Your TA, Section # and Section time:

"SOLUTIONS"

Your name:

ANDY RUINA

Cornell TAM 2030

No calculators, books or notes allowed.

3 Problems, 90^+ minutes total.

Prelim 3

April 14, 2009

Directions.

To ease your TA's grading and to maximize your score, please:

- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation**.
- ✓+ Be (I) neat, (II) clear and (III) well organized.
- ☐ TIDILY REDUCE and $\boxed{\text{box in}}$ your answers (Don't leave simplifiable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$\dot{\theta}_7$ = `18`" instead of, say, "`theta7dot = 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{n}} \ldots$) and signs ($\pm$) with sketches, equations or words.
- →| **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60–100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- ☐ Put your name on each extra sheet, fold it in, and refer to it at the relevant problem.
  Note the last page is **blank** for your use. Ask for more extra paper if you need it.

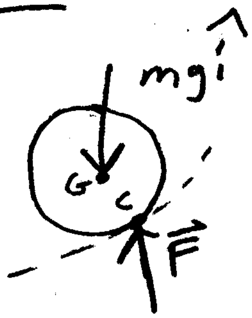
Includes alternate problem (8b)

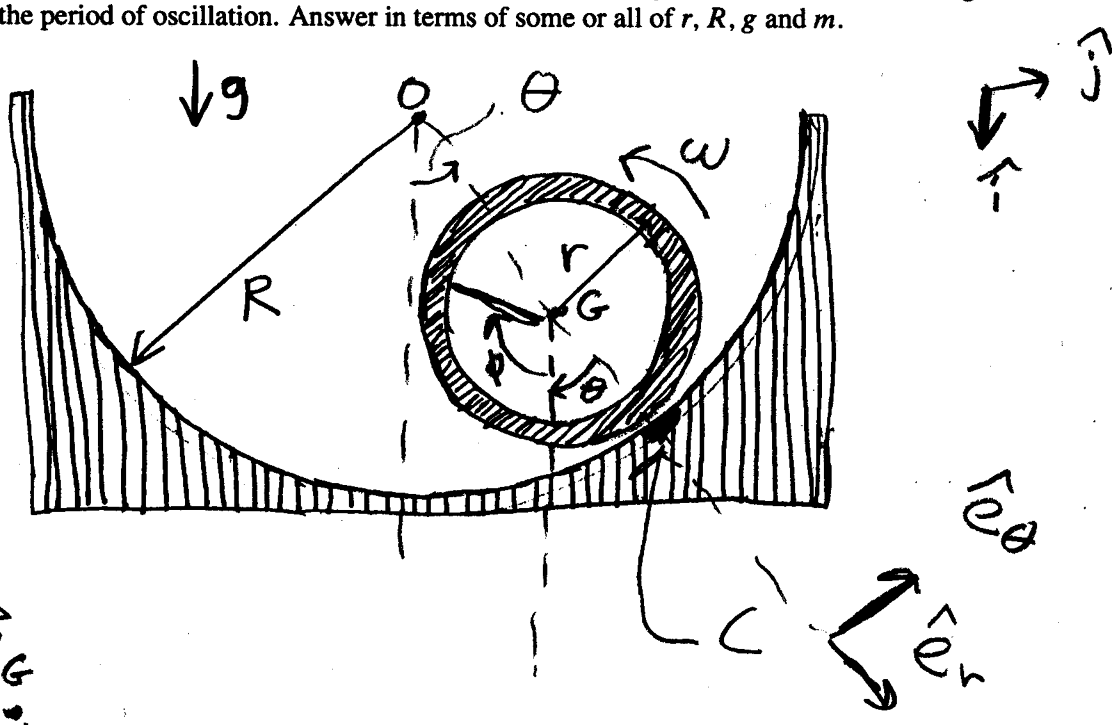
Student picks 8 or Alternate 8b

Problem 7:	/25
Problem 8:	/25
Problem 9:	/25

7) A thin-walled pipe with mass m and radius r rolls back and forth in a trough with radius R. Assuming small oscillations what is the period of oscillation. Answer in terms of some or all of r, R, g and m.

