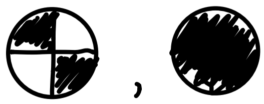
MAE 2030 Lecture #12:

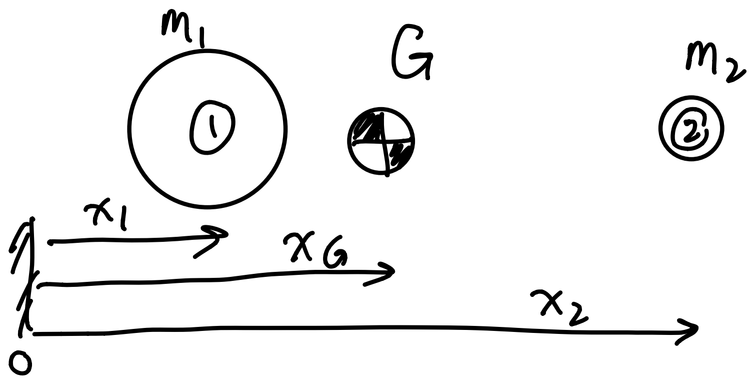
Today:
1. CoM continued
2. Collisions continued
3. ODE45

CoM: Center of Mass

G, 

= average position of mass in system

ex) 2 point masses



$m_{tot} = m_1 + m_2$

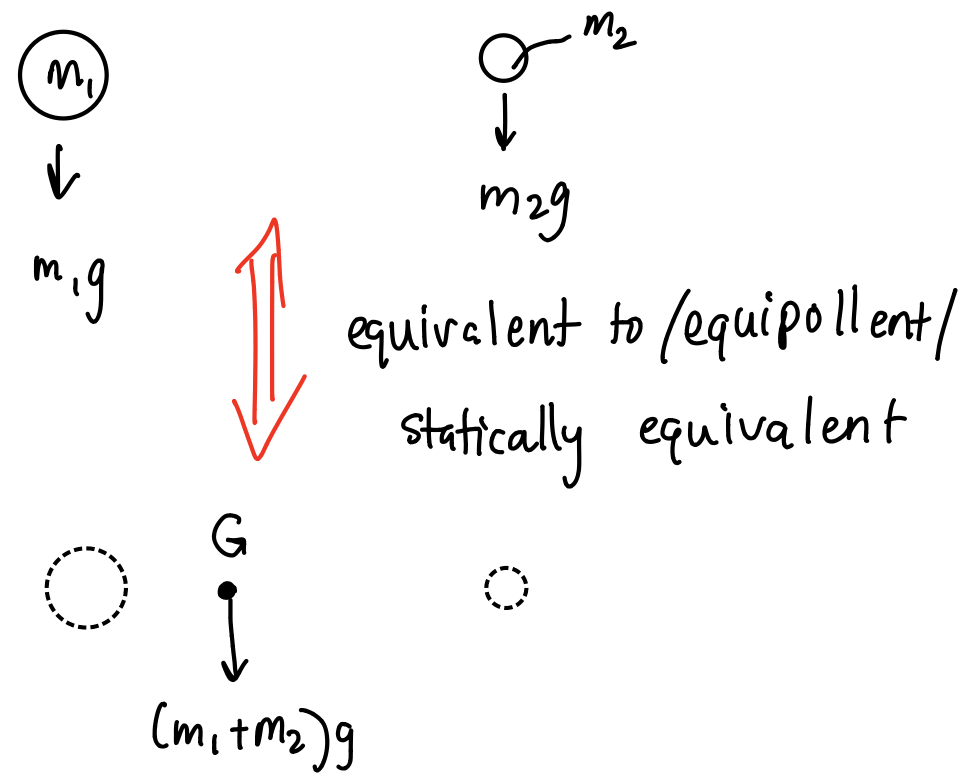
$$x_G m_{tot} = x_1 m_2 + x_2 m_2$$

$$x_G = \frac{x_1 m_1 + x_2 m_2}{m_{tot}}$$

Center of Gravity: that place where if you put the total gravitational force there, you get the correct moment about every point

- $m_{tot} g \hat{j}$ acting at CoG is equivalent to the total gravity effect

original system:



CoG is only a legit concept if g = constant in direction & magnitude

CoM in dynamics:

linear momentum: $\vec{L} = \dot{x}_1 m_1 \hat{i} + \dot{x}_2 m_2 \hat{i}$

$$\vec{L} = \dot{x}_g m_{tot}$$

kinetic energy: $E_K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

↳ 2 particles, 1D

$$= \frac{1}{2}\left[m_1\left(\underbrace{v_1 - v_G} + v_G\right)^2 + m_2\left(\underbrace{v_2 - v_G} + v_G\right)^2\right]$$

$v_{1/G} \equiv v_1 - v_G$
$= -v_{G1}$

$v_{2/G} \equiv v_2 - v_G$
$= -v_{G2}$

$$E_K = \frac{1}{2}m_1\left(v_{1/G} + v_G\right)^2 + \frac{1}{2}m_2\left(v_{2/G} + v_G\right)^2$$

$$= \frac{1}{2}m_1\left(v_{1/G}^2 + 2v_{1G}v_G + v_G^2\right) + \frac{1}{2}m_2\left(v_{2/G}^2 + 2v_{2G}v_G + v_G^2\right)$$

$$E_K = \frac{1}{2}\left[\left(v_{1/G}^2 m_1 + v_{2/G}^2 m_2\right) + v_G^2(m_1 + m_2) + 2\underbrace{\left(v_{1/G}m_1 + v_{2/G}m_2\right)}_{(*)}v_G\right]$$

Look at (*)

$$\left\{v_{1/G}m_1 + v_{2/G}m_2 = 0\right\} \quad v_{G/G}\, m_{tot}$$

"how fast is to CoM moving relative to the CoM?" zero!

$v_{1/G}m_1 \rightarrow v_1 - v_G \rightarrow \frac{v_1 m_1 + v_2 m_2}{m_1 + m_2}$

$v_{2/G}m_2 \rightarrow \vdots$

$$\therefore E_K = \left(\frac{1}{2}v_{1/G}^2 m_1 + \frac{1}{2}v_{2/G}^2 m_2\right) + \frac{1}{2}m_{tot}v_G^2$$

$$= E_{K/G} + E_{KG}$$

$E_{K/G}$: relative to CoM

E_{KG}: of CoM

Back to collisions:

Recall: $\vec{L}$ is conserved

$$\underbrace{m_1 v_1^+ + m_2 v_2^+}_{L_{after}} = \underbrace{m_1 v_1^- + m_2 v_2^-}_{L_{before}}$$

$$\Rightarrow \boxed{v_G^+ = v_G^-}$$

$$\Rightarrow E_{KG}^+ = E_{KG}^-$$

$\Rightarrow$ collisional dissipation maximized by $E_{K/G}^+ = 0$

$$\Rightarrow v_{2/G}^+ = v_{1/G}^+ \Rightarrow e = 0$$

demo: MATLAB ODE45