

—Print your netID on the top of every side of every sheet—  
—Print clearly (for computer text recognition)—

Net ID (don't include '@cornell.edu')

Your name:

ANDY RUINA

**SOLUTION**  
Cornell  
ME 2030

Prelim 2

March 19, 2019

3 Problems, 90 minutes (+ up to 90 minutes extra)

No calculators, books or notes allowed.

## How to get the highest score?

Please do these things:

- ✦ Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation**.
- A+ Be (I) neat, (II) clear and (III) well organized.
- TIDILY REDUCE and **box in** your answers (Don't leave simplifiable algebraic expressions).
- >> Make appropriate Matlab code clear and correct.  
You can use shortcut notation like " $\phi_7 = 2\pi$ " instead of, say, "`phi(7) = 2*pi;`".  
Small syntax errors will have small penalties.
- ↗ Clearly **define** any needed dimensions ( $\ell, h, d, \dots$ ), coordinates ( $x, y, r, \theta \dots$ ), variables ( $v, m, t, \dots$ ), base vectors ( $\hat{i}, \hat{j}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \dots$ ) and signs ( $\pm$ ) with sketches, equations or words.
- **Justify** your results so a grader can distinguish an informed answer from a guess. If you quote a fact that a grader might doubt your understanding of, explain it. Especially if it is not commonly used.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem). If you are referred here during the exam it means that answering your question would be telling you something you are being tested on.
- ≈ Work for **partial credit** (from 50%–100%, depending on the problem)
  - Put your answer in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** The last page is blank for you. Ask for more if you need it. Put your name on each sheet, and fold it in. Refer to extra pages (We have to find them on a computer scan.)

Problem 1:        /25

Problem 2:        /25

Problem 3:        /25

1) Towed trailer. 2D. There is gravity. A rope pulls with force  $F$  on the top of a trailer with mass  $m$ , length  $2\ell$ , and height  $h$ . The center of mass  $G$  is in the middle. There are good wheels under the center of mass at  $A$  and it skids with coefficient of friction  $\mu$  at  $B$ . The car is already moving to the right when  $F$  is applied. Positive acceleration  $a$  is to the right. Note whether applying  $F > 0$  speeds up or slows down the trailer compared to  $F = 0$ .

a) Find the acceleration of the block. If relevant, assume that  $\mu < \ell/h$ . Answer in terms of some or all of  $F, m, h, \ell, g$  and  $\mu$ .

b) Challenges: Find the acceleration of the block, and/or explain what is funny or interesting for various larger values of  $\mu$ , for example these three cases:

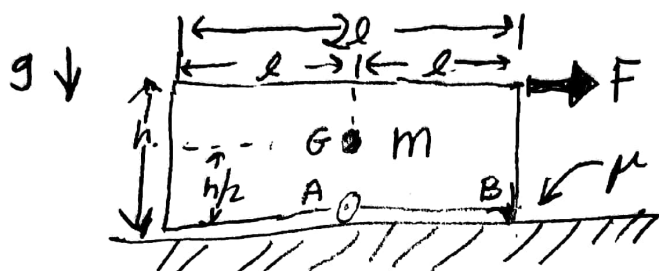
i)  $\mu = \ell/h$ ;

ii)  $\ell/h < \mu < 2\ell/h$ ;

iii)  $2\ell/h < \mu$ ;

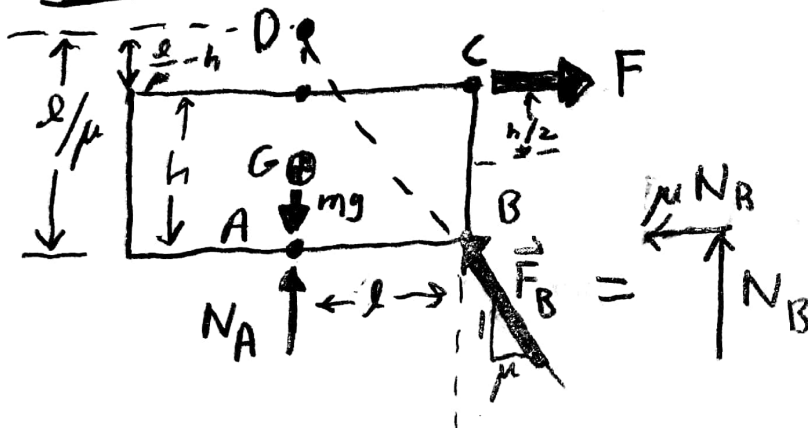
$$\vec{a}_G = a_G \hat{i}$$

ASSUME NO ROTATION



$v > 0$   
 $a = ?$

FBD



$$\text{AMB/D: } \left\{ \sum \vec{M}_{/D} = \vec{r}_{G/D} \times m \vec{a}_G \right\} \cdot \hat{k}$$

