MAE 2030 Lecture #17:

3/19/21

Today: ① Multi-particles wrap up
② Constrained motion (pulleys)

Multi-particle systems:

Each obeys $\vec{F}_i = m\vec{a}_i$

force laws

$$\vec{F}_i = \vec{F}_i(\vec{r}_1, \vec{r}_2 \ldots \vec{r}_i \ldots \vec{r}_n\ \vec{v}_1, \vec{v}_2 \ldots \vec{v}_i \ldots \vec{v}_n, t)$$

$$\dot{z} = \begin{bmatrix} \dot{\vec{r}}_i = \vec{v}_i \\ \underbrace{\dot{\vec{v}}_i = \vec{F}_i / m_i}_{\vec{F} = m\vec{a}} \end{bmatrix} \begin{matrix} \text{n vector eqns} \\ \text{n vector eqns} \end{matrix}$$

$$\begin{bmatrix} 4 \\ 6 \end{bmatrix} \cdot n \text{ scalar eqns}$$

2D: 4, 3D: 6

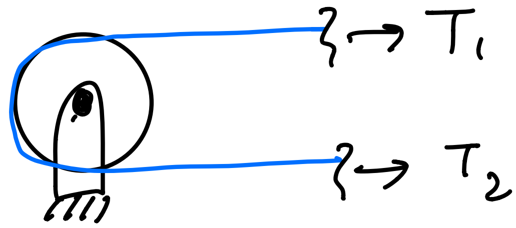
Laplace: tell me where everything is now and how fast it's going, and I can predict the future

Constraints:

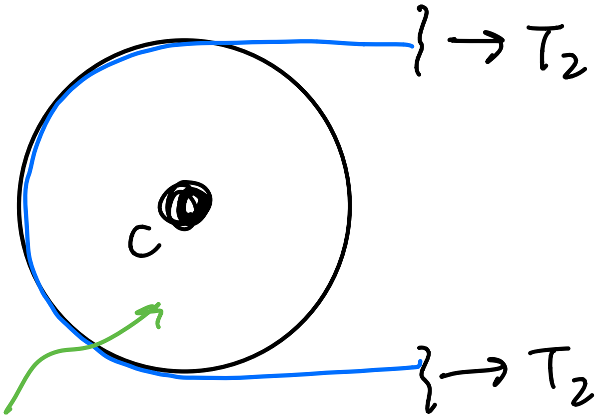
ex) pulley problems

demo: Andy demonstrate what a pulley is and does (ideal pulley)

ideal pulley:



FBD of pulleys & some rope:



low mass

↳ amount of torque it takes to turn it is near zero

$$\sum_i \vec{M}_{/c} = \vec{0}$$

statics approximation

$$-rT_1 + rT_2 + \text{friction of bearing} = 0$$

$\text{friction of bearing} \to 0$

$$\Rightarrow \boxed{T_1 = T_2}$$

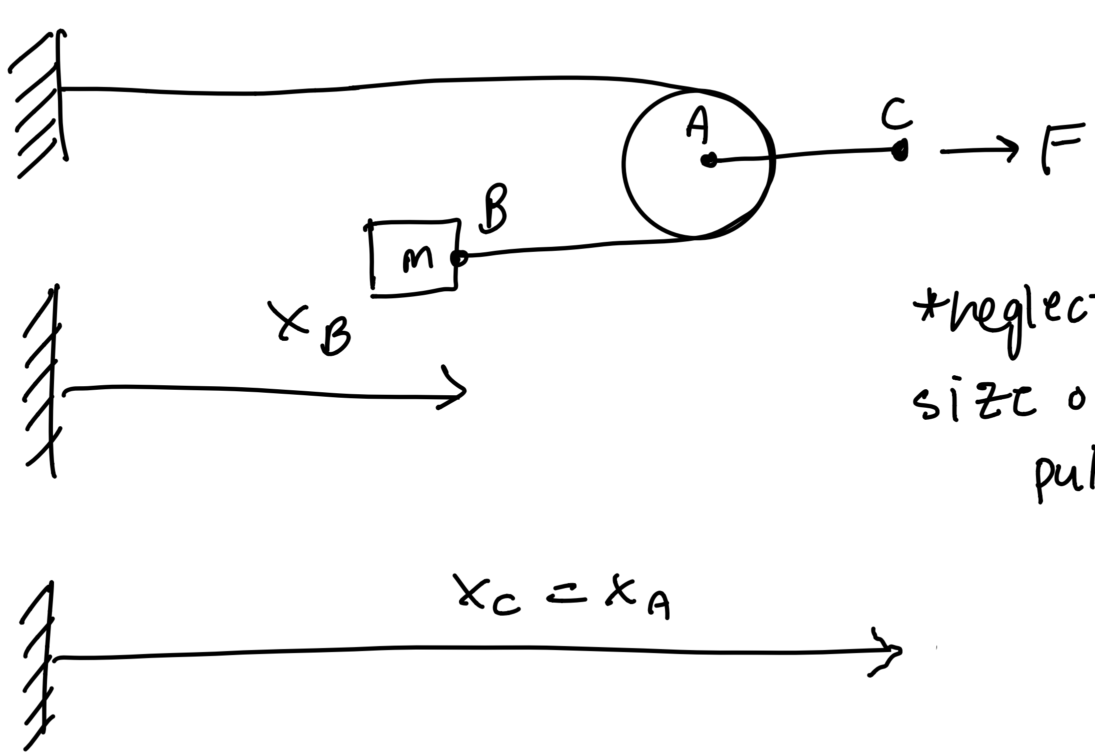
1st assumption

Assumptions: ① negligible mass
② negligible bearing friction
③ pulley is round

2nd assumption: length of rope/string is constant

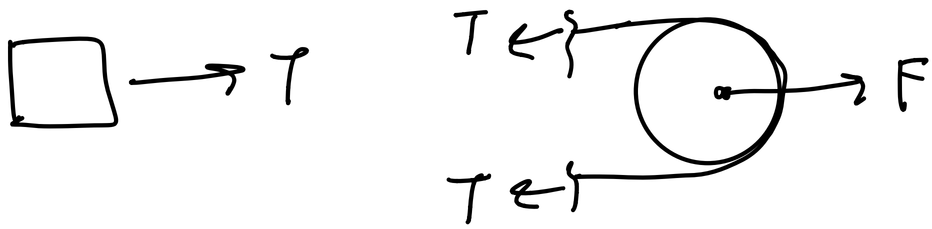
demo: Andy demonstrates 3rd grade pulley knowledge

↳ force and motion have a reciprocal relationship ex)



*neglecting size of pulleys

FBD:



$2T = F$

$T = F/2$

$$\text{Length} = L = \text{constant}$$
$$= x_A + (x_A - x_B)$$

$$\{\text{constant} = 2x_A - x_B\}$$

$$\frac{d^2}{dt^2}\{\;\} \rightarrow 2\ddot{x}_A - \ddot{x}_B = 0$$

$$\boxed{\ddot{x}_A = \ddot{x}_B/2}$$

LMB of B: $T = m\ddot{x}_B$

$$\frac{F}{2} = m\ddot{x}_B$$

$$\boxed{\ddot{x}_B = F/2m}$$

$$\boxed{\ddot{x}_A = F/4m}$$