FBD





Kinematics

$$\vec V_G = \vec V_G$$

$$\dot\theta(R-r) = \dot\phi r \qquad \dot\phi = -\omega = -\omega_{pipe}$$

$$\Rightarrow \ddot\theta = \frac{-\dot\omega_{pipe}\, r}{R-r} \quad (1)$$

AMB$_{/c}$:

$$\sum \vec M_{/c} = \dot{\vec H}_{/c}$$

$$I^G = mr^2$$

$$mgr\sin\theta\,\hat k = \vec r_{G/c} \times m\vec a_G + I^G\dot\omega\,\hat k$$

$$\vec r_{G/c} = -r\hat e_r \qquad \vec a_G = -(R-r)\dot\theta^2\hat e_r + (R-r)\ddot\theta\,\hat e_\theta$$

$$mgr\sin\theta\,\hat k = -r(R-r)m\ddot\theta\,\hat k + mr^2\dot\omega\,\hat k$$

$$\left\{\cancel{m}g\cancel{r}\sin\theta\,\hat k = -\cancel{r}(R-r)\cancel{m}\ddot\theta\, k \;\overset{(1)}{-}\; \cancel{mr}\frac{R-r}{\cancel{r}}\ddot\theta k\right\}$$

$$\{\}\cdot\hat k \Rightarrow \quad -g\sin\theta = \ddot\theta\, 2(R-r)$$

$$\theta \ll 1 \Rightarrow \quad \ddot\theta + \frac{g}{2(R-r)}\theta = 0 \Rightarrow \theta = A\cos\Big(\underbrace{\sqrt{\tfrac{g}{2(R-r)}}}_{2\pi}\, t - \beta\Big)$$

$$\boxed{t^* = \text{period} = 2\pi\sqrt{\frac{2(R-r)}{g}}}$$

8) A rectangular plate $\mathcal{P}$ rotates with constant counter-clockwise angular velocity $\omega_{\mathcal{P}}$ about the point O marked. A bug walks on the plate with constant speed v, relative to the plate, on the dotted circle shown (radius r, with center a distance R from O). At the instant of interest the center of the circle and the bug are both directly to the right of O.

a) What is the velocity (a vector) of the bug at this instant?

b) What is the acceleration (a vector) of the bug at this instant?