$$\Rightarrow F \cdot (\ell/\mu - h) = (\ell/\mu - h/2) m a_G$$

$$(a) \quad a_G = \frac{F}{m} \frac{(\ell/\mu - h)}{(\ell/\mu - h/2)}$$

for  $\ell/\mu > 0$  this is +.  $F$  changes  $a_G$  from 0 to greater than 0.

bi)  $\mu = \ell/h \Rightarrow$   
 $a_G = 0$   
Force has no effect  
on accel.

bii)  $\mu > \ell/h$   
 $\Rightarrow a_G < 0$

Pulling to right slows trailer down!!

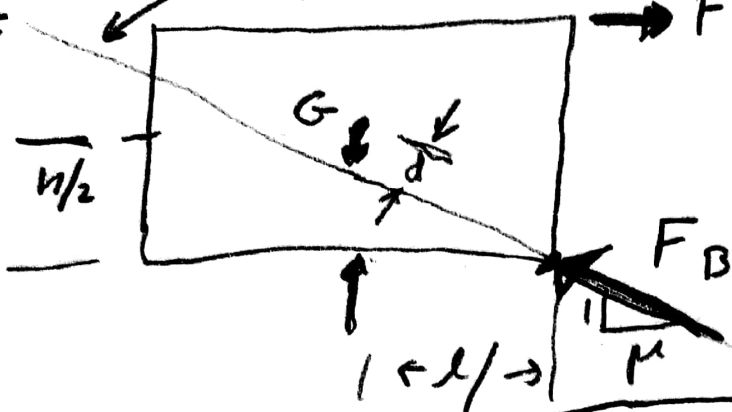
biii)  $\mu > 2\ell/h$   
changes sign again?  
Too Weird  $\Rightarrow$

biii) cont'd

$$\mu > 2l/h$$

passes below G

FBD



AMB/G

$$+ Fh/2 - dF_B = 0$$

$$\Rightarrow F_B < 0$$

Nonsense!

Summary

$(F=0 \Rightarrow \text{no accel.})$

$$\mu < l/h$$

F causes + a

$$\frac{l}{h} < \mu < 2l/h$$

F causes negative a  
 $\Rightarrow$  pulling causes slowing

$$\mu = 2l/h$$

stopping accel  $\rightarrow \infty$

$$\mu > 2l/h$$

NONSENSE

$\Rightarrow$  problem is nonsense  
What would happen?

ANS: Tipping

## Prelim 2, Q1: Post-Mission Briefing #303



Hey all! I graded Q1 of prelim 2 (the towed trailer problem), and I have returned to explain how I scored the problem and to share some common mistakes that students made while attempting it.

### Scoring

Part (a) was scored out of 15 points and each part of (b) was worth 4, for a total of 27 points possible if you did the entire problem perfectly. In general, people did very well on this problem; the median was 19, and over 40% of the class got a 23 or higher.

I was incredibly generous with partial credit for (b). This was because if I weren't, and a mistake in (a) meant an automatic 0 for part (b), then a small algebraic mistake or sign error in part (a) could cost you up to 16 points, which is ridiculous. As a result, I awarded some credit for a reasonable interpretation of your answers, even if your answer to part (a) was wrong. Even so, small mistakes in part (a) were more punishing than usual, because you not only didn't get the acceleration formula correct, but it was also impossible to get full marks for (b).

