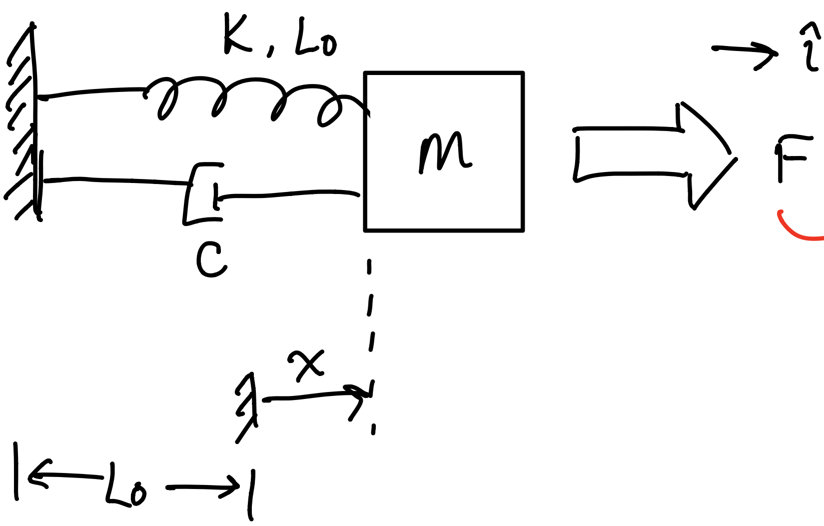
MAE 2030 Lecture #7: 2/22/21

Today: ① Forced, damped oscillator
② Midpoint method
③ Multiple masses

Forced damped harmonic oscillator:

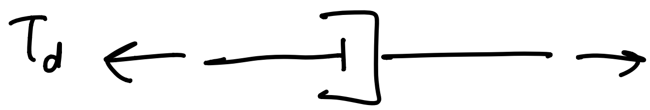
→ a famous often used model (very simple & covers lots of phenomena like things that vibrate)



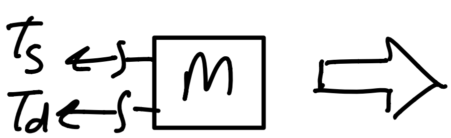
$\underbrace{F = F_0 \sin(\omega_0 t)}_{\text{Forcing}}$

FBD:

$T_s = kx$



$T_d = c\dot{x}$



$F_0 \sin(\omega_0 t)$

LMB: $\left\{ \sum \vec{F} = m\vec{a} \right\}$

$\left\{ \quad \right\} \cdot \hat{\imath} \Rightarrow -T_s - T_d + F_0 \sin(\omega_0 t) = m\ddot{x}$

$T_s \to kx \qquad T_d \to c\dot{x}$

Math Form: $\boxed{m\ddot{x} + c\dot{x} + kx = F_0 \sin(\omega_0 t)}$ $*$ *** Know this ***

Computer Form: $\ddot{x} = \left[F_0 \sin(\omega_0 t) - kx - cv\right] / m$

$\ddot{x} \to \dot{v} \qquad v \to \dot{x}$

1st order form
$$\boxed{\begin{aligned} \dot{x} &= v \\ \dot{v} &= \left[F_0 \sin(\omega_0 t) - kx - cv\right] / m \end{aligned}}$$
***

$$\dot{z} = f(t, z)$$

$$z = \begin{bmatrix} x \\ v \end{bmatrix}$$

Lots of special cases
Lots of phenomena with eqn (*) ⇒ read in book about vibrations

