"Solutions"

Your Name: ANDY RUINA

Section day and time: ______________

T&AM 203 Final exam

Tuesday May 14, 2002

Draft May 14, 2002

5 problems, 100 points, and 150 minutes (no extra).

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed. A blank page for tentative scrap work is provided at the back. Ask for extra scrap paper if you need it. If you want to hand in extra sheets, put your name on each sheet and refer to that sheet in the problem book for the relevant problem(s).

b) Full credit if

- →free body diagrams← are drawn whenever force, moment, linear momentum, or angular momentum balance is used;
- correct vector notation is used, when appropriate;
- ↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;
- ± all signs and directions are well defined with sketches and/or words;
- ⊣ reasonable justification, enough to distinguish an informed answer from a guess, is given;
- you clearly state any reasonable assumptions if a problem seems *poorly defined*;
- work is I.) neat,
  II.) clear, and
  III.) well organized;
- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);
- □ your answers are [boxed] in; and
- ≫ Matlab code, if asked for, is clear and correct. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot{\theta}_7$ = 18" instead of, say, "`theta7dot = 18`". You will be penalized, but not heavily, for minor syntax errors.

c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

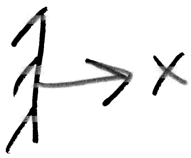
Problem 1:	/15
Problem 2:	/20
Problem 3:	/25
Problem 4:	/20
Problem 5:	/20
TOTAL:	**/100**

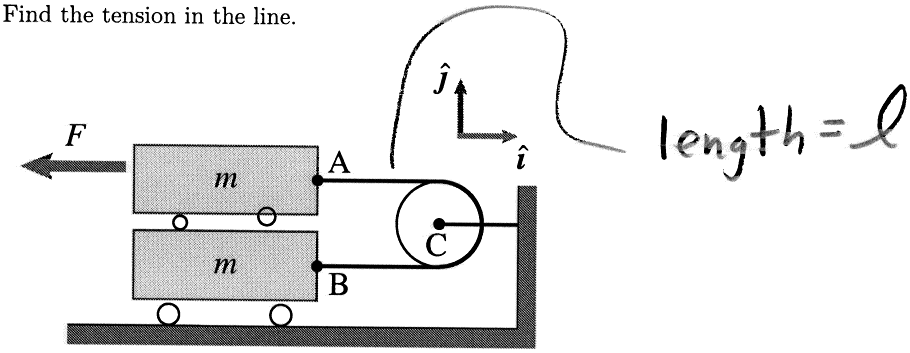
1) (15 pt) Two equal masses are stacked an tied together by the pully as shown. All bearings are frictionless. All rotating parts have negligible mass. The line is inextensible.

0) (~~10~~ 5 pt) for basic setup diagrams, assumptions, and equations needed to answer the questions below.

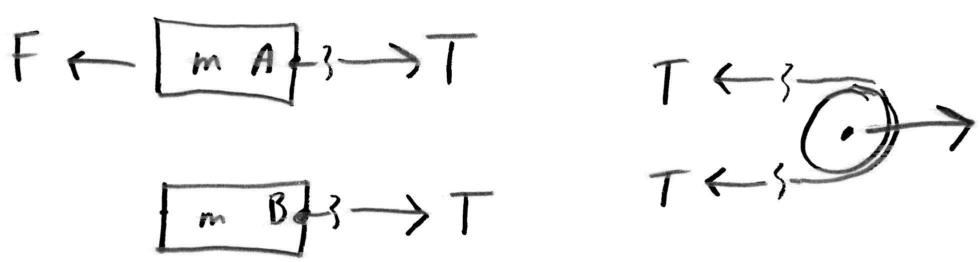
a) (5 pt) Find the acceleration of point A.

b) (5 pt) Find the tension in the line.





