"SOLUTIONS"

Your TA, Section # and Section time:

Your name: ANDY RUINA

Cornell TAM 2030

No calculators, books or notes allowed.

3 Problems, 90^+ minutes total.

Prelim 2

March 24, 2009

Directions. To ease your TA's grading and to maximize your score, please:

- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation**.
- ✓+ Be (I) neat, (II) clear and (III) well organized.
- □ TIDILY REDUCE and $\boxed{\text{box in}}$ your answers (Don't leave simplifiable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$\dot{\theta}_7$ = 18" instead of, say, "`theta7dot = 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \ldots$) and signs ($\pm$) with sketches, equations or words.
- →| **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems ***poorly defined***, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60–100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- Put your name on each extra sheet, fold it in, and refer to it at the relevant problem.
  Note the last page is **blank** for your use. Ask for more extra paper if you need it.

Problem 4: ______ /25

Problem 5: ______ /25

Problem 6: ______ /25

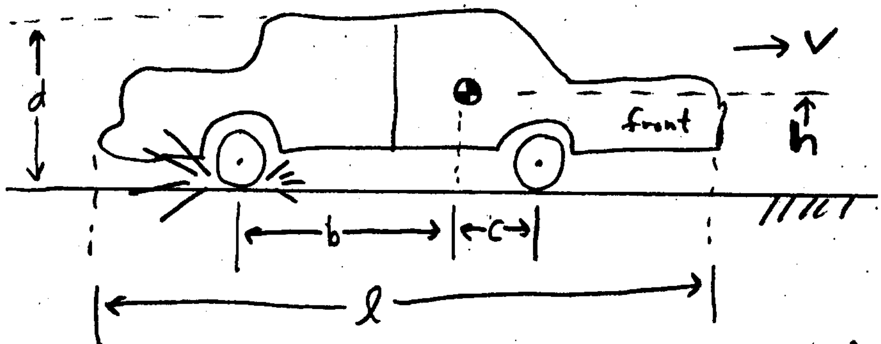
4) A car, moving to the right in the figure below, screeches to a stop, skidding the rear wheels (coefficient of friction $=\mu$, friction angle $= \phi$, with $\tan\phi = \mu$). The brakes are not applied to the light front wheels which roll easily.

What is the vertical force from the ground on the front wheels?

Answer in terms of some or all of the variables on this page. Extra credit for showing that your answer agrees with one or more special cases that you can evaluate more simply.

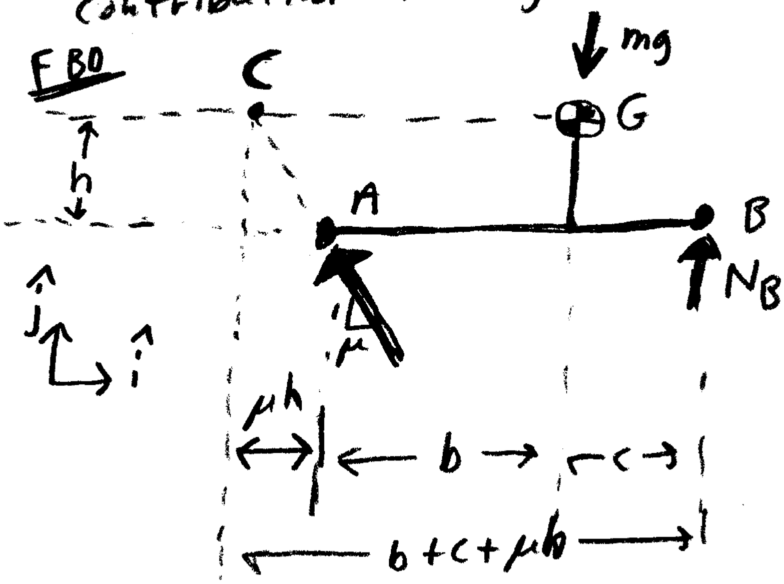
$\downarrow g$

m = mass
I^G = mom. of inertia about C.O.M



Assume car translation w/ no rotation,
" wheels & other moving parts have negligible contribution to Ang. mom.

