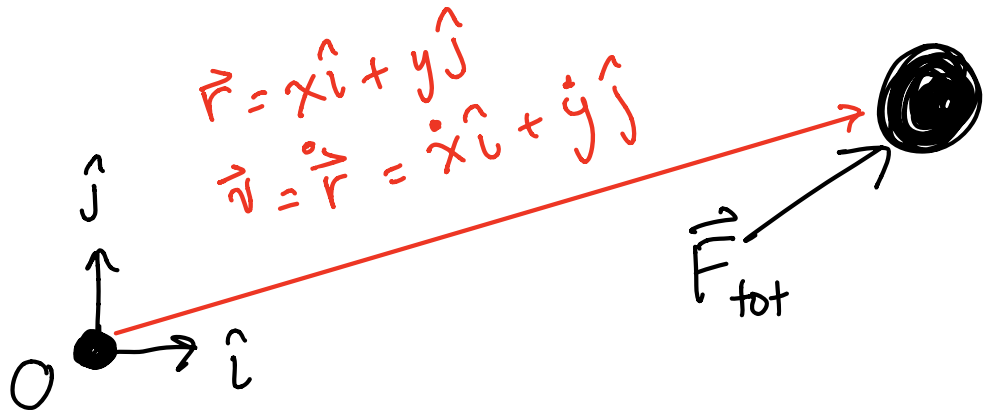
MAE 2030 Lecture #14: 3/12/21

Today: ① Dynamics of particles in 2D (& 3D)
② More MATLAB tricks ['Events']

(&3D)
2D dynamics of particles:



LMB: $\sum \vec{F}_i = \dot{\vec{L}}$

$$\vec{F}_{tot} = m\vec{a}$$

$$\Rightarrow \vec{a} = \frac{\vec{F}}{m}$$

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y \end{bmatrix} \\ \begin{bmatrix} \dot{v}_x \\ \dot{v}_y \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \end{bmatrix} / m \end{cases} \Longleftrightarrow \boxed{\begin{aligned} \dot{\vec{r}} &= \vec{v} \\ \dot{\vec{v}} &= \frac{\vec{F}_{tot}}{m} \end{aligned}}$$

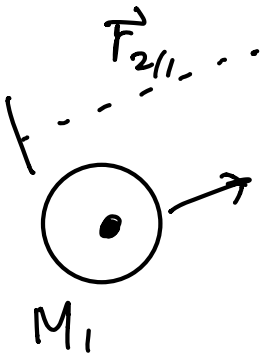
All forces of interest

$$\vec{F} = \vec{F}(t, \vec{r}, \vec{v})$$

ex) near earth gravity

$$\vec{F} = -mg\hat{j}$$

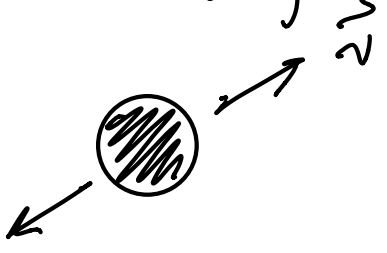
ex) inverse squared gravity



$$\vec{F} = -\frac{GM_1m_2}{|\vec{r}_{2/1}|^2} \cdot \frac{\vec{r}_{2/1}}{|\vec{r}_{2/1}|}$$

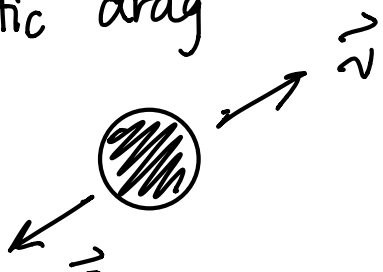
$$F = -\frac{GM_1m_2}{|r_{2/1}|^3} \cdot \vec{r}_{2/1}$$

ex) linear air drag



$$\vec{F}_D = C_{DL}|\vec{v}|\left(-\frac{\vec{v}}{|\vec{v}|}\right) = -C_{DL}\vec{v}$$

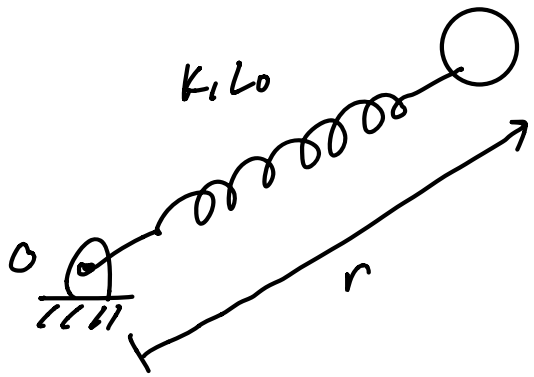
ex) quadratic drag

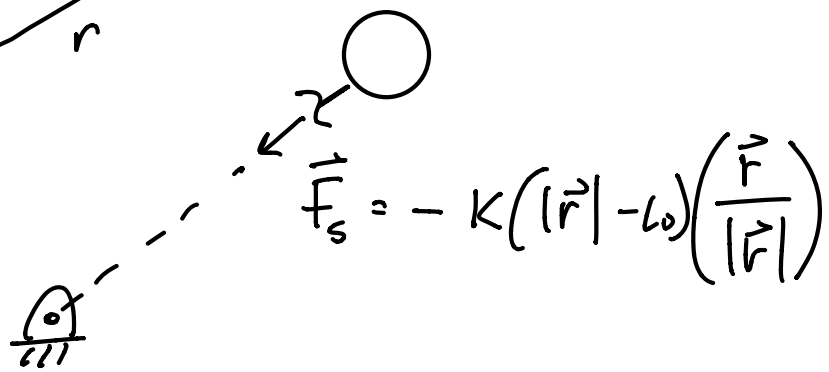


$$\vec{F}_{DQ} = -C_{DQ}|\vec{v}|^2\left(\frac{\vec{v}}{|\vec{v}|}\right)$$

$$\boxed{\vec{F}_{DQ} = -C_{DQ}|\vec{v}|\cdot\vec{v}}$$

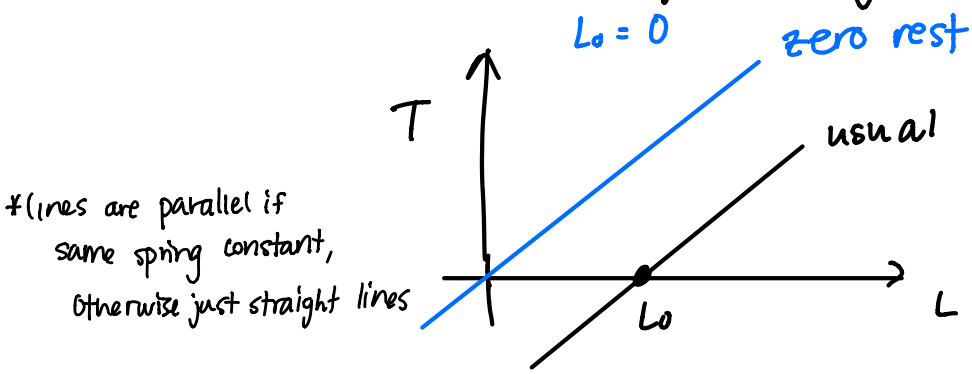
ex) linear spring





$$\vec{F}_s = -K\left(|\vec{r}| - L_0\right)\left(\frac{\vec{r}}{|\vec{r}|}\right)$$

ex) zero rest length spring



*(lines are parallel if same spring constant, otherwise just straight lines

$$\boxed{\vec{F}_s = -K\vec{r}}$$

demo: More fun MATLAB commands :)