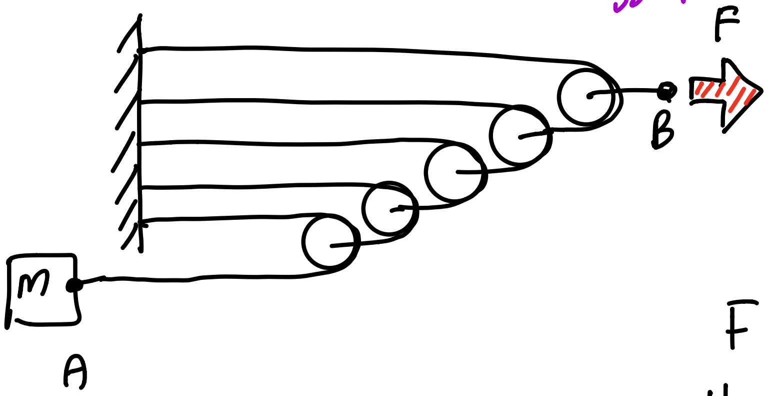
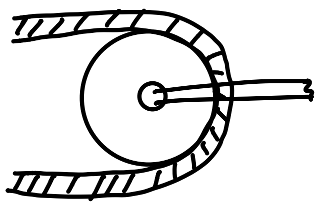
MAE 2030 Lecture #18:

3/22/21

Today: ① Pulley puzzle
② Theorems for many particles
③ Aladdin?

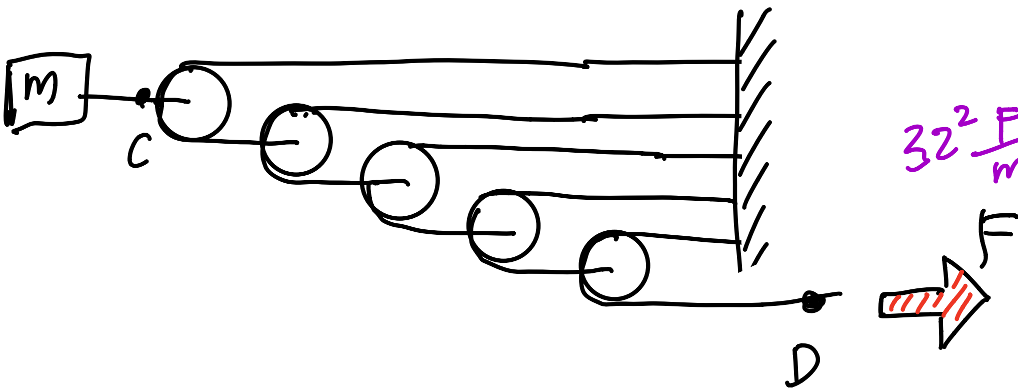
$\frac{1}{32^2}\frac{F}{m} = a_B$ close-up:





$F = F,\ m = m$

all ropes massless, inextensible
all pulleys massless & frictionless



$32^2\frac{F}{m} = a_D$

- Which point M has most acceleration? A, B, C, D?
- Which point L has least acceleration? A, B, C, D?
- $\frac{a_M}{a_L} = ?$ $\rightarrow$ $10^6 = \frac{32^2}{\frac{1}{32^2}}$

$a_A = 2^{-5}\frac{F}{m}$

$a_B = 2^{-10}\frac{F}{m}$

$a_C = 2^{5}\frac{F}{m}$

$a_D = 2^{10}\frac{F}{m}$

demo: Andy talks through logic of this problem

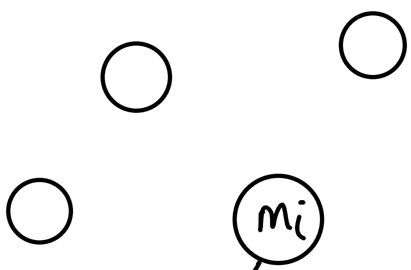
very important!! please listen & watch lecture

Theorems for many particles:

Lots of particles = n

each one is i

System



$\vec{F}_i^{\,tot}$

Linear Momentum!

$$\vec{F}_i^{\,tot} = m_i \vec{a}_i$$

$$\sum_i \vec{F}_i^{\,tot} = \sum_i m_i \vec{a}_i$$

$$\underbrace{\sum_i \vec{F}_i^{\,internal}}_{\text{assume } = \vec{0}} + \sum_i \vec{F}_i^{\,external} = \sum_i m_i \vec{a}_i$$

$$\Rightarrow \sum_i \vec{F}_i^{\,external} = \sum_i m_i \vec{a}_i$$

$$\boxed{\vec{F}^{\,ext}_{Tot} = m_G \vec{a}_G}$$

demo: Aladdin's COM when being pulled

Angular Momentum:

one particle

$$\vec{F}_i^{\,tot} = m_i \vec{a}_i$$

$$\left\{ \vec{r}_{i/c} \times \vec{F}_i^{\,tot} = \vec{r}_{i/c} \times (m_i \vec{a}_i) \right\}$$

any point

$\vec{F}_i^{\,int} + \vec{F}^{\,ext}$

$$\sum_i \{ \} \Rightarrow \sum_i \vec{r}_{i/c} \times \vec{F}_i^{\,ext} + \cancel{\underbrace{\sum_i \vec{r}_{i/c} \times \vec{F}_i^{\,int}}_{\text{internal torques add to zero}}} = \sum_i \vec{r}_{i/c} \times m_i \vec{a}_i$$

all particles

0

$$\sum_i \vec{r}_{i/c} \times \vec{F}_i^{\,ext} = \sum_i \vec{r}_{i/c} \times m_i \vec{a}_i$$

$$\vec{M}_{/c} = \dot{\vec{H}}_{/c}$$

$\frac{d}{dt}(\vec{H}_{/c})$

$$\vec{H}_{/c} \equiv \sum_i \vec{r}_{i/c} \times m_i \vec{v}_{i/c}$$

fixed point

energy: internal power does not drop out

↳ cannot neglect internal forces like we did for LM & AM

$$P^{int} + P^{ext} = \dot{E}_K$$

$P^{int} = \sum \vec{F}_i^{int} \cdot \vec{v}_i$

$P^{ext} = \sum \vec{F}_i^{ext} \cdot \vec{v}_i$

$\dot{E}_K = \frac{d}{dt} \sum \frac{1}{2} m_i \vec{v}_i \cdot \vec{v}_i$

Other things to know: Various ways of writing

E_K, $\vec{H}_{/C}$