FBDs:



$\{LMB \text{ for } A\} \cdot \hat{i} \Rightarrow \quad -F + T = m\ddot{x}_A$ ①

$\{LMB \text{ for } B\} \cdot \hat{i} \Rightarrow \quad T = m\ddot{x}_B$ ②

Kimematics:

$\ell = \text{const.} \Rightarrow \quad \ddot{x}_A = -\ddot{x}_B$ ③

① − ② ⇒ $-F = m\ddot{x}_A - \ddot{x}_B$

③ ⇒ $= 2m\ddot{x}_A$

$\Rightarrow \boxed{\underline{a}_A = -\frac{F}{2m}\hat{i}}$ (a)

① + ② ⇒ $-F + 2T = m(\ddot{x}_A + \ddot{x}_B)$

③ ⇒ $= 0$

$\Rightarrow \boxed{T = F/2}$ (b)

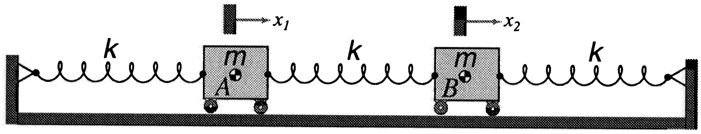
2) (20 pt) Two identical masses ($m = 2\,\text{kg}$) move in a straight line without friction. Three identical springs ($k = 7\,\text{N/kg}$) hold them in place (one between the left mass and a wall, one between the two masses, and one between the right mass and the wall). When the horizontal displacements x_1 and x_2 of the masses are zero all three springs are relaxed.

The system is released from rest at $t = 0$ with $x_1(0) = 0.3\,\text{m}$ and $x_2(0) = -.3\,\text{m}$.

a) (15 points) Write Matlab code where the final output will be the position of mass one at $t = 10\,\text{s}$. Your code should be general enough to handle arbitrary initial conditions. [Do not just use Matlab to evaluate the solution from (b) below.]

b) (5 points) Write a formula for the answer above. That is, evaluate an analytic solution of the resulting differential equations at $t = 10\,\text{s}$. [Hint: Using ideas from the lab makes this problem much easier than blindly grinding through the methods of Math 293, 294].

$\hat{j}$, $\hat{i}$



$x_1 = x_A$

$x_2 = x_B$

FBDs

$$T_1 = kx_1,\quad T_2 = k(x_2 - x_1),\quad T_3 = -kx_3$$

$\{\text{LMB for } A\}\cdot\hat{i} \Rightarrow$

$$-T_1 + T_2 = m\ddot{x}_1$$

$$-kx_1 + k(x_2 - x_1) = m\ddot{x}_1$$

$$-2kx_1 + kx_2 = m\ddot{x}_1 \qquad (1)$$

$\{\text{LMB for } B\}\cdot\hat{i} \Rightarrow$

$$T_3 - T_2 = m\ddot{x}_B$$

$$-kx_2 - (k(x_2 - x_1)) = m\ddot{x}_2$$

$$kx_1 - 2kx_2 = m\ddot{x}_2 \qquad (2)$$

(1) & (2) $\Rightarrow$

$$m\begin{bmatrix}\ddot{x}_1\\ \ddot{x}_2\end{bmatrix} = k\begin{bmatrix}-2 & 1\\ 1 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Given ICs:

$x_1(0) = .3$
$x_2(0) = -.3$
} (using consistent mks units)

$v_1(0) = 0$
$v_2(0) = 0$
} defining $v_1 = \dot{x}_1$, $v_2 = \dot{x}_2$

(2 cont'd)

```
X10=.3; X20=-.3; V10=0; V20=0;
Z0=[X10 X20 V10 V20];
tspan=[0 10];
[t z]= ode23('twoblocks', tspan, Z0);
answer = z(end,1)        % note, no semicolon
```

} driver file

```
function zdot = twoblocks(t,z)
K=7; m=2;
pos=[z(1) z(2)]';
vel=[z(3) z(4)]';
posdot= vel;
veldot= K*[-2 1;
           1 -2]*pos/m;
zdot= [posdot' veldot'];
```

