MAE 2030 Lecture #15:

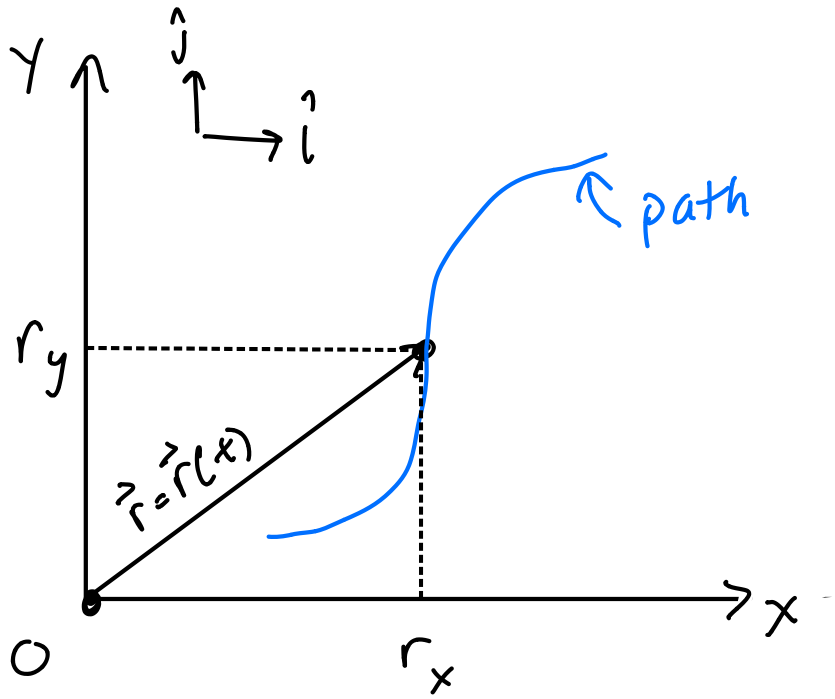
3/15/21

Today: ① 2D particles continued

$\rightarrow \vec{r}, \vec{v}, \vec{a}$

② Theorems

③ Multi-particle systems



$$\vec{r} = r_x \hat{i} + r_y \hat{j}$$

$$\vec{v} = \dot{\vec{r}} = \dot{r}_x \hat{i} + \dot{r}_y \hat{j}$$

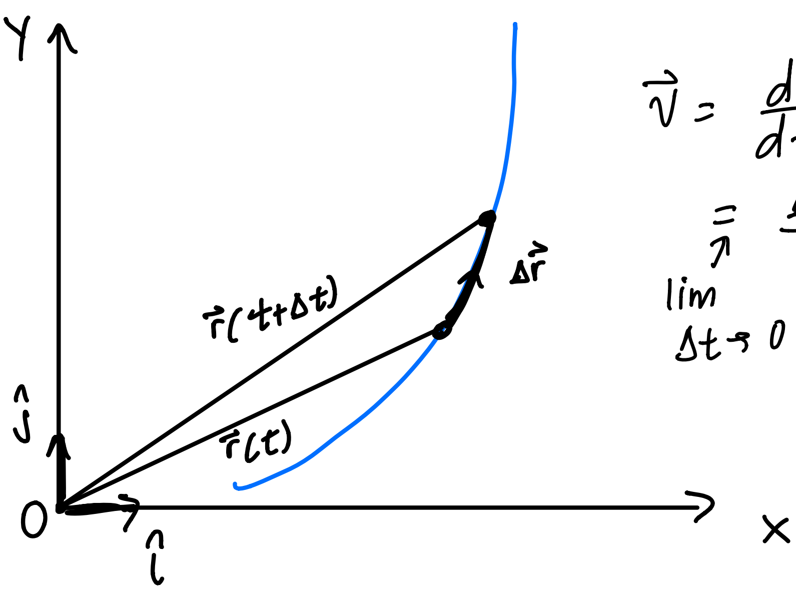
$$= v_x \hat{i} + v_y \hat{j}$$

$$\vec{a} = \dot{\vec{v}} = \dot{v}_x \hat{i} + \dot{v}_y \hat{j}$$

$$= \ddot{r}_x \hat{i} + \ddot{r}_y \hat{j}$$