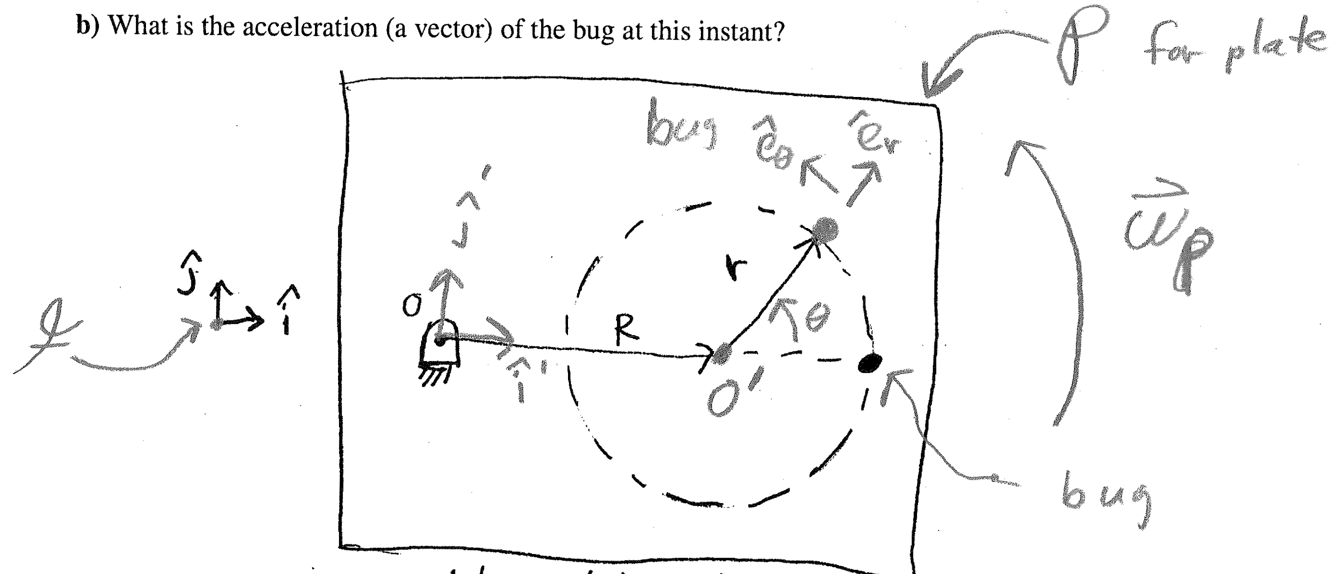


plate rotates about O

Lots of (legitimate) choices!

$$\vec{V}_b = \underbrace{\vec{V}_{O'/\mathcal{F}}}_{=\omega_p R\hat{J}} + \vec{V}_{/\mathcal{P}} + \underbrace{\vec{\omega}\times\vec{r}_{/O'}}_{\omega_p r\hat{J}}$$

$\vec{V}_b = \vec{V}_{bug}$

$\vec{V}_{/\mathcal{P}} = v\hat{e}_\theta = v\hat{J}$

$$\boxed{=(\omega_p(R+r)+v)\hat{J} = \vec{V}_{bug}}\ (a)$$

$$\vec{a}_b = \vec{a}_{O'/O} + \vec{a}_{/\mathcal{P}} - \omega_p^2\vec{r}_{/O'} + \cancel{\dot{\vec{\omega}}}\times\vec{r}_{/O'} + 2\vec{\omega}_p\times\vec{V}_{/\mathcal{P}}$$

$\vec{a}_{O'/O} = -\omega^2 R\hat{i}$, $\vec{a}_{/\mathcal{P}} = -\frac{v^2}{r}\hat{i}$, $\vec{r}_{/O'} = r\hat{i}$, $\dot{\vec{\omega}} = \vec{0}$, $\vec{\omega}_p = \omega_p\hat{K}$, $\vec{V}_{/\mathcal{P}} = v\hat{J}$

$$\boxed{\vec{a}_b = \left[-\omega_p^2(R+r) - \frac{v^2}{r} - 2\omega_p v\right]\hat{i}}\ (b)$$

Sanity check:
when $R=0 \Rightarrow \vec{a}_b = -\underbrace{\left(\omega_p + \frac{v}{r}\right)}_{=\text{ the net "}\dot{\theta}\text{"}}{}^2\, r\hat{i}$ ✓

8b) Assume r and θ are measured in the standard way relative to an xy coordinate system. A particle motion is described with polar coordinates with

$$r = r_0 \cos\theta \quad \text{and} \quad \dot\theta = \omega = \text{constant}.$$

