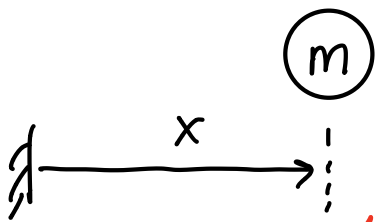
MAE 2030 Lecture #5: 2/17/21

Today: 1D dynamics continued
(Work, Energy, Momentum)

1D dynamics : $\rightarrow \hat{i}$



$\rightarrow \vec{F} = F\hat{i}$

laws of mechanics III

LMB: $\underbrace{\vec{F} = m\vec{a}}_{\text{laws of mechanics}} = m\ddot{x}\hat{i}$

II Kinematics $\left\{ \vec{a} = \dot{\vec{v}}, \quad \vec{v} = \dot{x} \right\}$

$\vec{F} = \vec{F}\underbrace{(t, x, v)}$

Springs, dashpots, gravity, friction, external forces

material properties I
Force laws
constitutive laws

$\left\{\vec{F} = m\vec{a}\right\} \cdot \hat{i}$ $\qquad \vec{F} = F\hat{i}$

$\Rightarrow F = ma \quad (*)$

* try not to set it up and then figure out the sign after
↳ sort out the sign while doing the problem

Find consequences of (*)

$$\int \{*\}\,dt \rightarrow \int F\,dt = \int m\dot{v}\cdot dt$$

$$\int F\,dt = m\int \underbrace{\dot{v}\,dt}_{dv}$$

$$\int F\,dt = mv$$

$$\boxed{\dot{v}\,dt = \frac{dv}{dt}\cdot dt = dv \qquad \frac{\Delta v}{\Delta t}\cdot \Delta t = \Delta v}$$

$$\int_{t_1}^{t_2} F(t)\,dt = m\underbrace{(v_2 - v_1)}_{\Delta v} = L_2 - L_1$$

L_2: @ $t = t_2$

L_1: @ $t = t_1$

$L = mv =$ momentum

$$\int F\cdot dt \equiv \text{Impulse} = p \quad \text{"}I\text{"}$$

Principle of impulse & momentum:

$$\boxed{p = \underbrace{\Delta L}_{\text{change in momentum}}}$$

p: impulse

principle is satisfied automatically by sol'ns of $F = ma$

What it's good for:

① Check on your sol'ns
② Shortcut for some problems

a) Look at sol'n
b) Calculate $p = \int_{t_1}^{t_2} F \cdot dt$
c) Calculate L_1 & L_2
d) Check $p \stackrel{?}{=} L_2 - L_1$

How to numerically calculate p:

$$\dot{p} = F$$

Known at any given time

New set of ODEs:

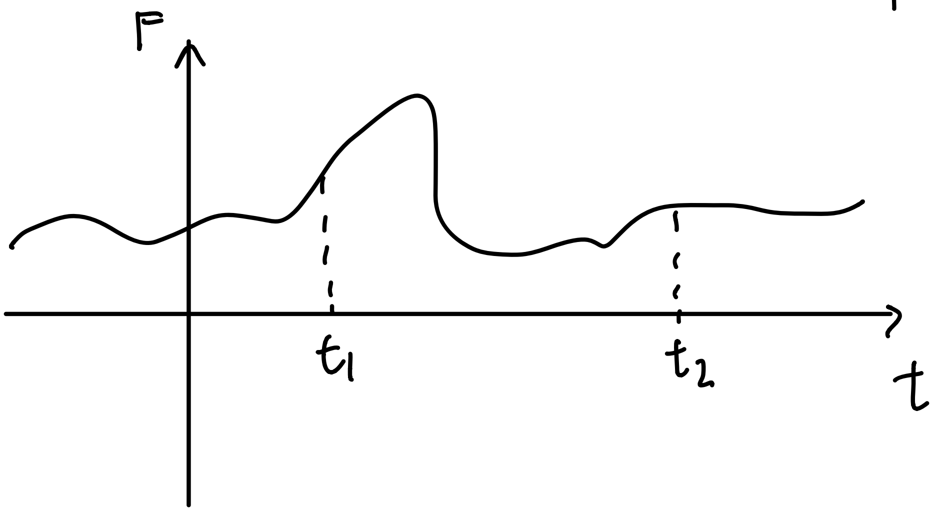
3 ODEs

$$\begin{cases} \dot{x} = v \\ \dot{v} = F/m \\ \dot{p} = F \end{cases}$$

$F(t, x, v)$

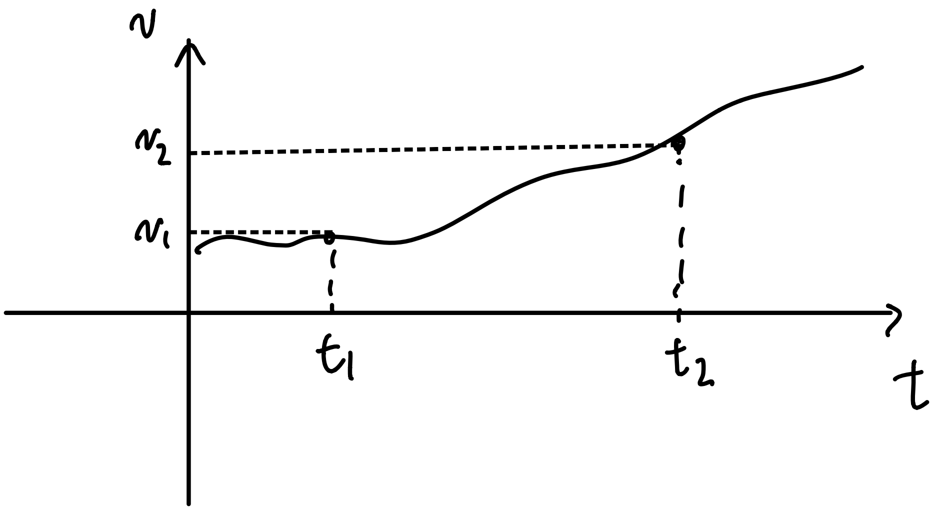
$$z = \begin{bmatrix} x \\ v \\ p \end{bmatrix}, \quad z_0 = \begin{bmatrix} x_0 \\ v_0 \\ 0 \end{bmatrix}$$

P(end) = net impulse



p is not a state variable

↳ p is associated with an interval in time



Power, Work, & Energy:

$$F = ma$$

$$\{*\}\cdot v \qquad \underbrace{Fv}_{\text{Power } P} = ma v, \quad a = \frac{dv}{dt}$$

Notice:

$$\frac{d}{dt}\left(\frac{v^2}{2}\right) = \frac{2v}{2}\frac{dv}{dt}$$

*chain rule

$$= \frac{dv}{dt} \cdot v$$

$$= av$$

$$\underbrace{Fv}_{\text{Power}} = \frac{d}{dt}\underbrace{\left(\frac{1}{2}mv^2\right)}_{\substack{\text{Kinetic Energy} \\ E_K}}$$

Power & Kinetic Energy:

$$\boxed{P = \dot{E}_K} \quad (**)$$

Power = rate of change of kinetic energy

$$P = Fv, \quad E_K = \frac{1}{2}mv^2$$

$$\int \{(**)\}\, dt \Rightarrow \underbrace{\int P\, dt}_{\text{Work}} = \underbrace{\int \dot{E}_K\, dt}_{\Delta E_K}$$

$$\text{Work} = \Delta E_K$$

Think about work:

$$W = \int P \cdot dt$$

$$= \int F \cdot \underbrace{v}_{\frac{dx}{dt}} \cdot dt$$

$$\boxed{W = \int F \cdot dx}$$

$$\boxed{\begin{aligned} W &= \Delta E_k \\ &= \int P \cdot dt \\ &= \int F \cdot dx \end{aligned}}$$

Who cares?

1. Shortcut for some problems
2. Check on calculations → $\Delta E_k \stackrel{?}{=} W_{12}$

How to numerically use it:

1. Evaluate $\Delta E_k = \underbrace{\tfrac{1}{2} m v_2^2}_{\text{end of simulation}} - \underbrace{\tfrac{1}{2} m v_1^2}_{\text{start of simulation}}$
2. Evaluate $W = \int P \cdot dt$

$$\dot{W} = P = Fv$$

Changes our ODEs:

$$\dot{x} = v$$
$$\dot{v} = F/m$$
$$F(t, x, v)$$
$$\dot{W} = Fv$$

ICs: $x(0) = x_0$
$v(0) = v_0$
$W(0) = 0$

$W(end)$ = net work

Potential Energy: associated with certain Force laws

$$F(t, x, v) = \underbrace{F(x)}_{}$$

Force only depends on position

ex) $F = -kx$ spring ✓

ex) $F = mg$ gravity ✓

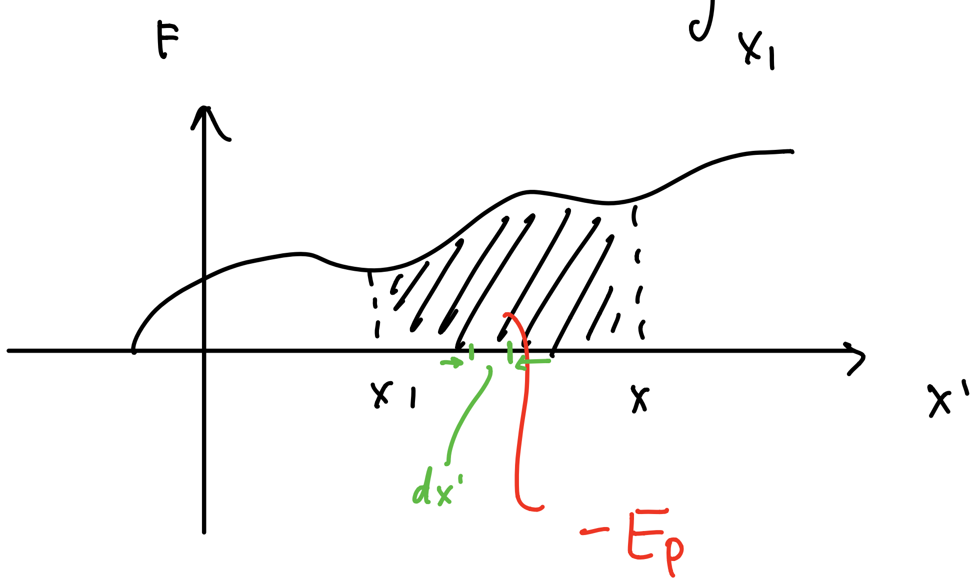
ex) $\underbrace{F = -cv}_{\text{not in discussion now}}$ viscous friction ✗

Work & Energy: $W = \Delta E_k$

$$\int_{x_1}^{x_2} F(x)\cdot dx = E_{k_2} - E_{k_1}$$

→ define $E_p(x) = -\int_{x_1}^{x} F(x')\,dx'$

dummy variable x'



$$\left.\begin{aligned} W &= -\Delta E_p \\ W &= \Delta E_k \end{aligned}\right\} \quad 0 = \Delta(E_k + E_p)$$

$$\boxed{0 = \Delta E_T}$$

Conservation of Energy

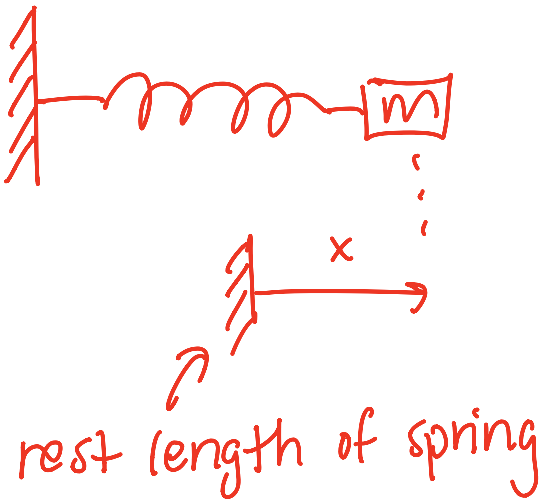
$E_T = E_k + E_p$

* only applies if $F(t, x, v) = F(x)$

Who cares?

① Shortcut for some problems
② Check on sol'n of some problems

→ ex) Harmonic oscillator



$$F = -kx$$

$$E_p = -\int_0^x F(x')\,dx'$$

$$= \int_0^x kx'\,dx'$$

$$\boxed{E_p = \frac{kx^2}{2}}$$

potential energy for spring

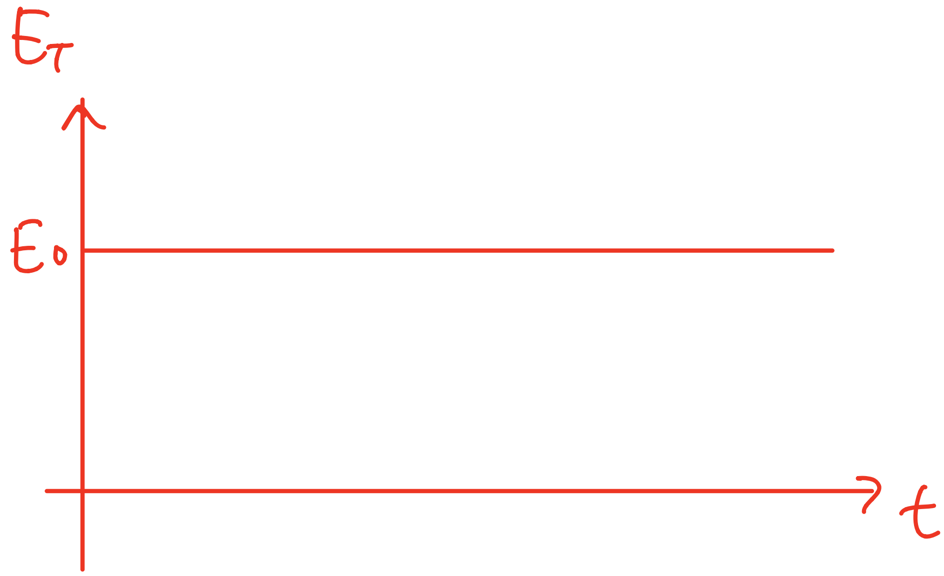
For oscillator:

$E_p \neq$ constant

$E_k + E_p =$ constant

$$\underbrace{\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}}_{\text{during motion}}$$

Check:



Better check:

$$E_T = E_T(0)$$