} twoblocks.m

By inspection a normal mode of this system has blocks moving equally and oppositely. Appling $x_2 = -x_1$ to ① $\Rightarrow$

$$m\ddot{x}_1 = -3k\,x_1 \Rightarrow x_1 = A\cos\sqrt{\tfrac{3k}{m}}\,t + B\sin\sqrt{\tfrac{3k}{m}}\,t$$

Init. cond. $\Rightarrow$ $A = .3$, $B=0$

So, in consistent units, at $t=10$

$$\boxed{x_1 = A\cos\sqrt{\tfrac{3k}{m}}\,t = .3\cos\left(10\sqrt{\tfrac{21}{2}}\right)}$$ (b)

3) (25 pt) A suitcase on level ground with wheels in front and skids in back is pulled by its handle with a forward force F. The handle is directly above the front wheels. In all cases you are given these quantities and can use them in your answer:

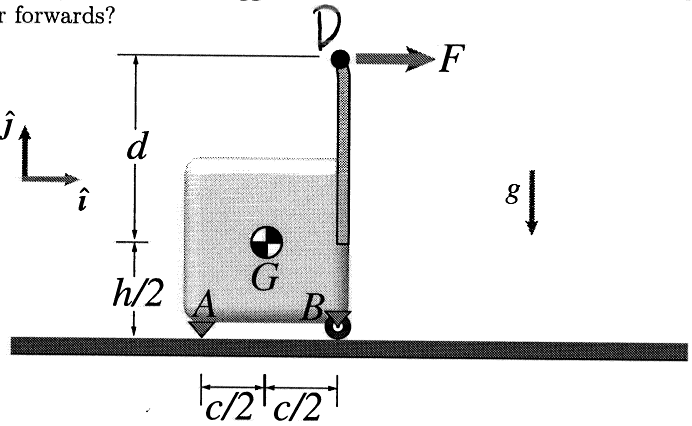
- $m =$ total mass of suitcase;
- $I^G =$ the polar moment of inertia about an axis through the center of mass G and in the ussual zz direction (perpendicular to the plane on which a side view of the suitcase is drawn);
- $h/2 =$ the height of G above the ground;
- $c/2 =$ the distance G is forward of the rear ground contact at A;
- $c/2 =$ the distance G is behind the front ground contact at B;
- $d =$ the height of the handle at D above the center of mass of the suitcase;
- $\mu =$ the coefficient of friction between the rear skids and ground;
- $g =$ the gravity constant.

0) (10 points) for the basic diagrams, stated assumptions, and mechanics equations needed to solve the parts below.

a) (5 points) Given F and that F is big enough for non-zero forward acceleration, and that F is not so big as to cause the suitcase to tip, what is the acceleration of the suitcase?

b) (5 points) What is the smallest F that can make the suitcase move at all?

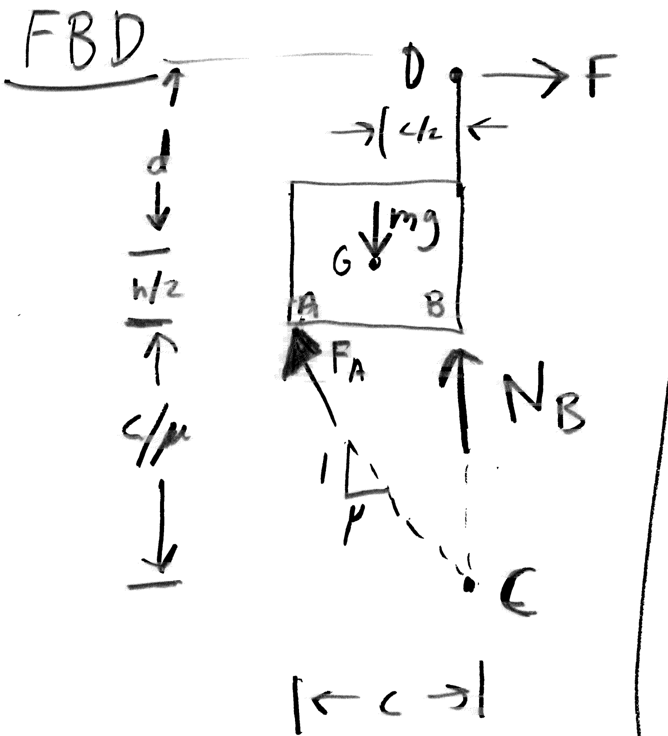
c) (5 points) What is the biggest acceleration that can be achieved without the suitcase tipping over forwards?