ex) $x_g = x_h + x_p$

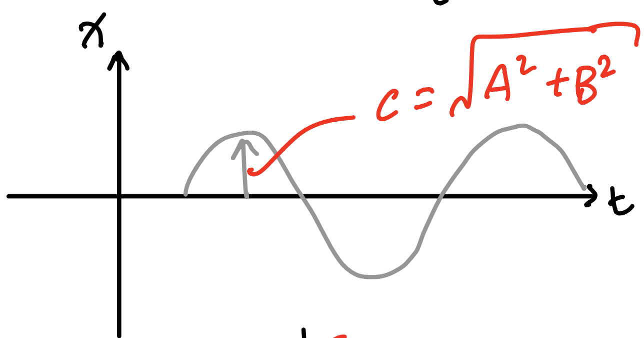
ex) Long term behavior

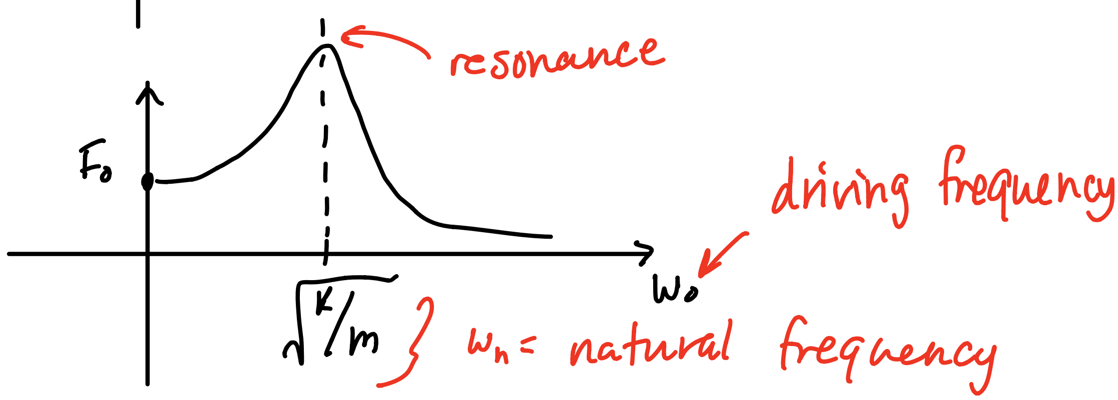
- Assume that $c > 0$
  (might be very small)

  $$x_h(t) \xrightarrow[t \to \infty]{} 0$$

  $$\Rightarrow \boxed{x_p(t)} \quad \text{"steady state soln"}$$

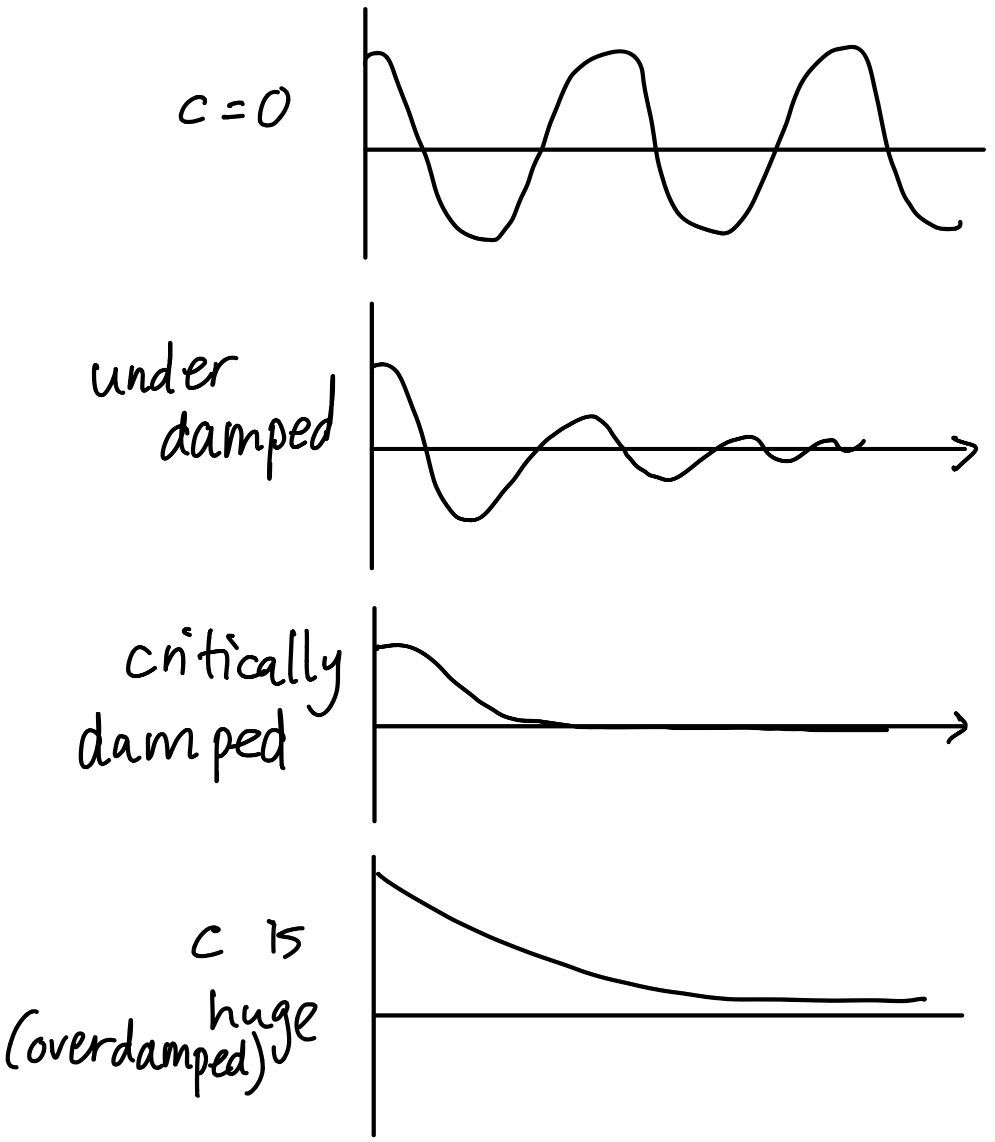
  $$x_p = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$





