"SOLUTIONS"

Your Name: ___Ruina___

TA's name, Section # and Section time: _____

# T&AM 203    Prelim 1
## Tuesday February 23, 2009
Draft February 24, 2009

3 problems, $25^+$ points each, and $90^+$ minutes.

**Please follow these directions to ease grading and to maximize your score.**

a) No calculators, books or notes allowed. A blank page for tentative scrap work is provided at the back. Ask for extra scrap paper if you need it. If you want to hand in extra sheets, put your name on each sheet and refer to that sheet in this problem book at the location of the relevant problem.

b) Full credit if

- →free body diagrams← are drawn whenever force, moment, linear momentum, or angular momentum balance are used;
- correct vector notation is used, when appropriate;
- any dimensions, coordinates, variables and base vectors that you add are clearly defined;
- all signs and directions are well defined with sketches and/or words;
- reasonable justification, enough to distinguish an informed answer from a guess, is given;
- you clearly state any reasonable assumptions if a problem seems *poorly defined*;
- work is     I. ) neat,
              II. ) clear, and
              III.) well organized;

- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);
- □ your answers are boxed in; and
- ≫ Matlab code, if asked for, is clear and correct. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot{\theta}_7$ = 18" instead of, say, "theta7dot = 18". You will be penalized slightly for minor syntax errors.

c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve. Unless specifically stated otherwise, substantial partial credit if you provide Matlab code which would generate the desired answer.

Problem   1:   _25_/25

Problem   2:   _25_/25

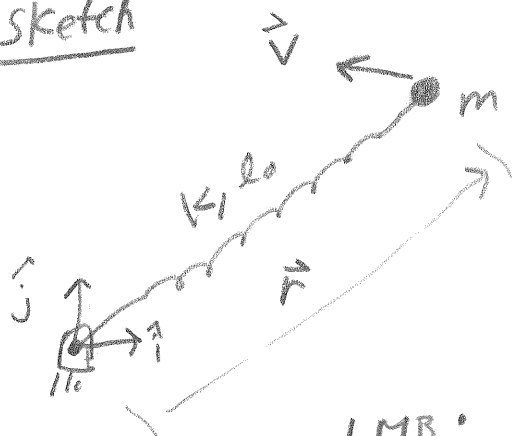Problem   3:   _25_/25

**1) (25 pt)** A mass $m$ moves on a frictionless plane. There are two forces on it

  i) $\vec{F}_s$ the force of a linear spring. One end of the spring is anchored at the origin, the other is attached to the mass. The spring has rest length $\ell_0$ and spring constant $k$.

  ii) $\vec{F}_d$ the force of a linear viscous drag. The drag force opposes the motion and is proportional, with constant $c$, to the speed.

  The Matlab code below is intended to give the differential equation to a Matlab solver like ODE45. It is incomplete. On the opposite page, provide the missing lines and box them in.
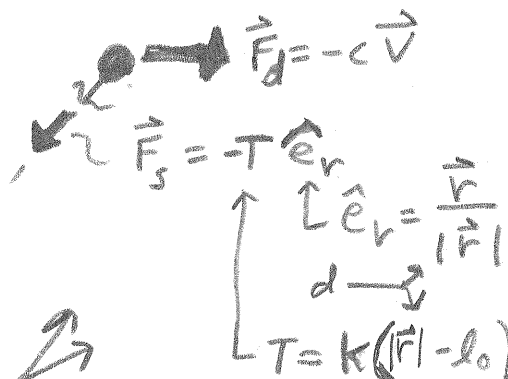
```
function zdot = rhs(t,z,m,k,Lo,c)
r = z(1:2); % two components of position
v = z(3:4); % two components of velocity
d = sqrt(r(1)^2 + r(2)^2); % radius, use if convenient
s = sqrt(v(1)^2 + v(2)^2); % speed, use if convenient
```

