MAE 2030 Lecture #13:

3/8/21

Today: ① Collisions recap & demos
② 2D & 3D particle dynamics

Particle collisions (1D so far):

Assumptions: * collision time is short $\quad t^+ - t^- \equiv$ small relative to other times in system

$t_2 \quad t_1$

* collision forces >> all non-collision forces

$\Rightarrow$ collisional FBD ignores non-collisional forces

$\Rightarrow \quad x^+ = x^- \Big\}$ no change in position during collision

Only deal with 2-object interactions

Equations: * $\vec{L}_{system} = \text{constant}$

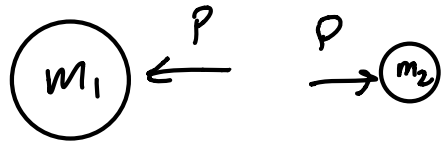
$$m_1 v_1^+ + m_2 v_2^+ = m_1 v_1^- + m_2 v_2^-$$

* Restitution eqn
(not so accurate)

Separation velocity = e approach velocity

$$v_2^+ - v_1^+ = -e(v_2^- - v_1^-)$$

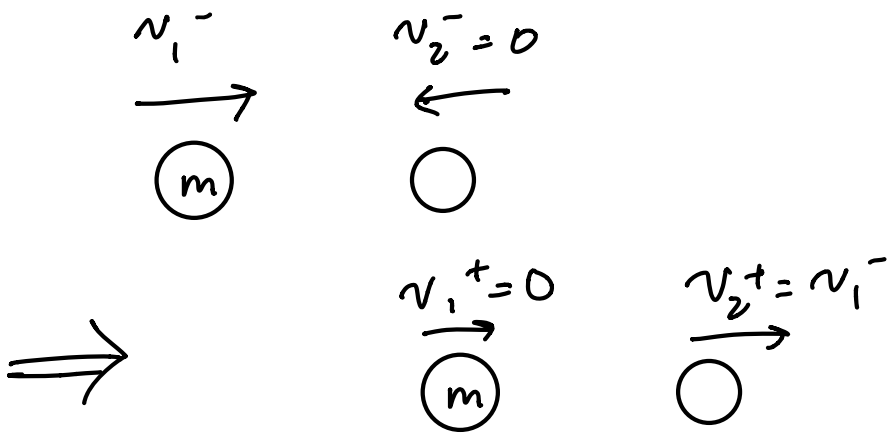
Collisional FBD:



$$m_2 v_2^+ = m_2 v_2^- + P$$

$$m_1 v_1^+ = m_1 v_1^- - P$$

Special case: $m_1 = m_2$



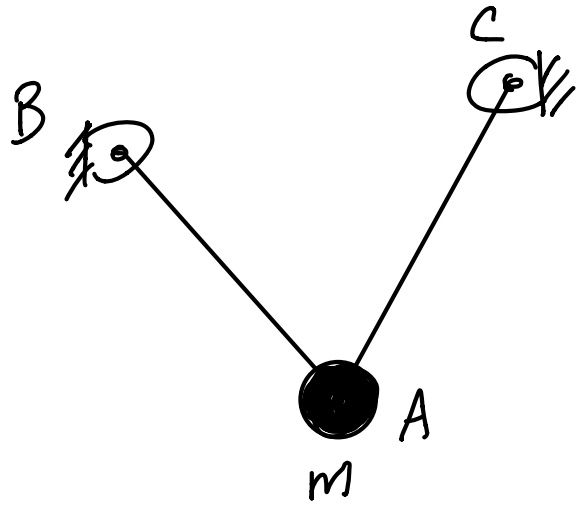
demo: replicate examples with toys

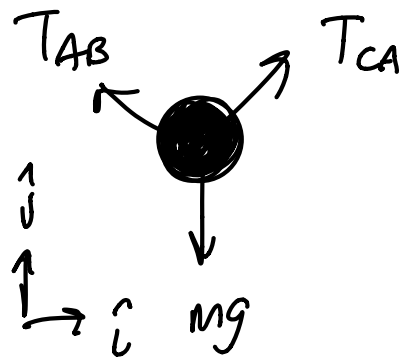
2D & 3D Particle Dynamics:

Instantaneous dynamics: $\boxed{\sum_i \vec{F}_i = m\vec{a}}$ (*)

given some information about $\vec{F}_i$ & some about $\vec{a}$

→ Plug what you know into (*) & solve for some info about force & acceleration





ex) Given $\vec{r}_A, \vec{r}_B, \vec{r}_C$

Given T_{AB}, T_{BC}

$\vec{a} = ?$

$\sum \vec{F} = m\vec{a}$

$$(**) \quad T_{AB}\hat{\lambda}_{AB} + T_{AC}\hat{\lambda}_{AC} - mg\hat{j} = m\vec{a}$$