FBD



$AMB_{/C}$

$$\sum \vec{M}_{/C} = \dot{\vec{H}}_{/C}$$

$$\vec{r}_{B/C} \times N_B\hat{j} + \vec{r}_{G/C} \times (-mg\hat{j}) = \vec{r}_{G/C} \times m\vec{a} + \overset{\vec{0}}{\overbrace{I\alpha\hat{k}}}$$

$(\mu h + b)\hat{i}$, $a\hat{i}$

$\Rightarrow$

$$[(\mu h + b + c)N_B - mg(\mu h + b)]\hat{k} = \vec{0}$$

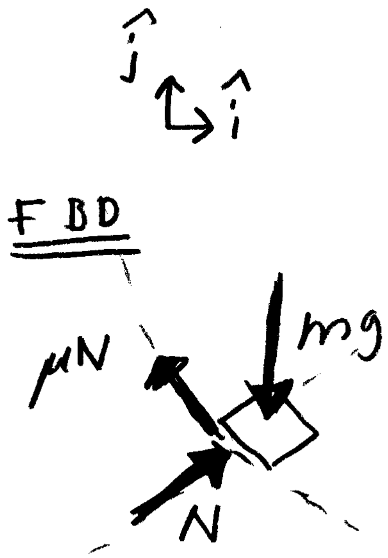
$$\Rightarrow \boxed{N_B = mg\frac{\mu h + b}{\mu h + b + c}} \; *$$

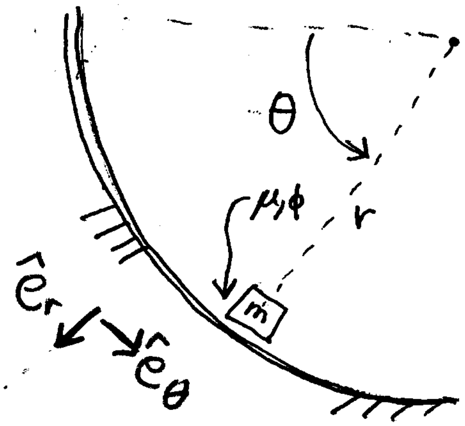
Special cases

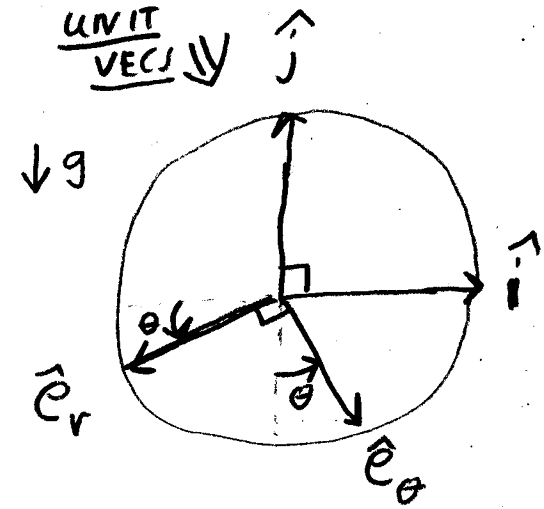
1) $\mu = 0 \Rightarrow$ statics $\Rightarrow N_B = mg\frac{b}{b+c}$ ✓
2) $h = 0 \Rightarrow$ no tipping from decelation $\Rightarrow$ same N_B as for statics ✓
3) $h = 0, b = 0 \Rightarrow$ No load on front wheel ever ✓
4) $h = 0, c = 0 \Rightarrow$ All load on front wheels $\Rightarrow N_B = mg$ ✓

5) A small block slides down a circular chute. You are given $\dot{\theta}$ and the other variables shown. *Find* $\ddot{\theta}$.

Extra credit for showing that your answer agrees with one or more special cases that you can evaluate more simply.







LMB

$$\Sigma \vec{F} = m\vec{a}$$

$\vec{a}$ for circular motion

$$\left\{ -mg\hat{j} - N\hat{e}_r - \mu N \hat{e}_\theta = m\left[r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r\right] \right\}^* \quad (1)$$

$$\{(1)\}\cdot\hat{e}_r \Rightarrow -mg\underbrace{\hat{j}\cdot\hat{e}_r}_{-\sin\theta} - N = -mr\dot{\theta}^2 \Rightarrow N = mr\dot{\theta}^2 + mg\sin\theta \quad (2)$$

$$\{(1)\}\cdot\hat{e}_\theta \Rightarrow -mg\underbrace{\hat{j}\cdot\hat{e}_\theta}_{-\cos\theta} - \mu N = mr\ddot{\theta} \quad (3)$$

N: $mr\dot{\theta}^2 + mg\sin\theta$ from (2)