### Common Mistakes

- Angular Momentum Balance:** the most common mistake, by far, was to set the sum of the moments about a point equal to 0 when you shouldn't have. In 2D, the formula for AMB is this:  $\sum \vec{M}_{/C} = \vec{r}_{G/C} \times m\vec{a}_G + I^G \alpha \hat{k}$  where G is the center of mass and C can be any point of your choice. In this problem,  $\alpha$  (angular acceleration) is 0, but you still have to deal with the rest of the right hand side. If you choose  $C = G$ , or if you chose C to be another point on the horizontal line containing G, then the right hand side simplifies to 0, but if you chose C to be A or B, then it does not.
- Cross Products:** don't get lazy with your cross products! Many of you attempted to "eyeball it" without writing out the unit vectors and got a sign error for your trouble. Because of the sequential nature of this problem - you needed to do (a) perfectly to get full credit on (b) - you paid a heavy price for your mistake, probably on the order of 5-7 points.
- Counter-Clockwise is Positive:** additionally, it is worth noting that  $\hat{i} \times \hat{j}$  equals positive  $\hat{k}$ ; therefore, in this problem, where positive  $\hat{i}$  points to the right and positive  $\hat{j}$  points up, a positive moment ( $\hat{k}$ ) must be counter-clockwise, and a clockwise moment will be negative ( $-\hat{k}$ ). Some of you defined clockwise moments as positive and counter-clockwise as negative. I didn't take any points off this time because the equations you wrote were technically still correct (if you just flip the signs on every term, the equation doesn't change), but in the future, especially in 3D problems, you might end up writing an equation that isn't true if you make that kind of assumption.

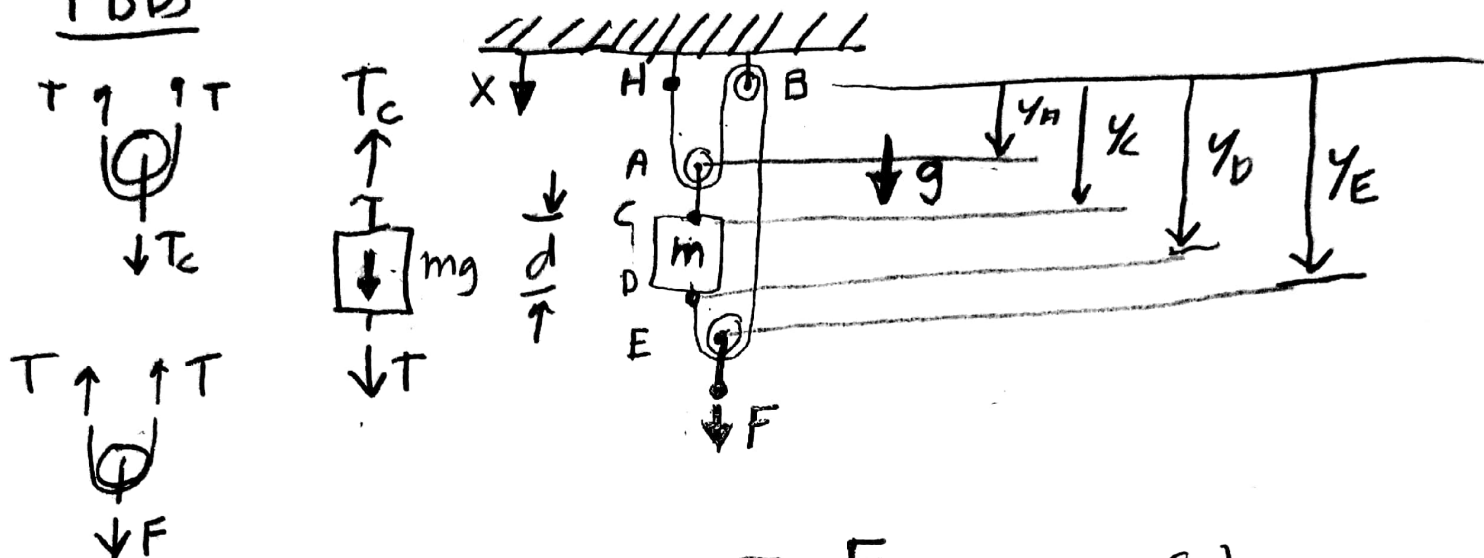
**2) Pulleys.** 1D. There is gravity. All parts have negligible mass and weight except  $m$ . Make the usual assumptions about pulleys and ropes.

**a) Find  $F$  for this system to be in static equilibrium?**

Radii  $r_1 = R$

b) For other values of  $F > 0$ , find  $a_F$ .