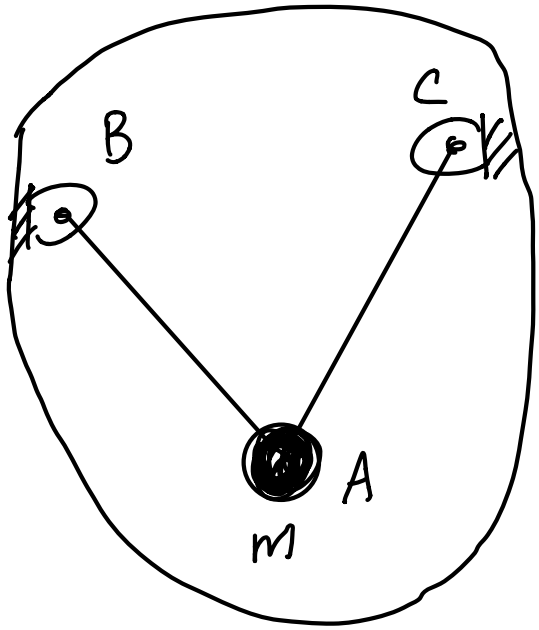
$$\frac{\vec{r}_{AB}}{|\vec{r}_{AB}|}$$

$$\lambda_{AC} = \frac{\vec{r}_{AC}}{|\vec{r}_{AC}|}$$

$\vec{r}_C - \vec{r}_A$

$$\Longrightarrow \vec{a} = \frac{\sum F_i}{m}$$

ex)



$\vec{r}_A, \vec{r}_B, \vec{r}_C$

$m, \vec{a}$

$T_{AB} = ?$ $T_{AC} = ?$

$$** \quad \overset{?}{T_{AB}}\,\overset{\checkmark}{\hat{\lambda}_{AB}} + \overset{?}{T_{AC}}\,\overset{\checkmark}{\hat{\lambda}_{AC}} - mg\hat{j} = \overset{\checkmark}{m}\overset{\checkmark}{\vec{a}}$$

<u>Method #1:</u> $\hat{i}\cdot\{\ \}$, $\hat{j}\cdot\{\ \}$

$\Rightarrow$ 2 eqns, 2 unknowns

<u>Method #2:</u> (MATLAB)

$$r_{AB} = r_A - r_B$$

$$\begin{bmatrix} * \\ * \end{bmatrix} \qquad \begin{bmatrix} * \\ * \end{bmatrix}$$

$$r_{AC} = r_A - r_C$$

$$\lambda_{AB} = r_{AB} / \text{norm}(r_{AB})$$

$$\lambda_{AC} = r_{AC} / \text{norm}(r_{AC})$$

$$grav = \begin{bmatrix} 0 \\ -mg \end{bmatrix}$$

$$A = \left[\lambda_{AB} , \lambda_{AC} \right]$$

$$A \begin{bmatrix} T_{AB} \\ T_{AC} \end{bmatrix} = m \begin{bmatrix} a_x \\ a_y \end{bmatrix} + \begin{bmatrix} 0 \\ mg \end{bmatrix}$$

$$T = A \backslash \left(grav + m \begin{bmatrix} a_x \\ a_y \end{bmatrix} \right)$$

$$\begin{bmatrix} T_A \\ T_B \end{bmatrix}$$

done

<u>Method #3:</u>

$\left\{ \sum \vec{F} = m\vec{a} \right\}$ • any vector that is perpendicular to $\hat{\lambda}_{AB}$ or $\hat{\lambda}_{AC}$

ex) $\hat{k} \times \vec{r}_{AB}$

unit vector in z direction

2 types of problems:

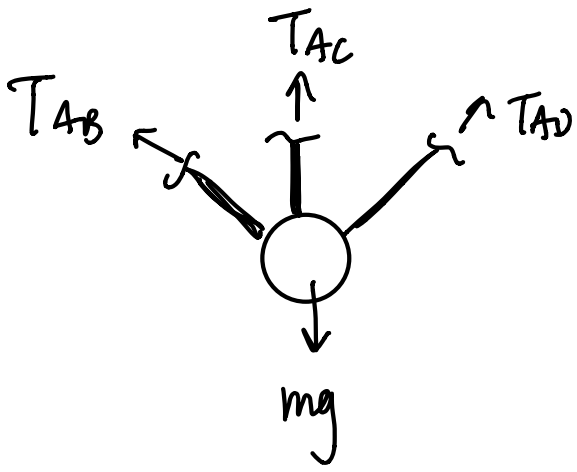
① Forward dynamics: given info about forces, find motion

→ involves solving ODEs

② Inverse dynamics: given motion, find the forces

ex) harder inverse instantaneous dynamics problem

3D



$m\vec{a}$ = given

$\hat{\lambda}_{AB}, \hat{\lambda}_{AC}, \hat{\lambda}_{AD}$ given

Method 1:

LMB: $\sum \vec{F} = m\vec{a}$

$$\hat{\lambda}_{AB} \overset{?}{T_{AB}} + \ldots - mg\hat{j} = m\vec{a}$$

$$\underbrace{\left[\hat{\lambda}_{AB} \;\vdots\; \hat{\lambda}_{AC} \;\vdots\; \hat{\lambda}_{AD}\right]}_{3\times 3 \text{ matrix}} \cdot T = m\vec{a} + mg\hat{j}$$

<u>Method 2:</u> $T = A \backslash \left(grav + m \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \right)$

(MATLAB) $\uparrow \begin{bmatrix} T_{AB} \\ T_{AC} \\ T_{AD} \end{bmatrix}$

<u>Method 3:</u> $(\vec{r}_{AB} \times \vec{r}_{AC}) \cdot \left\{ \sum \vec{F} = m\vec{a} \right\} \Rightarrow$ 1 eqn for T_{AD}

$$(\vec{r}_{AB} \times \vec{r}_{AC}) \cdot \hat{\lambda}_{AD} T_{AD} = (\vec{r}_{AB} \times \vec{r}_{AC}) \cdot [mg\hat{j} + m\vec{a}]$$