Solve (3) for $\ddot{\theta}$ $\Rightarrow$ $\boxed{\ddot{\theta} = \frac{g}{r}\cos\theta - \frac{\mu}{r}\left[r\dot{\theta}^2 + g\sin\theta\right]}$

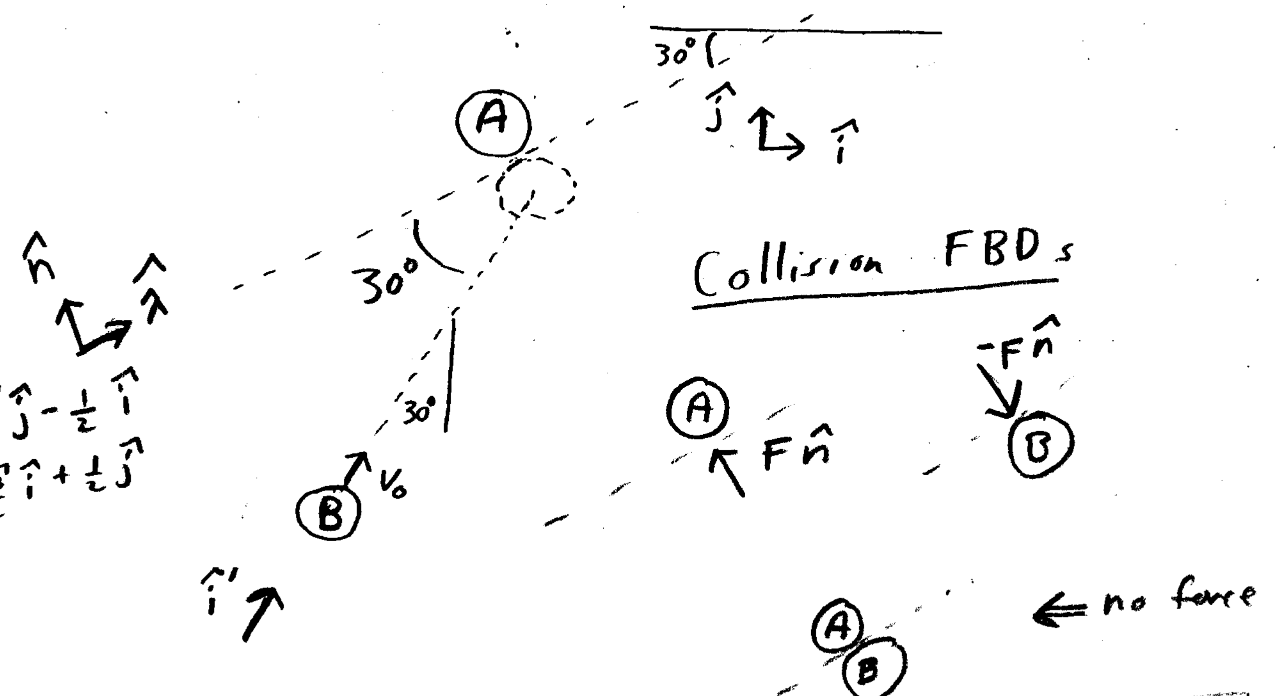
Special cases
1) $\mu = 0 \Rightarrow$ simple pendulum ($\cos\theta$ is like the usual $\sin\theta$) ✓
(only the expected centrip term) ✓
2) $g = 0 \Rightarrow \ddot{\theta} = -\mu\, r\dot{\theta}^2$ ($a_\theta = r\ddot{\theta}$ is like sliding on level) ✓
3) $\dot{\theta} = 0, \theta = \pi/2$, $\ddot{\theta} = \mu\, g/r$

*To solve with only one eqn. in one unknown: $\{(1)\}\cdot(\mu\hat{e}_r - \hat{e}_\theta)$
This "kills" N & μN.

6) Two balls on a plane have equal mass m. One is initially still and the other is moving at speed v_0 in the direction shown. They have a frictionless collision with coefficient of restitution $e = 1$.

Find the velocity (a vector) of either ball (your choice) after the collision.

Answer in terms of some or all of $m, v_0, \hat{\imath}, \hat{\jmath}$ and various numbers. [Note: $\sin 30° = 1/2$ and $\cos 30° = \sqrt{3}/2$.]