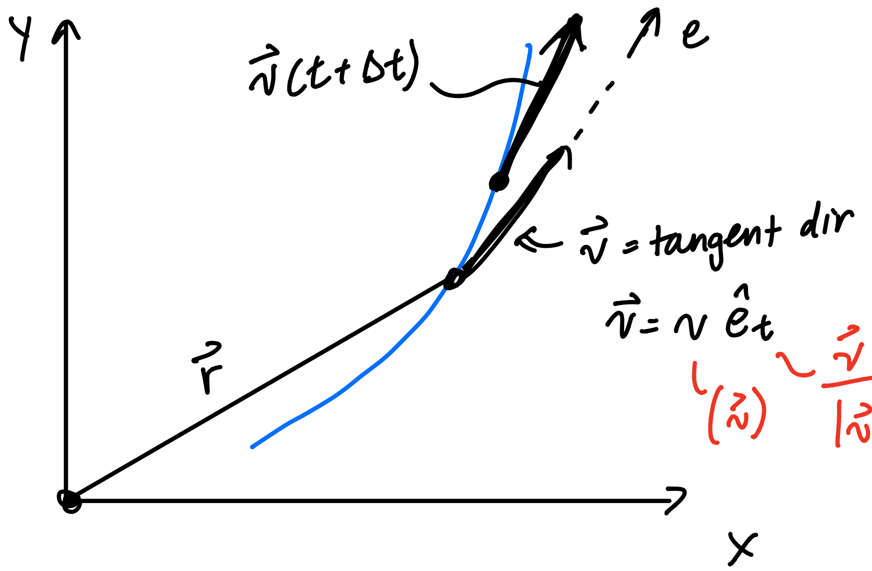
$$= a_x \hat{i} + a_y \hat{j}$$

More geometrical

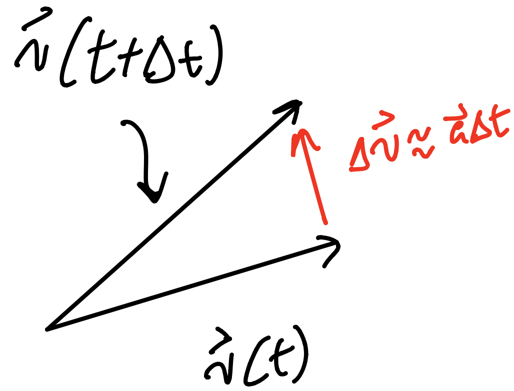


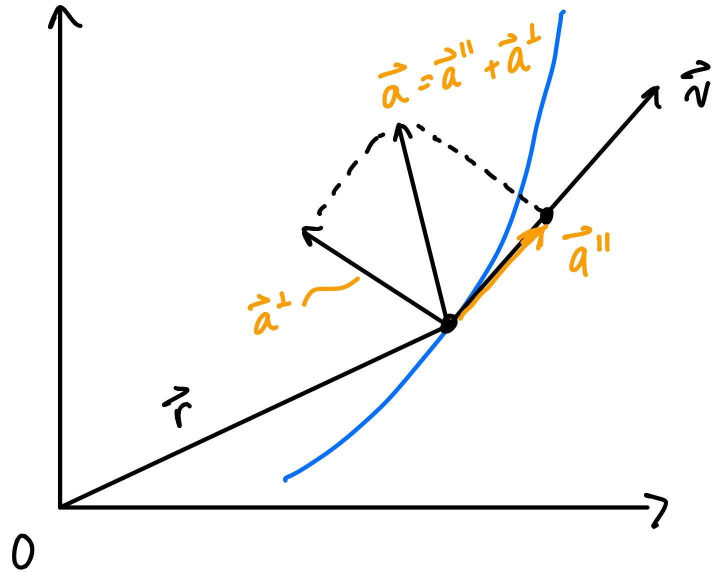
$$\vec{v} = \frac{d}{dt}\vec{r} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{v} = \dot{\vec{r}} = \lim_{\Delta t \to 0} \frac{\overbrace{\vec{r}(t+\Delta t) - \vec{r}(t)}^{\Delta\vec{r}}}{\Delta t}$$



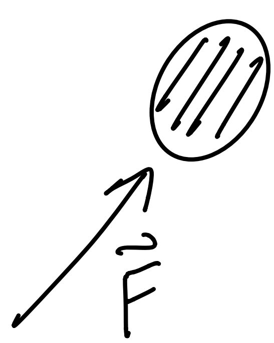
$$\vec{a} = \dot{\vec{v}} = \frac{\Delta\vec{v}}{\Delta t}$$





Theorems:

FBD:



LMB: $\boxed{\vec{F} = m\vec{a}}$ *

Linear Momentum: $\int \{*\}\, dt = \int \vec{F}\, dt = \int m\vec{a}\, dt$

$$\Rightarrow \int_{t_1}^{t_2} \vec{F}\, dt = m(\vec{v}_2 - \vec{v}_1)$$

$$\boxed{\vec{P}_{12} = \underbrace{m(\vec{v}_2 - \vec{v}_1)}_{\Delta\vec{L}}}$$

impulse

Energy: $\{*\} \cdot \vec{v} \Rightarrow \vec{F} \cdot \vec{v} = m\, \vec{a} \cdot \vec{v}$

Note: $\frac{d}{dt}(\vec{v} \cdot \vec{v}) = 2\vec{v} \cdot \vec{a}$

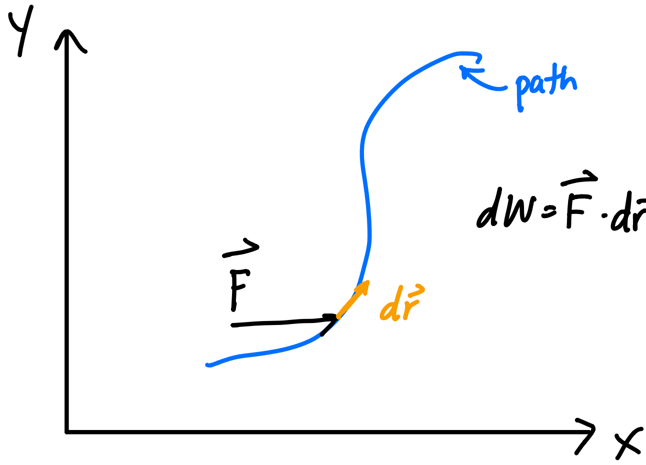
Power — $P = \frac{d}{dt}\left(\frac{1}{2} m v^2\right)$

$\vec{v} \cdot \vec{v}$

$$\int \{\ \}\, dt \to \int_{t_1}^{t_2} \vec{F} \cdot \vec{v}\, dt = \int_{t_1}^{t_1} \dot{E}_K\, dt$$

$$\int \vec{F} \cdot \frac{d\vec{r}}{dt}\, dt = E_{K_2} - E_{K_1}$$

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = E_{K_2} - E_{K_1}$$



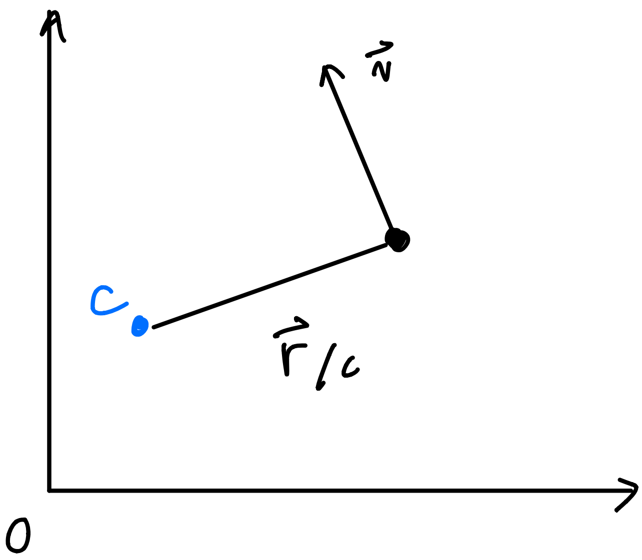
$$dW = \vec F \cdot d\vec r$$

$$\boxed{\int \underbrace{dW}_{\vec F \cdot d\vec r} = \Delta E_K}$$

Angular Momentum: $\vec F = m\vec a$

$$\vec r_{/c} \times \vec F = \vec r_{/c} \times m\vec a$$

any point



Assume C is fixed

$$\vec r_{P/C} \times \vec F = \vec r_{P/C} \times \vec a_P\, m$$

Look at $\frac{d}{dt}\left(\vec r_{P/C} \times \vec v_P\right)$

$$= \dot{\vec r}_{P/C} \times \vec v_P + \vec r_{P/C} \times \dot{\vec v}_P$$

$$\hookrightarrow \dot{\vec r}_P - \cancel{\dot{\vec r}_C}^{\,\vec 0}$$

$$= \cancel{\vec v_P \times \vec v_P}^{\,\vec 0} + \vec r_{P/C} \times \dot{\vec v}_P$$

$$\longrightarrow \underbrace{\vec r_{P/C} \times \vec F}_{\vec M_{/C}} = \frac{d}{dt}\big(\underbrace{\vec H_{/C}}_{\vec r_{P/C} \times m\vec v_{P/C}}\big)$$