ex) overdamped & underdamped

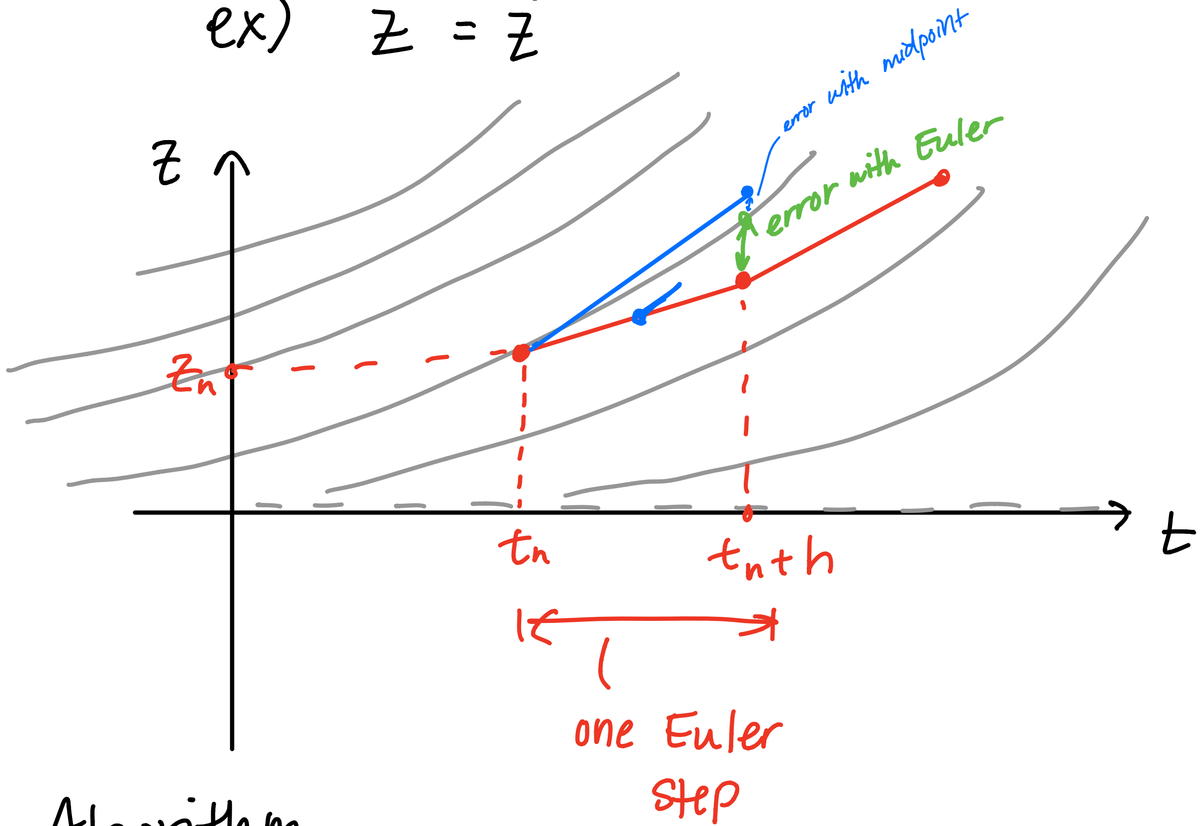
x_h



* Read book, check phenomena w/ computer simulation

Midpoint Method for numerical sol'n of ODEs:

ex) $\dot{z} = \overset{f(z)}{z}$



Algorithm

$$\dot{z} = \underbrace{f(t, z)}_{\text{r.h.s. function}}$$

given t_n, z_n, h

$$\dot{z}_{1/2} = f\Big(\overbrace{t_n + \tfrac{1}{2}h}^{\text{half a time step}}, \big(z_n + \dot{z}_n \tfrac{h}{2}\big)\Big)$$

