MAE 2030 Lecture #9:

2/26/21

Today: ① Linear algebra
② Normal modes (continued)

Linear Algebra:

- Linear equations (algebraic)

$$\left.\begin{aligned} a x_1 + c x_2 &= b_1 \\ d x_1 + e x_2 &= b_2 \end{aligned}\right\} \quad (*)$$

$\underbrace{a, c, d, e, b_1, b_2}_{\text{given constants}}$

find x_1, x_2

Most common case:

\# eqns = # unknowns

$\Rightarrow$ usually can find a solution

$\Rightarrow$ usually unique

Matrix notation:

$$\underbrace{\begin{bmatrix} a & c \\ d & e \end{bmatrix}}_{[A]} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{[X]} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}}_{[b]}$$

* ⇒ given $\underbrace{[A] \text{ and } [b]}$, find $[X]$

$$AX = b, \quad X = A \backslash b$$

Can solve with MATLAB:

$$\boxed{X = A \backslash b}$$

demo: MATLAB related to normal modes

dot notation in MATLAB:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} .\times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae & bf \\ cg & dh \end{bmatrix}$$

"excel" element by element

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

Matrix

$$Ax = b$$

$$\underbrace{A^{-1}A}_{I}x = A^{-1}b$$

$$I = A^{-1}b$$

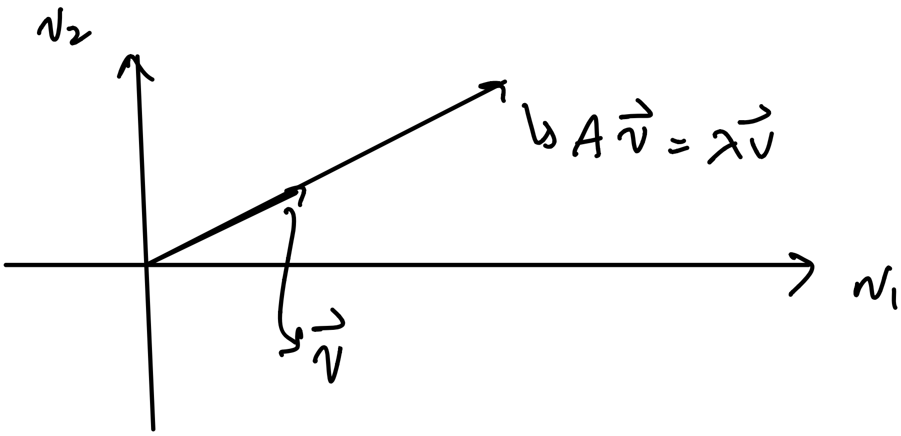
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boxed{\rightarrow x = A^{-1}b}$$

Eigenvalues & Eigenvectors:

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$ has special characteristic vectors v^1, v^2 and values λ_1, λ_2

$$[A][V] = \lambda[V]$$



demo: eigenvalues & eigenvectors on MATLAB ex) $[v] = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

check: $\underbrace{\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} -1 \\ 1 \end{bmatrix}}_{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \underbrace{-1}_{\lambda_1} \underbrace{\begin{bmatrix} -1 \\ 1 \end{bmatrix}}_{v^1}$

ex) $[v] = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Check: $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \overbrace{3}^{\lambda_2} \overbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}^{v^2}$

$\boxed{Av = \lambda v}$ Characteristic vector

Normal Modes:

* Any collection of springs & masses

* small motions

EOM $\Rightarrow$ $\underbrace{[M]}_{n \times n} \underbrace{\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \vdots \end{bmatrix}}_{n \times 1} + [K] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \end{bmatrix}$

zero if no force

$$\boxed{M\ddot{x} + Kx = F}$$

w/ damped

$$\boxed{M\ddot{x} + C\dot{x} + Kx = [0]}$$

Special case : no damping $[C] = [0]$

no forces $[F] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\boxed{M\ddot{x} + Kx = 0} \quad (*)$$

How to solve?

→ guess $x = \sin(\underset{\text{constant}}{\omega} t)\,\underset{\text{constant}}{[V]}$

mode shape

eigenvector

→ plug guess into $(*)$, multiply by M^{-1}

~skipped algebra~

$$\Rightarrow \quad \underset{M^{-1}K}{A}\, v = \lambda v$$

eigen value $= \omega^2$

eigenvector

mode shape