FBDs



statics:  $T = F/2$  &  $T_c = 2T = F$

a) Equilib :  $T + mg - T_c = 0 \Rightarrow$   
 $\uparrow_{F/2} \quad \quad \quad \uparrow_F$

(a)

$$F = 2mg$$

↑ equilibrium

b) Kinematics

$$\underbrace{\text{Kinematics}}_{\left\{ 2 Y_A + (\pi R) \cdot 3 + Y_E + Y_E - Y_D = 0 \right\}} \quad (1)$$

$$y_p - y_c = d = \text{const}, \quad y_c - y_A = \frac{\text{const}}{\dots}$$

$$y_D - y_C = d = \text{const}, \quad y_C - y_A = \text{const}$$

$$\frac{d^2}{dt^2} \{ \} \Rightarrow 2\ddot{y}_C + 2\ddot{y}_E - \ddot{y}_C = 0 \Rightarrow \boxed{\ddot{y}_C = -2\ddot{y}_E}^*$$

$$L = \ddot{y}_D$$

LMB for mcs)

$$\sum F_y = m \ddot{y}$$

$$mg + \underset{\substack{\uparrow \\ F/2}}{T} - T_c = m \ddot{y}_c \Rightarrow$$

$$\ddot{y}_c = g - F/2m$$

$$* \Rightarrow \dot{y}_E' = -\frac{\ddot{y}_c}{2}$$

$$a_E = \ddot{y}_E = -\frac{g}{2} + F/4m \quad (b)$$

Same as lecture  
rotated 90°, gravity added.

check against part (a)

$$\underbrace{F = 2mg}_{\text{equilib soln}} \Rightarrow a_E = -\frac{g}{2} + \frac{g}{2} = 0$$



## Prelim 2 Q2: Summary #308



The whole class did really well on question 2. Everybody drew their free body diagrams. One third of the class got the problem fully correct. The mean is 20.49; the median is 23.0; the standard deviation is 4.86. The grading rubric can be found on Gradescope. There are only a few things to pay attention to:

1. **Consistency of coordinate systems.** Make sure your coordinate system is consistent throughout the problem. A lot of students took positive  $x$  as pointing downward when they wrote their string length equation, but flipped the  $x$  coordinate when they wrote their linear momentum balance equation (LMB) and ended up with an inverted sign error. I deducted 2 points for each flip of the coordinate system. Remember:  $\ddot{x} = \vec{a}$ , so your positive  $x$ , positive  $\ddot{x}$ , and positive  $a$  should all point in the same direction.
2. **Motion of massless objects.** Since the mass of the pulley is negligible, you can't use linear momentum balance (LMB, i.e.  $\vec{F} = m\vec{a}$ ) on it. To treat this kind of problems, you need to relate the motion of the massless object directly to the motion of a mass by considering the linearity between them. In this problem, we get  $\ddot{x}_A = \ddot{x}_m$  because pulley A is rigidly connected to the mass and  $\ddot{x}_E = -\ddot{x}_m/2$  by taking the second derivative of the constant string length equation.



3) **Three masses in space. 2D.** Assume consistent units. The only concern is one instant in time.

- All three masses are mutually attracted to each other with inverse-square gravity  $G$ .
- A linear negligible-mass dashpot  $c$  connects  $m_1$  and  $m_2$ .
- A given external force  $\vec{F}_3^{ext}$  acts on mass 3.
- There are no other forces.
- **Short problem statement:** Write matlab commands to find the acceleration  $\vec{a}_1$  of  $m_1$ .

**Given.** For the instant of interest, assume that these things have been defined (*i.e.*, assigned numerical values):

- **The system state**, the positions and velocities of the three masses (each one is a 2-element MATLAB column vector):  $r_1, r_2, r_3, v_1, v_2$ , &  $v_3$ , relative to an inertial fixed origin;
- **The three masses:**  $m_1, m_2$ , &  $m_3$ ;
- **Other parameters:** The universal gravity constant  $G$ ; the constant  $c$  of the one dashpot; and the two-component applied force as a column vector  $F_{3\_ext}$ .

Your solution can use any or all of these.

**Full problem statement:** Find the components of  $\vec{a}_1$  by filling in this MATLAB function:

```
function [a1] = my_three_body(m1,m2,m3,r1,r2,r3,v1,v2,v3,F3_ext,G,c)
...
end
```

(Note: the accelerations  $\vec{a}_2$  and  $\vec{a}_3$  are of no concern.)