Note: in all cases $F = \mu N$

(slip is, or is about to, occur)

FBD



AMB/C

$\sum \underline{M}_{/C} = \dot{\underline{H}}_{/C}$

$\left\{ \frac{mgc}{2}\hat{k} - F\left(\frac{c}{\mu} + \frac{h}{2} + d\right)\hat{k} = \underline{r}_{G/C} \times m\underline{a}_G + I^G\dot{\omega}\hat{k} \right\}$

$\underline{a}_G = a\hat{i}$, $\dot{\omega} = 0$, $\underline{r}_{G/C} = \left(\frac{c}{\mu} + \frac{h}{2}\right)\hat{j} - \frac{c}{2}\hat{i}$

$\{\}\cdot\hat{k} \Rightarrow \frac{mgc}{2} - F\left(\frac{c}{\mu} + \frac{h}{2} + d\right) = -ma\left(\frac{c}{\mu} + \frac{h}{2}\right)$

(a)

$\Rightarrow a = \left[-\frac{mgc}{2} + F\left(\frac{c}{\mu} + \frac{h}{2} + d\right)\right] \Big/ m\left(\frac{c}{\mu} + \frac{h}{2}\right)$

$\underline{a} = a\hat{i}$

Sanity check: when $d = h = 0$ 3, Cont'd

$$\Rightarrow \quad a = \frac{\left(-mg\,c/2 + F\,\frac{c}{\mu}\right)}{m\,\frac{c}{\mu}}$$

$$a = F/m - \mu mg/2 \quad \text{(as expected, good)}$$

(b) One approach would be to do prob. again as a statics prob. Another is to set $\underline{a = 0}$ in soln. above.

$$\Rightarrow \quad \frac{mgc}{2} = F_{min}\left(\frac{c}{\mu} + \frac{h}{2} + d\right)$$

$$\boxed{F_{min} = \frac{mgc}{2\left(\frac{c}{\mu} + \frac{h}{2} + d\right)}} \quad \text{(b)}$$

Sanity check: when $d = h = 0$ $\Rightarrow F_{min} = \mu mg/2$ (as expected)

(c) In this case $F_A = 0$ (just barely)

$\underline{AMB/_D} \Rightarrow \quad \sum \underline{M}_{/D} = \dot{\underline{H}}_{/D}$

$$\left\{ \frac{mgc}{2}\hat{k} = \underline{r}_{G/D} \times m\,\underline{a}_G \right\}$$

where $\underline{r}_{G/D} = -d\hat{j} - \frac{c}{2}\hat{i}$ and $\underline{a}_G = a\hat{i}$