(*)→

```
% Missing lines of code here.  You can use vectors or scalars
% so long as the final code would run properly.
% Please define intermediate variables, like d and s above,
% to simplify the readability of your code.

zdot = [rdot;vdot]';
end
```

sketch

FBD

$$\vec{F}_d = -c\vec{v}$$
$$\vec{F}_s = -T\hat{e}_r$$
$$\hat{e}_r = \frac{\vec{r}}{|\vec{r}|}$$
$$d$$
$$T = k(|\vec{r}| - \ell_0)$$

LMB: $\sum \vec{F} = m\vec{a}$

$$\boxed{\vec{F}_s + \vec{F}_d = m\vec{v}}$$

$$\boxed{\begin{array}{l} eR = r/d; \quad \% \text{ unit vector in } \vec{r} \text{ dir.} \\[4pt] T = k*(d-Lo); \quad \% \text{ spring tension} \\[4pt] Fs = -T*eR; \quad \% \text{ spring force, 2 element vector} \\[4pt] Fd = -c*v; \quad \% \text{ drag force, 2 element vector} \\[4pt] vdot = (Fs + Fd)/m \quad \% \text{ the main ODEs} \\[4pt] rdot = v; \qquad\qquad \% \quad \dot{\vec{r}} = \vec{v} \text{ (easy)} \end{array}}$$ (*)

**2) (25 pt)** Four equal masses $m$ are in a line between two walls. There are 5 springs separating the masses from each other and the walls. All springs have constant $k$ but for the middle spring which has constant $2k$. Clearly describe one normal mode of vibration (with words and equations), giving both the shape and the frequency.
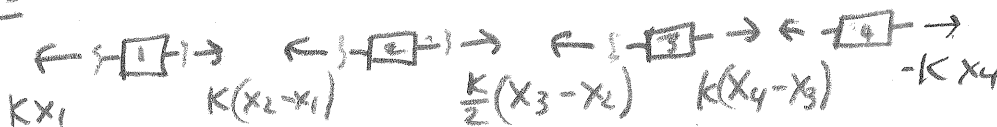
$$k_{eff} = k/2$$

$$\underset{k}{\overset{k}{\sim\!\!\sim}} \; \underset{}{\sim\!\!\sim} \; = \; \underset{k/2}{\sim\!\!\sim}$$
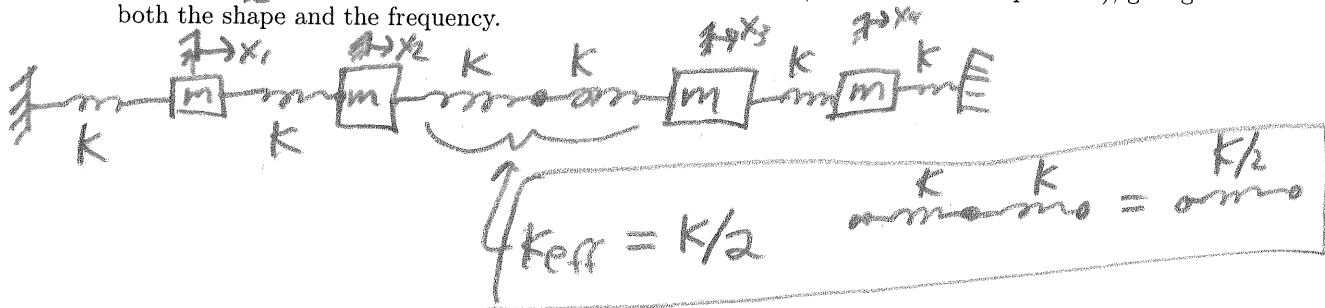
**By inspection:** if two masses on left go right by $\Delta$
and " " " " right " left " "
then each mass has restoring force $k\Delta$.

$$\Rightarrow \text{mode shape} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \quad \& \quad \lambda = \sqrt{k/m}.$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \left[ A\cos\left(\sqrt{\tfrac{k}{m}}\, t\right) + B\sin\left(\sqrt{\tfrac{k}{m}}\, t\right) \right] \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \quad -(*)$$

done.
(good enough)