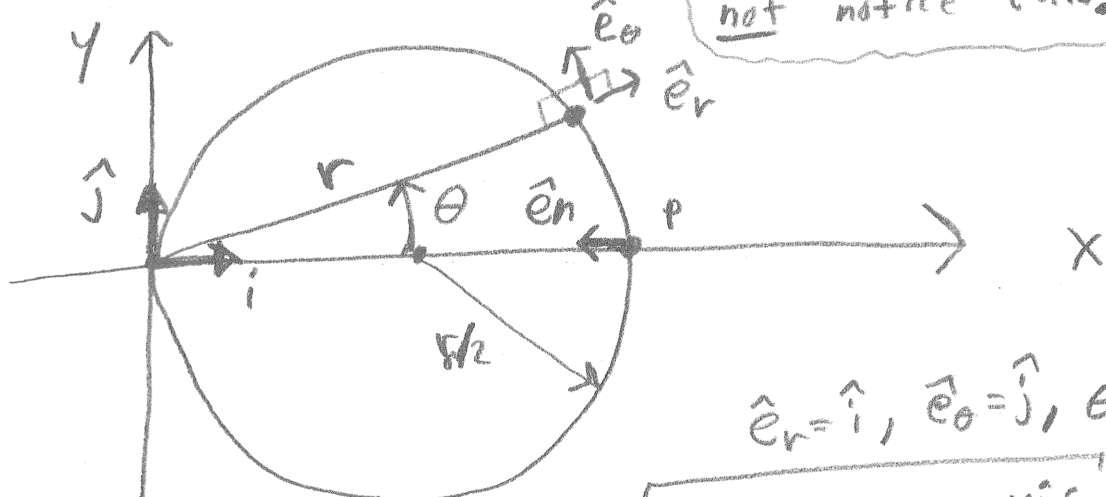
We are interested in the instant that the particle passes through the x axis at $\vec r = r_0 \hat e_r = r_0 \hat\imath$. Answer in terms of some or all of $r_0, \omega, \hat\imath$ and $\hat\jmath$.

Happens to be polar coord, formula for a circle. But you need not notice this.

a) What is the velocity of the particle at this instant?

b) What is the acceleration of the particle at this instant?

c) What is the the radius of curvature of the particle path at this instant?



$\hat e_r = \hat\imath,\ \hat e_\theta = \hat\jmath,\ \theta = 0$

Take $\theta = \omega t$ so at $t=0$ P is on x axis

$$\vec V = \dot r \hat e_r + r\underbrace{\dot\theta}_{\omega} \hat e_\theta = \boxed{r_0 \omega \hat\jmath = \vec V(t=0)} \quad (a)$$

$r = r_0 \cos\theta = r_0$

$$\dot r = \frac{d}{dt}(r_0 \cos\theta) = -r_0 \cancelto{0}{\sin\theta}\, \dot\theta$$

$$\vec a = (\ddot r - r\underbrace{\dot\theta^2}_{\omega^2})\hat e_r + (r\underbrace{\ddot\theta}_{0} - 2\underbrace{\dot r}_{0}\dot\theta)\hat e_\theta = \boxed{-2 r_0 \omega^2 \hat\imath = \vec a} \quad (b)$$

$$\ddot r = \frac{d}{dt} r_0(\sin\theta)\omega = -r_0\underbrace{\cos\theta}_{1}\,\omega^2$$

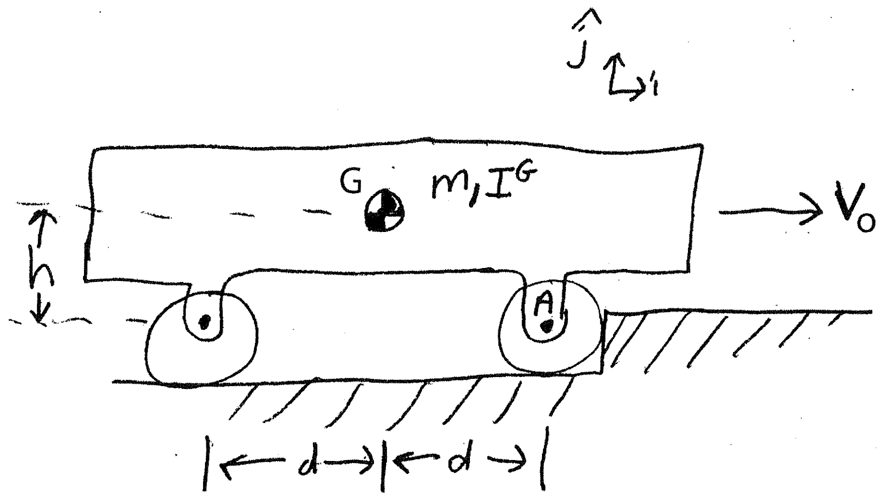
Check: $|\vec a| = v^2/(r_0/2)$

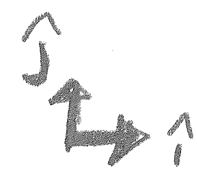
(consistent w/ circular shape)