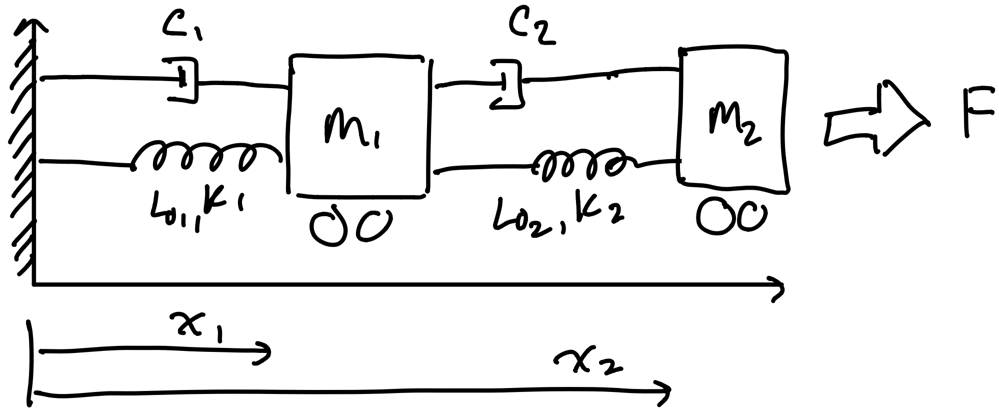
$$z_{n+1} = z_n + h\,\dot{z}_{1/2}$$

└ adds ~2 lines of code

→ ODE2, keep ODE1

Multi - DoF : many degrees of freedom

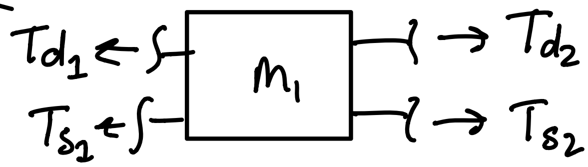
ex)



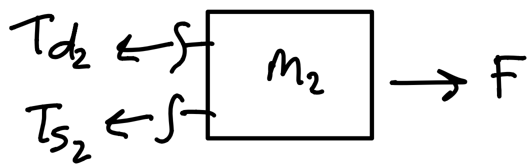
Goal: ODEs we can solve

FBDs:

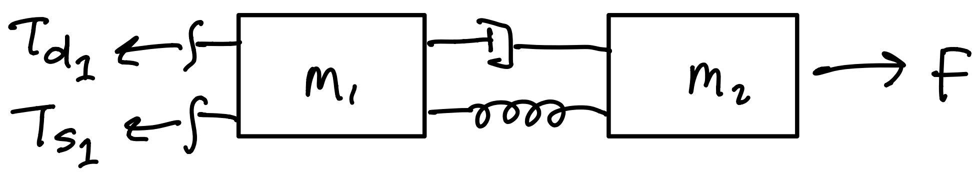
mass 1 :



mass 2 :



System:



LMB

mass 1 : $\boxed{m_1 \ddot{x}_1 = T_{d_2} + T_{s_2} - T_{d_1} - T_{s_2}}$

mass 2 : $\boxed{m_2 \ddot{x}_2 = -T_{d_2} - T_{s_2} + F}$

*need to figure out tensions

spring 1 : $T_{s_1} = k(\underbrace{L}_{x_1} - L_{0_1})$

*be careful of signs!!

dashpot 1 : $T_{d_1} = c_1 \dot{L}_1 = c_1 \dot{x}_1$

spring 2 : $T_{s_2} = k_2(\underbrace{L_2}_{x_2 - x_1} - L_{0_2})$

dashpot 2 : $T_{d_2} = c_2 \dot{L}_2 = c_2(\dot{x}_2 - \dot{x}_1)$