$$\{\ \}\cdot\hat{k} \Rightarrow \quad \frac{mgc}{2} = d\,m\,a$$

$$a = \frac{gc}{2d}$$

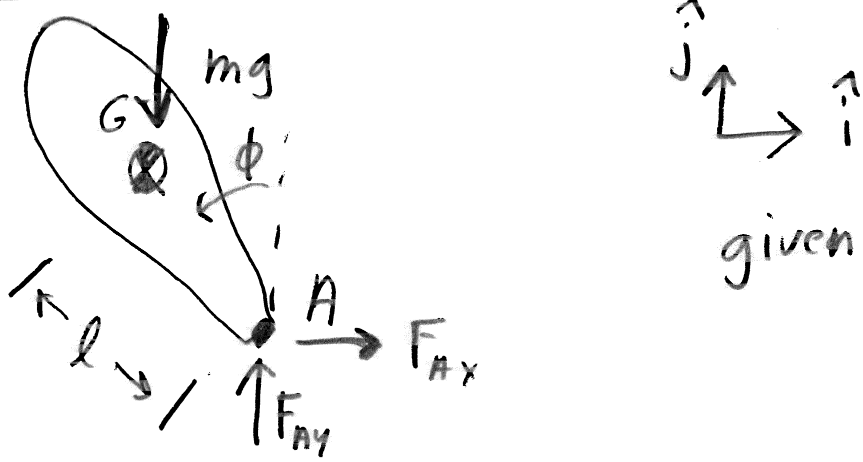
$$\boxed{\underline{a} = a\hat{i} = \frac{gc}{2d}\hat{i}} \quad \text{(c)}$$

4) (20 pt) An inverted pendulum is supported at one end A by a hinge that moves up and down and, at the instant of interest, has an upwards acceleration a. The pendulum mass is m and its moment of inertia about the center of mass G is I^G. G is a distance ℓ from the end at A. At the instant in question the pendulum is tipped counter-clockwise from the vertical an angle ϕ and is tipping at the rate $\dot{\phi}$. Gravity g is pointing down.

Find $\ddot{\phi}$ in terms of some or all of a, ℓ, m, I^G, g, ϕ, and $\dot{\phi}$.

[Hint: you can check to see if your answer reduces to something you know well when $a = 0$ and $I^G = 0$. Another check is to set $a = -g$.]

FBD



given $\underline{a}_A = a\hat{j}$

$\underline{\omega} = \dot{\phi}\hat{k}$

AMB/A $\Rightarrow$ $\sum \underline{M}_{/A} = \dot{\underline{H}}_{/A}$

$\underline{r}_{G/A} \times (-mg\hat{j}) = \underline{r}_{G/A} \times (m\underline{a}_G) + I^G \ddot{\phi}\hat{k}$

$\underline{r}_{G/A} = \ell(\cos\phi\hat{j} - \sin\phi\hat{i})$

$\underline{a}_G = \underline{a}_A + \underline{a}_{G/A}$

$= a\hat{j} + \dot{\underline{\omega}} \times \underline{r}_{G/A} - \omega^2 \underline{r}_{G/A}$

$\dot{\underline{\omega}} = \ddot{\phi}\hat{k}$

$\omega^2 \underline{r}_{G/A}$ term: will drop out in the end ($\underline{r}_{G/A} \times \underline{r}_{G/A} = \underline{0}$)

$\Rightarrow \{ mg\ell\sin\phi\hat{k} = \ell(\cos\phi\hat{j} - \sin\phi\hat{i}) \times$

$m[a\hat{j} + \ddot{\phi}\ell(-\cos\phi\hat{i} - \sin\phi\hat{j}) - \omega^2 \underline{r}_{G/A}] + I^G\ddot{\phi}\hat{k}$

$[\cos^2 + \sin^2 = 1] \Rightarrow$ $= -m\ell a\sin\phi\hat{k} + m\ell^2\ddot{\phi}\hat{k} + I^G\ddot{\phi}\hat{k} \}$

$\{ \} \cdot \hat{k} \Rightarrow m(g+a)\ell\sin\phi = (I^G + m\ell^2)\ddot{\phi}$

$$\Rightarrow \boxed{\ddot{\phi} = \frac{(g+a)m\ell\sin\phi}{I^G + m\ell^2}}$$

checks: 1) $a = 0$, $I^G = 0$
$\Rightarrow \ddot{\phi} = \frac{g}{\ell}\sin\phi$ (simple inverted pendulum)