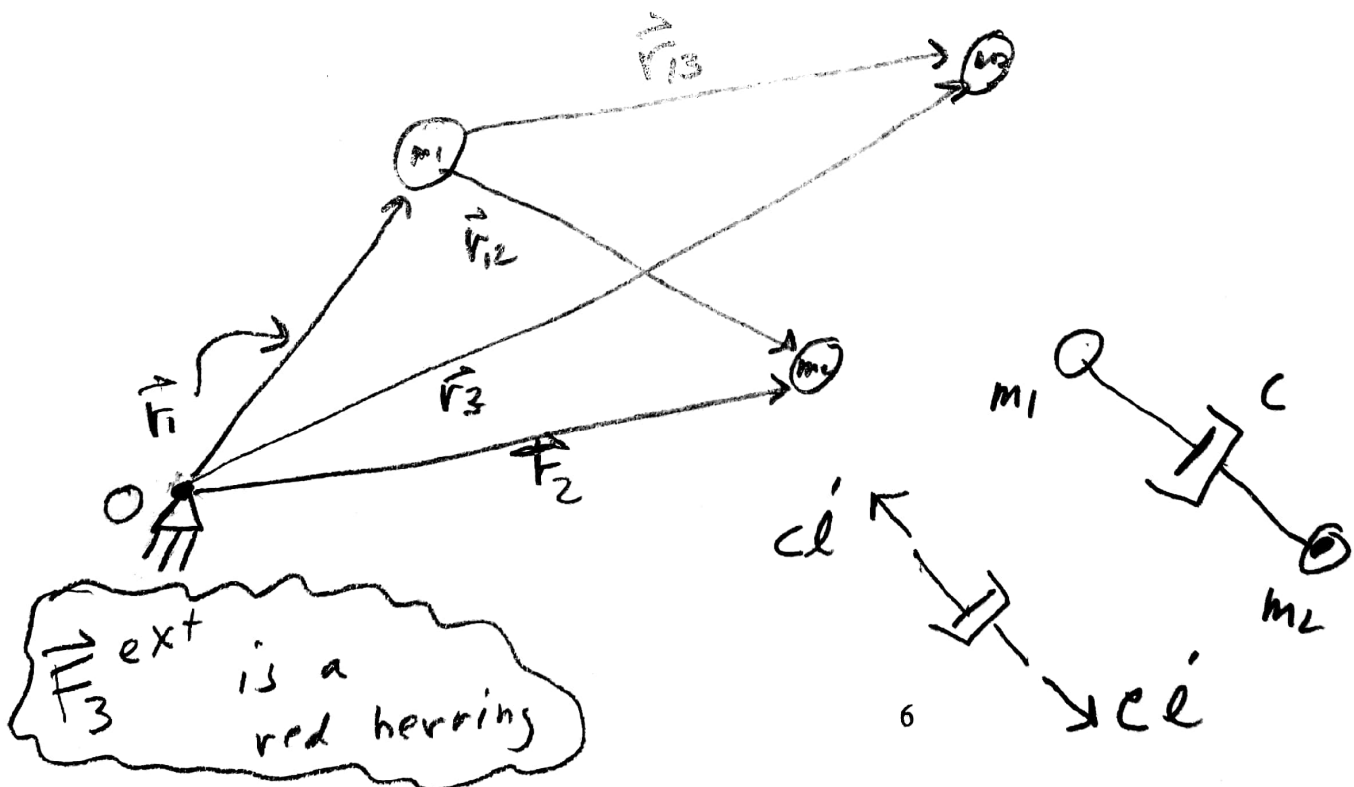
**Problem restatement.** Given all of the parameters listed, write MATLAB commands to find the acceleration  $a_1$  of mass 1. The last line of your code should start with:

$a_1 =$  ,

where  $a_1$  is a 2 element MATLAB column vector.

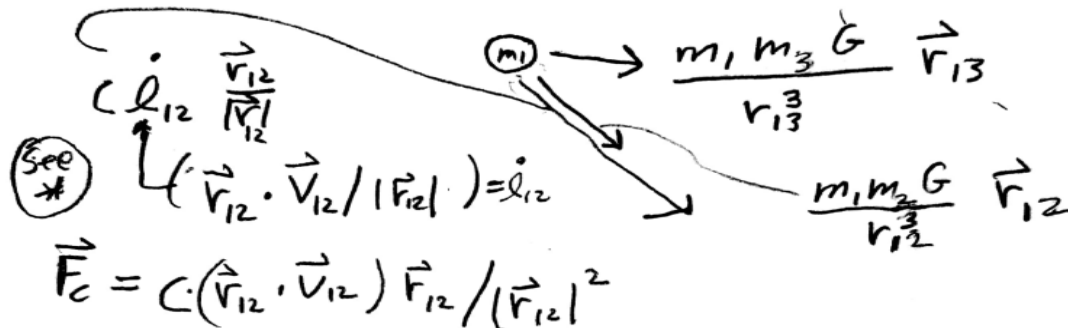
#### Clarifications:

- \* Write easy-to-understand short lines of code; don't write long complex lines of code.
- \* Don't make any explicit use of vector components; only partial credit if you refer to, say,  $r_1(1)$  or  $r_1(i)$ .
- \* Variables on the right side of  $=$  signs must be given (above), or previously defined by you.
- \* Partial credit for missing, incomplete or incorrect code will be based on: sketches, geometry, free body diagrams, math (*e.g.*, eqns using  $\vec{a}_1, \vec{r}_3, \dots$ ), *etc.*





## FBD of mass 1



```
function [a1] = my_three_body(m1,m2,m3,r1,r2,r3,v1,v2,v3,F3_ext,G,c)

    r_13 = r3 - r1; % position from 1 to 3
    r_12 = r2 - r1; % position from 1 to 2

    v_12 = v2 - v1; % velocity of 2 wrt 1

    dist_13 = norm(r_13); % distance from 1 to 3
    dist_12 = norm(r_12); % distance from 1 to 2

    unit_r_12 = r_12 / dist_12; % unit vector pointing from 1 to 2
    unit_r_13 = r_13 / dist_13; % unit vector pointing from 1 to 3

    dot_dist_12 = dot(unit_r_12, v_12); % change in distance from 1 to 2

    mag_f_grav_12 = m1 * m2 * G / (dist_12^2); % gravitational force magnitude between 1 and 2
    mag_f_grav_13 = m1 * m3 * G / (dist_13^2); % gravitational force magnitude between 1 and 3
    mag_f_c_12 = c * dot_dist_12; % damping force magnitude between 1 and 2

    f_grav_12 = mag_f_grav_12 * unit_r_12; % gravitational force vector between 1 and 2
    f_grav_13 = mag_f_grav_13 * unit_r_13; % gravitational force vector between 1 and 3
    f_c_12 = mag_f_c_12 * unit_r_12; % damping force vector between 1 and 2

    a1 = (1 / m1) * (f_grav_12 + f_grav_13 + f_c_12); % Newton's 2nd Law
end
```

\*Note:  $\{l_{12}^2 = \vec{r}_{12} \cdot \vec{r}_{12}\}$  (as per lecture)

$$\frac{d}{dt} \{ \} \Rightarrow 2 l_{12} \dot{l}_{12} = 2 \vec{r}_{12} \cdot \dot{\vec{r}}_{12}$$

$$\dot{l}_{12} = \frac{\vec{r}_{12} \cdot \dot{\vec{r}}_{12}}{l_{12}} = \frac{\vec{r}_{12} \cdot \vec{v}_{12}}{|\vec{r}_{12}|}$$

### I. FINAL THOUGHTS: PROBLEM 3

This problem was graded on a two-tiered grading system. The first tier is for students who made small number of mistakes in their code. The second tier is for students who wrote problematic code or no code at all. In these cases, I carefully looked at the free body diagram and equations of motion to assess understanding on a case by case basis.

#### A. If you scored 16 or more points:

Most students lost 4 points for incorrectly calculating the damping force. The damper opposes the *change in length* between masses 1 and 2. As covered in lecture, there are multiple ways of deriving this change in length. One way is:

$$l^2 = r_{12} \cdot r_{12} \quad (1)$$

$$2l\dot{l} = 2r_{12} \cdot \dot{r}_{12} \quad (2)$$

$$\dot{l} = \frac{r_{12} \cdot \dot{r}_{12}}{l} \quad (3)$$

The damping force magnitude is  $c\dot{l}$  and the damping force vector is  $c\dot{l}r_{12}$ .

About half of students lost 4 points for calculating unit vectors incorrectly in MATLAB. The MATLAB command norm does  $\sqrt{x \cdot x}$  or  $\sqrt{x^T x}$ . In MATLAB one calculates a unit vector by doing  $x/\text{norm}(x)$ . This was the only common MATLAB related issue.

#### B. If you scored 15 or fewer points:

If you got 15 points it means you got the equations right, but paid a price for your principled stand against MATLAB. If you got below 10-12 points your work likely indicated that you are confused between scalars and vectors. For the purposes of this class, being confused between scalars and vectors in an emergency. If the words scalar/vector are written on your test, or if you think this might be the case, I implore you to come talk to Dr. Ruina, Ryan Elandt, Duan Li or Walker Lee.