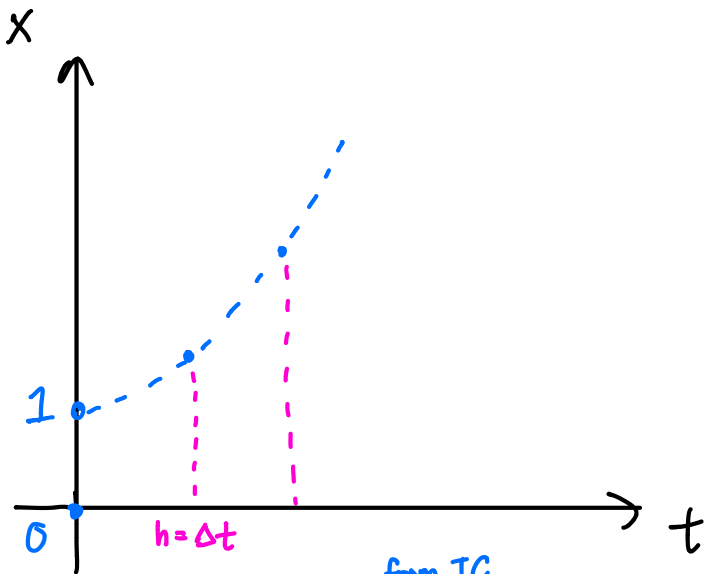


How to do on computer?

Can evaluate r.h.s. $= f(x,t)$

$$\dot{x} = \underbrace{f(x,t)}$$

ex) $f(x,t) = x$



If you know the rules of a system, and the present value, you can predict the future

$x(0) = x_0 = 1$ ↙ from IC

$x(h) = x_0 + h\, \underbrace{f(x,t)}_{x=1}$ (↗ 0)

$x(nh) = x[(n-1)h] + h \cdot f[(n-1)h]$

Euler's Method

$$x(t+h) = x(t) + h\,\dot{x}[t, x(t)]$$

- $t+h$: a little bit in the future
- $x(t)$: present value
- h: time interval
- $\dot{x}[t, x(t)]$: present rate of change

First order form: ex) $\ddot{x} + x = 0$ ①

define $v = \dot{x}$

$\Rightarrow \dot{x} = v$ ②

① $\Rightarrow \ddot{x} = -x$

$\dot{v} = -x$ ③

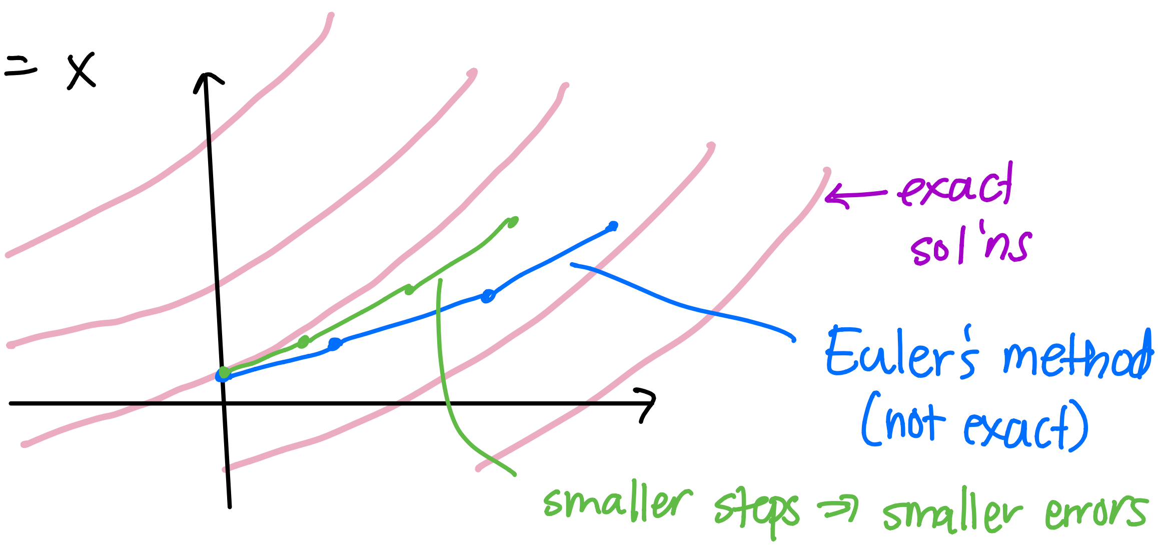
$$\ddot{x} + x = 0 \Rightarrow \boxed{\begin{aligned} \dot{x} &= v \\ \dot{v} &= -x \end{aligned}}$$

Present state of system: $z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \end{bmatrix}$

ex) $z_1 = x$
$z_2 = y$

ODE : $\boxed{\dot{z} = f(t, z)}$

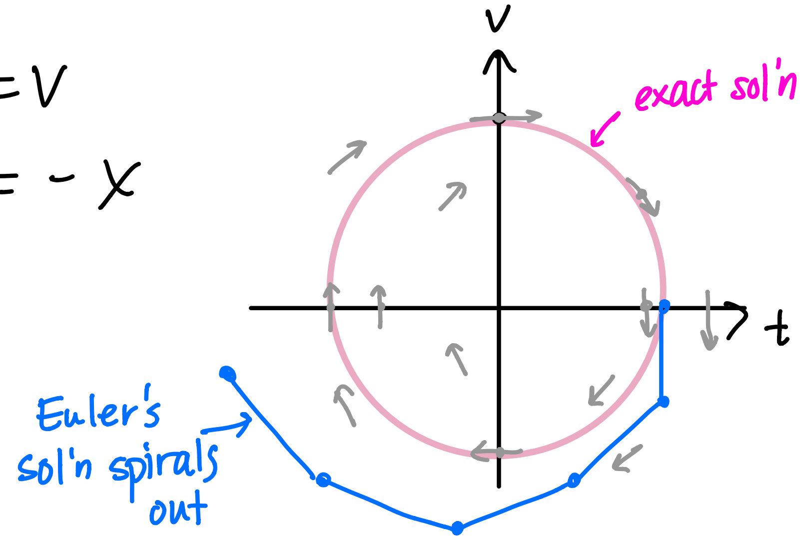
Error: $\dot{x} = x$



Phase plane:

ex) $\dot{x} = v$

$\dot{v} = -x$



Phil & Sally's emotional state:

$\dot{S} = P$

$\dot{P} = -S$

} just like the harmonic oscillator

$z = \begin{bmatrix} P \\ S \end{bmatrix}$

↗ state of the system

Solving ODEs on MATLAB:

demo: see posted code & Andy's walkthrough