2) $a = -g$
$\Rightarrow \ddot{\phi} = 0$ (pendulum in falling elevator. Like outer space)

5) (20 pt) A "centripital gun" consists of a rod hinged at one end at A and a frictionless collar that slides on the rod at the moving position C. The gun is powerred by the applied torque T. Neglect gravity. At the instant of interest you are given

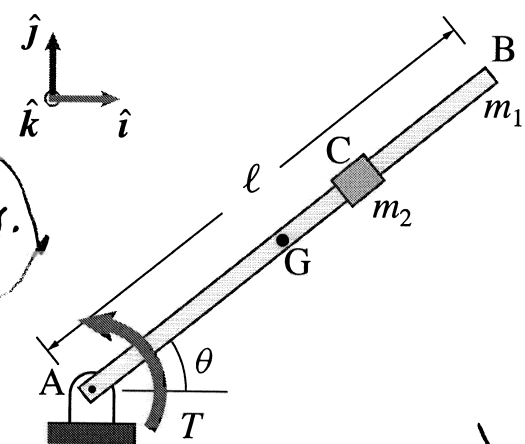
$T =$ the applied torque (counterclockwise is positive);
$m_1 =$ mass of the rod;
$m_2 =$ mass of the colar;
$I^G = m_1\ell^2/12 =$ the polar moment of inertia of the rod about an axis through the center of mass G and in the ussual zz direction (perpendicular to the plane on which a side view of the suitcase is drawn);
$\ell/2 =$ the distance from A to G (from end of rod to COM);
$R = R(t) =$ the distance from A to C (the radius of the collar);
$\dot{R} = \dot{R}(t) =$ the rate of change of distance from O with time;
$\theta = \theta(t) =$ the counterclockwise angle of the rod relative to a fixed $+x$ axis.
$\dot{\theta} = \dot{\theta}(t) = \frac{d}{dt}\theta$;

Find $\ddot{R}$ in terms of some or all of $T, R, \dot{R}, \theta, \dot{\theta}, m_1, m_2, \ell$, and I^G .

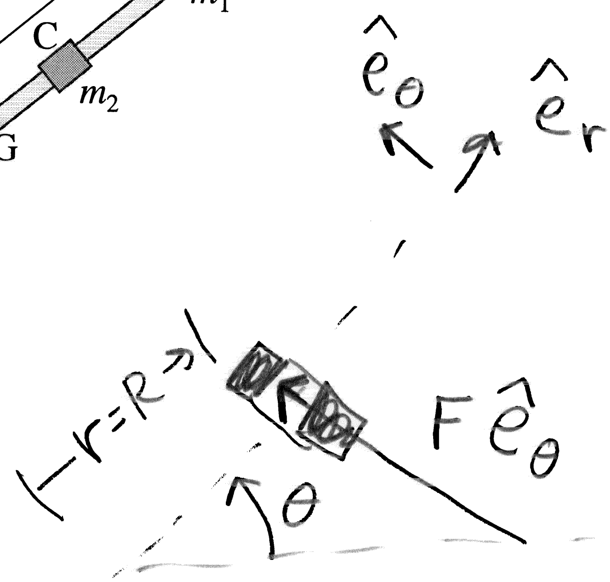
[Simplify your answer until it looks simple.]

Can also draw FBD of rod & look at its dynamics, but no need.

There are longer solutions. Here's a quick one.



FBD of collar!



LMB for collar

$$\sum \underline{F} = m\underline{a}$$

$$\left\{ F\hat{e}_\theta = m\left[(\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta\right]\right\}$$

$$\{\ \}\cdot\hat{e}_r \Rightarrow \quad \ddot{r} - r\dot{\theta}^2 = 0$$

$$\boxed{\ddot{r} = r\dot{\theta}^2}$$

That's it!

Note: T has no effect! but to make $\dot{\theta}$ bigger for later on.