$\hat{n} = \frac{\sqrt{3}}{2}\hat{j} - \frac{1}{2}\hat{i}$

$\hat{\lambda} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

$\{\text{LMB block } A\} \cdot \hat{\lambda} \Rightarrow \cancel{m}\vec{V}_A^{+} \cdot \hat{\lambda} = \cancel{m}\underbrace{\vec{V}_A^{-}}_{=\vec{0}} \cdot \hat{\lambda} \Rightarrow \boxed{\vec{V}_A^{+} \cdot \hat{\lambda} = 0}$ ①

$\{\text{LMB block } B\} \cdot \hat{\lambda} \Rightarrow \cancel{m}\vec{V}_B^{+} \cdot \hat{\lambda} = \cancel{m}\underbrace{\vec{V}_B^{-}}_{V_0\hat{i}'} \cdot \hat{\lambda} \Rightarrow \vec{V}_B^{+} \cdot \hat{\lambda} = V_0 \cos 30°$

$\boxed{\vec{V}_B^{+} \cdot \hat{\lambda} = V_0\sqrt{3}/2}$ ②

$\{\text{LMB system}\} \cdot \hat{n} \Rightarrow \cancel{m}(\vec{V}_A^{+} + \vec{V}_B^{+}) \cdot \hat{n} = \cancel{m}(\vec{V}_A^{-} + \vec{V}_B^{-}) \cdot \hat{n}$

$\boxed{\vec{V}_A^{+} \cdot \hat{n} + \vec{V}_B^{+} \cdot \hat{n} = V_0/2}$ ③

$\{\text{Restitution eqn.}\} \quad (\vec{V}_A^{+} - \vec{V}_B^{+}) \cdot \hat{n} = -(\underbrace{\vec{V}_A^{-}}_{\vec{0}} - \vec{V}_B^{-}) \cdot \hat{n} \qquad (e=1)$

$\Rightarrow \boxed{\vec{V}_A^{+} \cdot \hat{n} - \vec{V}_B^{+} \cdot \hat{n} = V_0/2}$ ④

③ − ④ $\Rightarrow \boxed{\vec{V}_B^{+} \cdot \hat{n} = 0}$ ⑤

③ or ④ $\Rightarrow \boxed{\vec{V}_A^{+} \cdot \hat{n} = V_0/2}$ ⑥

$\hat{n}$ & $\hat{\lambda}$ comps of $\vec{V}_A^{+}$ & $\vec{V}_B^{+}$

①, ②, ⑤, ⑥ $\Rightarrow$ all the final velocities

For example:

$\vec{V}_A^{+} \cdot \hat{\lambda}$ is $\hat{\lambda}$ comp. of $\vec{V}_A^{+}$

$\Longrightarrow$ next page

6) (con't'd)

$$\vec{V}_B^{\,+} = \frac{\sqrt{3}}{2} V_0 \,\hat{\lambda}$$

$\hat{\lambda} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

No change of $\vec{V}$ in $\hat{\lambda}$ dir., Dead stop in $\hat{n}$ direction.

B: $$\boxed{\begin{aligned} \vec{V}_B^{\,+} &= \frac{\sqrt{3}}{2} V_0 \left[\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right] \\ \vec{V}_B^{\,+} &= V_0\left[\frac{3}{4}\hat{i} + \frac{\sqrt{3}}{4}\hat{j}\right] \end{aligned}}$$

$$\vec{V}_A^{\,+} = \frac{V_0}{2}\,\hat{n}$$

$\hat{n} = \frac{\sqrt{3}}{2}\hat{j} - \frac{1}{2}\hat{i}$

A: $$\boxed{\begin{aligned} \vec{V}_A^{\,+} &= \frac{V_0}{2}\left[\frac{\sqrt{3}}{2}\hat{j} - \frac{1}{2}\hat{i}\right] \\ \vec{V}_A^{\,+} &= V_0\left[-\frac{1}{4}\hat{i} + \frac{\sqrt{3}}{4}\hat{j}\right] \end{aligned}}$$

only gets pushed in $\hat{n}$ dir. so only goes in $\hat{n}$ dir.

<u>Note</u>: You can do this problem in your head. In $\hat{\lambda}$ direction B keeps its velocity ($\frac{\sqrt{3}}{2}V_0$) and A doesn't pick up any. In $\hat{n}$ direction B gives up its velocity of $V_0/2$ and gives it to A. In $\hat{n}$ dir. its an elastic collision between balls of equal mass $\Rightarrow$ balls trade velocities.