Summary of Theorems:

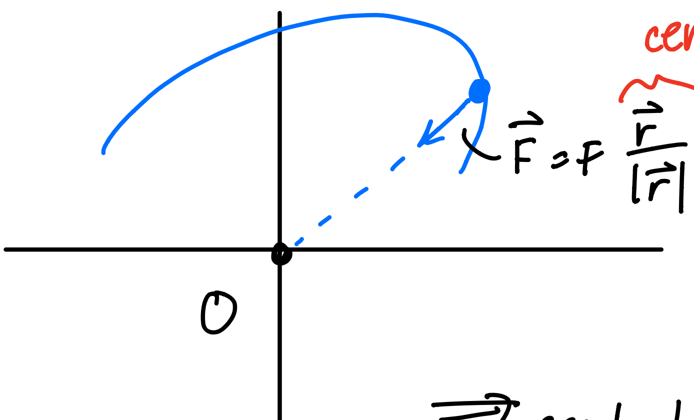
$$\overrightarrow{\text{Impulse}} = \Delta \vec{L}$$

$$P = \dot{E}_K$$

$$\left. W = \Delta E_K \right] \xrightarrow[\text{force}]{\text{conservative}} \boxed{-\Delta E_p = W}$$

$$\vec{M}_{/C} = \dot{\vec{H}}_{/C}$$

ex) Central Force motion

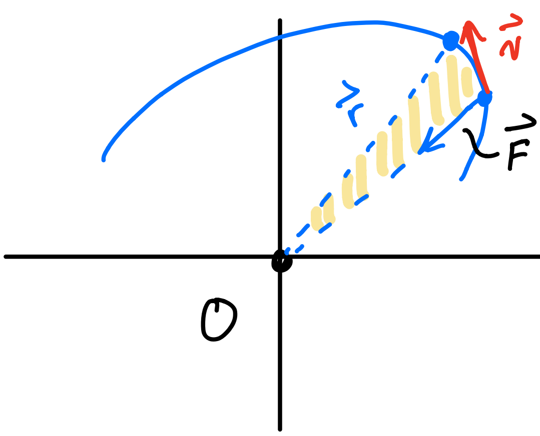


$\vec{M}_{/0} = 0$

↑ central force

$\Rightarrow$ central force $\Rightarrow$ $\vec{H}_{/0}$ = constant

$\Rightarrow$ $\vec{r}_{/0} \times m\vec{v}$ = constant



$$|\vec{r} \times \vec{v}| = 2\dot{A}$$

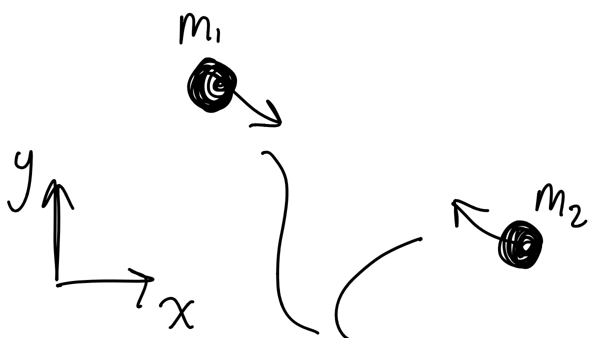
↖ swept area

"equal areas in equal times"

Multi-particle problems:

use $\vec{F} = m\vec{a}$ for each particle

ex) 2 particles



$$F = \frac{m_1 m_2 G}{r_{12}^2}$$

LMB$_1$: $\vec{F} = m_1 \vec{a}_1$

$$\frac{G m_1 m_2}{|\vec{r}_{12}|^2} \cdot \frac{\vec{r}_{12}}{|\vec{r}_{12}|}$$

$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$

LMB$_2$: $\vec{F} = m_2 \vec{a}_2$

$$\frac{-G m_1 m_2 \vec{r}_{12}}{|\vec{r}_{12}|^3}$$

Key: use state of all particles to find force on any one of them ex) 3 particles in 2D

$$\begin{array}{ll} \text{position} & x \\ & y \\ \text{velocity} & \dot{x} \\ & \dot{y} \end{array}\left.\vphantom{\begin{array}{l} x \\ y \\ \dot{x} \\ \dot{y} \end{array}}\right\} 4 \times 3 \text{ particles} = 12 \text{ ODEs}$$