$$\vec a = \frac{v^2}{\rho}\hat e_n = \frac{(r\omega_0)^2}{\rho}(-\hat\imath) \; *$$

$$\vec a = \vec a \Rightarrow \frac{(r_0\omega)^2}{\rho} = 2 r_0 \omega^2 \Rightarrow \frac{r_0^{\cancel{2}}\cancel{\omega^2}}{\rho} = 2\cancel{r_0}\cancel{\omega^2} \Rightarrow \boxed{\rho = \frac{r_0}{2}} \quad (c)$$

9) A rigid cart (mass m, moment of inertia I^G) with light well-lubricated wheels is rolling on level ground at constant speed v_0 when the front wheel suddenly gets completely stuck against a curb. Just after this collision what is the velocity of G? Answer in terms of some or all of v_0, m, I^G, d, h and g.



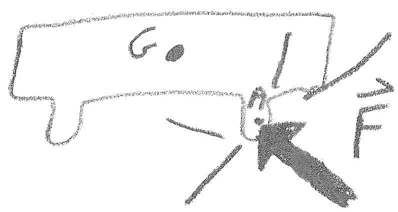


$-$ = just before collision: $\vec{\omega}^- = \vec{0}$

$+$ = just after collision: $\vec{V}_A^+ = \vec{0}$

Collisional FBD

neglects non-colli forces



AMB$_{/A}$

$$\Delta \vec{H}_{/A} = \int \vec{M}_{/A}\, dt = \vec{0}$$

$$\Rightarrow \vec{H}_{/A}^{\,+} = \vec{H}_{/A}^{\,-}$$

$$\Rightarrow \vec{r}_{G/A} \times m\vec{V}_G^{\,+} + I^G \vec{\omega}^{\,+} = \vec{r}_{G/A} \times m\vec{V}_G^{\,-}$$

$\vec{r}_{G/A} = -d\hat{\imath} + h\hat{\jmath}$; $\vec{V}_G^{\,+} = \omega^+\hat{k} \times \vec{r}_{G/A}$; $\vec{\omega}^{\,+} = \omega^+\hat{k}$; $\vec{V}_G^{\,-} = V_0\hat{\imath}$

$$\Rightarrow \{ m(h^2+d^2)\omega^+\hat{k} + I^G\omega^+\hat{k} = -hmV_0\hat{k} \}$$

$$\{\}\cdot\hat{k} \Rightarrow \quad \omega^+ = \frac{-hmV_0}{I^G + m(h^2+d^2)}$$

$$\vec{V}_G^{\,+} = \vec{\omega}^{\,+} \times \vec{r}_{G/A} = \frac{-hmV_0}{I_G + m(h^2+d^2)}\hat{k} \times (-d\hat{\imath} + h\hat{\jmath})$$

$$\boxed{\vec{V}_G^{\,+} = \frac{mV_0 h}{I_G + m(h^2+d^2)}(h\hat{\imath} + d\hat{k})}$$

Sanity checks:

$h = 0 \Rightarrow \vec{V}_G^{\,+} = 0$ ✓

$d = 0$ & $I^G = 0 \Rightarrow \vec{V}_G^{\,+} = V_0\hat{\imath}$ ✓

$I^G \to \infty \Rightarrow \vec{V}_G^{\,+} \to 0$ ✓

Units match: [V] on both sides ✓