MAE 2030 Lecture #11: 3/3/21

Today: 1D collisions

1D collisions:

Collision : interaction

* short in time
* ignore the details of $F(t)$
* primary interest in impulse

$$P = \int_{t^-}^{t^+} F(t)\,dt$$

before: $\vec{v}_a^-$ $\vec{v}_b^-$

$v_a^- > v_b^-$

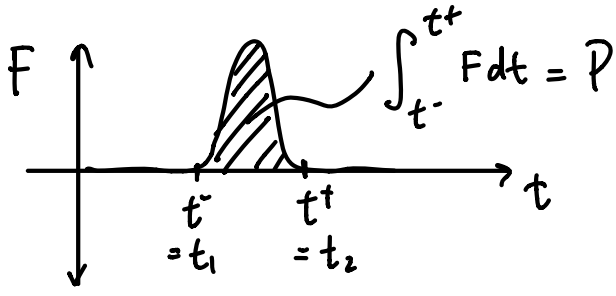
$- =$ "before" collision

$+ =$ "after" collision

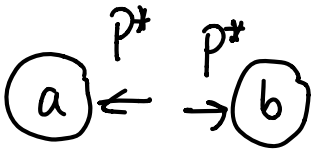
during collision: $\rightarrow \hat{i}$

after collision: $\vec{v}_a^+$ $\vec{v}_b^+$

$v_b^+ \geq v_a^+$



Collisional FBDs:



Momentum:

particle a: $\sum \vec{F} = m\vec{a}$

$$\Rightarrow F = m_a \ddot{x}_a$$

$$\Rightarrow \int_{t^-}^{t^+} F dt = m_a \Delta \dot{x}_a$$

$$\boxed{P = m_a \Delta v_a} \quad (1)$$

particle b: $\sum \vec{F} = m\vec{a}$

$$-F = m_b \ddot{x}_b$$

$$\boxed{-P = m_b \Delta v_b} \quad (2)$$

demo: physical demo of particle collisions

(1) + (2) $\Rightarrow \Delta L = 0$

$L = m_a v_a + m_b v_b$

Conservation of Linear Momentum

$$\boxed{L = \text{constant} = m_a v_a + m_b v_b}$$

Position x_a^- vs x_a^+

$$x_a^+ - x_a^- = \Delta x = \int_{t^-}^{t^+} v\,dt$$

assume $\Delta t = t^+ - t^-$ is very small

note: v does not $\to \infty$ as $\Delta t = t^+ - t^- \to 0$

$$\boxed{\Delta x \to 0}$$

Collision: $F \to \infty$

$\Delta v \to$ finite

$\Delta t \to 0$

$\Delta x \to 0$

Problem: Given v_a^-, v_b^-, m_a, m_b

Find v_a^+, v_b^+ ?

2 unknowns

3rd unknown: P

given

$$m_a v_a^- + m_b v_b^- = m_a v_a^+ + m_b v_b^+ \quad *$$

1 eqn, 2 unknowns $\Rightarrow$ no unique sol'ns

lots of solutions

demo: physical demo with collisions, conservation of momentum

Need info about materials:

ex) sticky collision

dead collision

plastic collision

$$\Rightarrow \boxed{v_a^+ = v_b^+} \quad **$$

(* + **) are 2 eqns for v_a^+ and v_b^+

$$\Rightarrow v_a^+ = v_b^+ = \frac{m_a v_a^- + m_b v_b^-}{m_a + m_b}$$

ex) happy ball

conservation of energy

$$E^+ = E^-$$

$$\frac{1}{2} m_a v_a^{-2} + \frac{1}{2} m_b v_b^{-2} = \frac{1}{2} m_a v_a^{+2} + \frac{1}{2} m_b v_b^{+2}$$

skipping algebra $\Rightarrow$

$$v_a^+ - v_b^+ = |v_a^- - v_b^-|$$

$$= -(v_a^- - v_b^-) \quad ***$$

interesting case

2 soln's a) boring soln (no contact)

$$\Delta v_a = \Delta v_b = 0$$

$(*) + (**) \Rightarrow v_a^+, v_b^+$

Everything in between:

$$v_a^+ - v_b^+ = -e(v_a^- - v_b^-)$$

e = coefficient of restitution (measures "liveliness")

$$\boxed{0 \le e \le 1}$$

Allow passing through $-1 \le e \le 1$

All 1D collision problems:

momentum system: $L^- = L^+$

$$m_a v_a^- + m_b v_b^- = m_a v_a^+ + m_b v_b^+$$

impulse: $p = -m_a \Delta v_a = m_b \Delta v_b$

restitution: $v_a^+ - v_b^+ = -e\,(v_a^- - v_b^-)$

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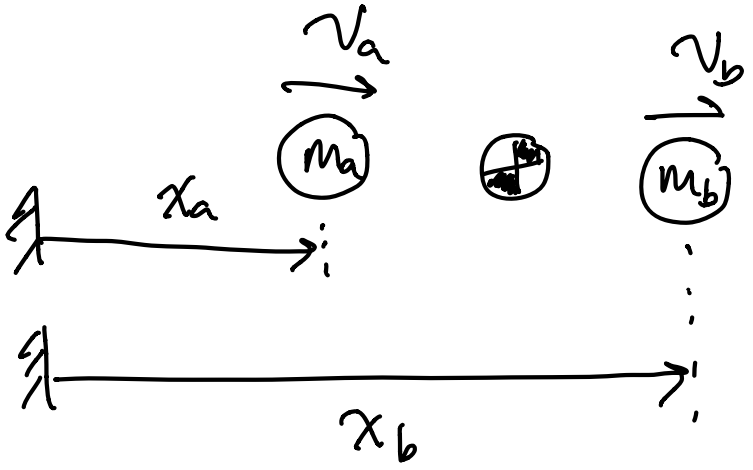
All probs are (****) w/ this or that given or unknown

method: solve eqns

Center of mass G



$$x_G = \frac{x_a m_a + x_b m_b}{(m_a + m_b)}$$

$$v_G = \frac{v_a m_a + v_b m_b}{m_a + m_b}$$

L = constant

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v_G = constant