**FBDs**

$$\leftarrow\!{\underset{kx_1}{\boxed{1}}}\!\rightarrow \quad \leftarrow\!{\underset{k(x_2-x_1)}{\boxed{2}}}\!\rightarrow \quad \leftarrow\!{\underset{\frac{k}{2}(x_3-x_2)}{\boxed{3}}}\!\rightarrow \quad \leftarrow\!{\underset{k(x_4-x_3)}{\boxed{4}}}\!\rightarrow \; -kx_4$$

To find all 4 modes one needs eigenvalues & eigenvectors. Too hard to solve 4th order polynomial on exam!

**LMB**

$$m\ddot{x}_1 = -2kx_1 + kx_2$$
$$m\ddot{x}_2 = kx_1 - \tfrac{3k}{2}x_2 + \tfrac{k}{2}x_3$$
$$m\ddot{x}_3 = \tfrac{k}{2}x_2 - \tfrac{3k}{2}x_3 + kx_4$$
$$m\ddot{x}_4 = kx_3 - kx_4$$

$$\Rightarrow m\begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \ddot{x}_4 \end{bmatrix} = -k\underbrace{\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 3/2 & -1/2 & 0 \\ 0 & -1/2 & 3/2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}}_{A}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (**)$$

**Note:** $*$ solves $**$ because

$$A\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

Alt. soln: $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$

$$\lambda = \sqrt{3k/m}, \quad A\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} = 3\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$
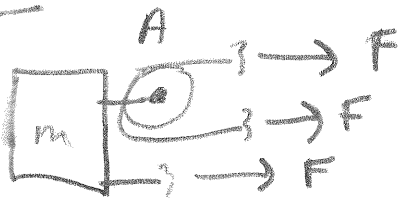
**3) (25 pt)** A mass $m$ is to be accelerated using pulleys and ropes and a force $F$. Design a pulley system (using as many pulleys as you like) so that the acceleration of the mass is $3F/m$. For this pulley system, what is the acceleration at the point where the force is applied?

Need rope tension on mass 3 times over:



(a)
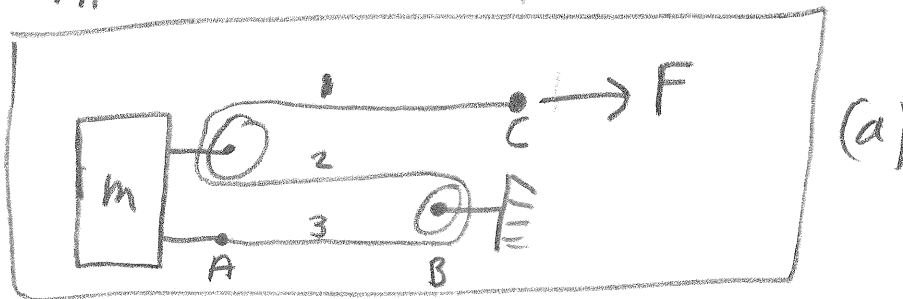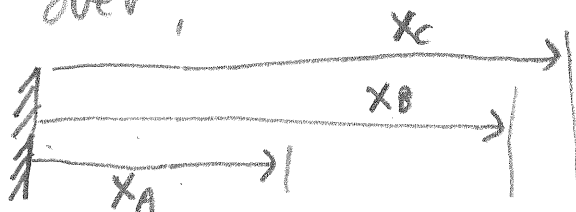
FBD



tension = const along rope

LMB:     $3F = ma_A$

$$\boxed{a_A = 3F/m}$$

L = String length = const = $(x_C - x_A) + 2(x_B - x_A) + 2(\pi R)$

$0 = \ddot{L} = \ddot{x}_C - \ddot{x}_A + 2\ddot{x}_B - 2\ddot{x}_A + 0$

$\ddot{x}_B \to 0$

$$\boxed{\ddot{x}_C = 3\ddot{x}_A = 9F/m}$$ (b)