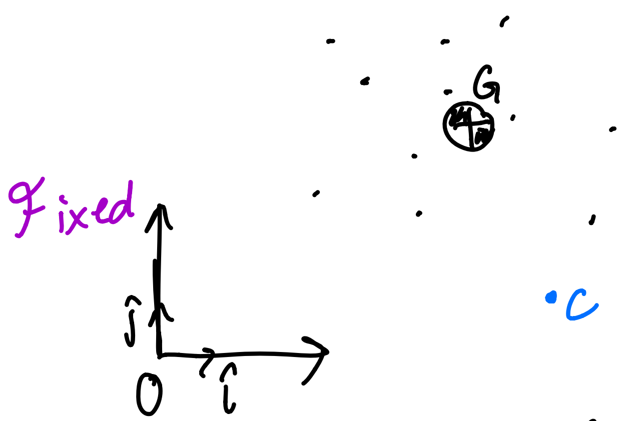
MAE 2030 Lecture #19: 3/24/21

Today: ① Angular Momentum Theory
② Car accelerating

Angular Momentum: $\sum \vec{M}_{/c} = \dot{\vec{H}}_{/c}$

Rate of change of angular momentum with respect to point C

C must be fixed



$$\vec{H}_{/c} = \sum_{i=1}^{n} \vec{r}_{i/c} \times m_i \vec{v}_{i/c}$$

$(\vec{r}_i - \vec{r}_c)$ $\quad$ $\vec{v}_i - \vec{v}_c$

$$\dot{\vec{H}}_{/c} = \sum_{i=1}^{n} \vec{r}_{i/c} \times m_i \vec{a}_{i/\mathcal{F}}$$

Fixed reference frame (denoted as fancy "F")

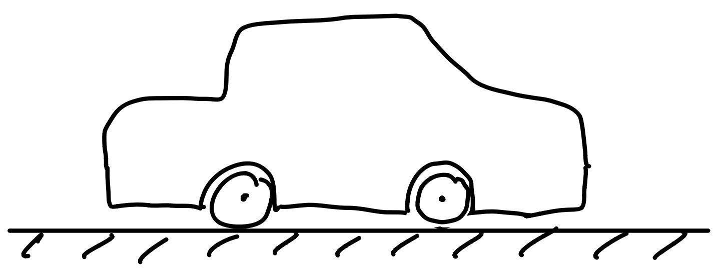
$$\boxed{\vec{M}_{/c} = \sum \vec{r}_{i/c} \times m_i \vec{a}_{i/c}}$$

Another case also works: $C = G$ ← c_{COM}

$$\sum \vec{M}_{/G} = \dot{\vec{H}}_{/G} = \frac{d}{dt} \sum \vec{r}_{i/G} \times m_i \vec{v}_{i/G}$$

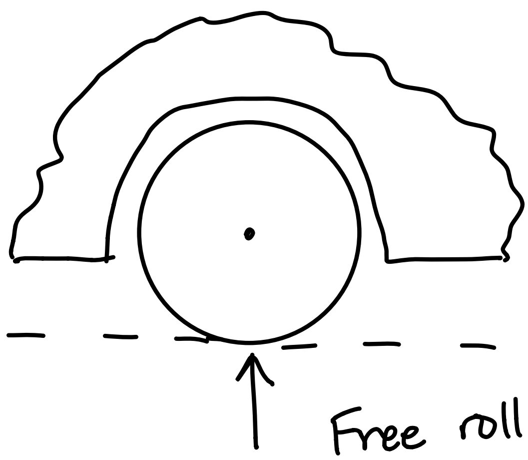
Simplest Motion: A rigid object does not rotate & moves in a straight line "straight line motion"

ex) car

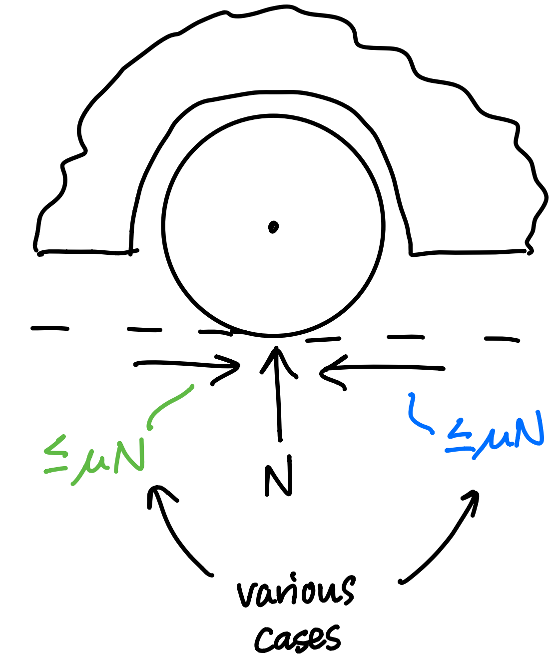


- rigid
- stiff suspension (no rotation, no up & down motion)

partial FBD:



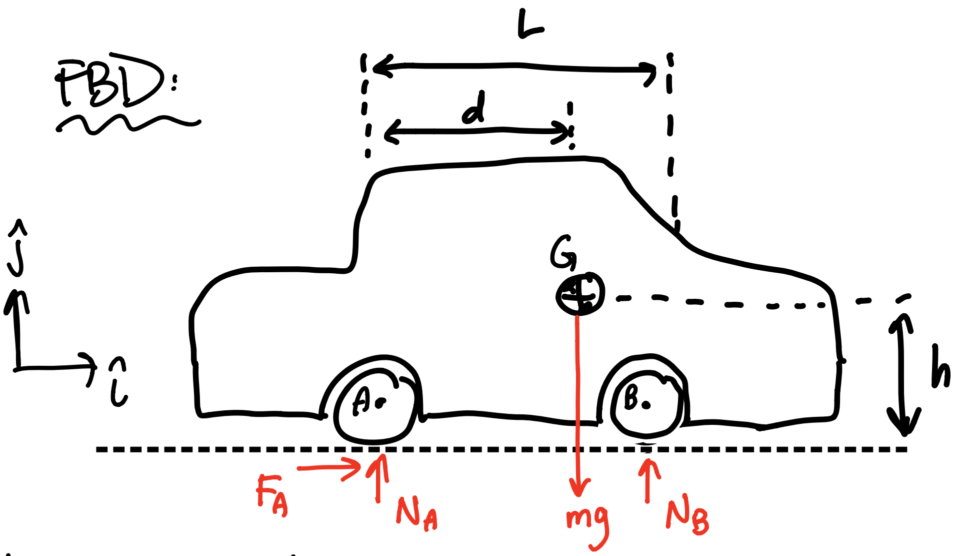
Free rolling (ideal round, rigid wheel with no bearing friction)



explained more in depth in textbook

ex) Max acceleration of rear wheel drive car? (gigantic engine)

FBD:



$F_A \leq \mu N_A$

$\Rightarrow F_A = \mu N_A$

blob works just as well

LMB: $\sum \vec{F}_i = m\vec{a}_G$

$a_G \hat{i}$

2 scalar eqns

$$\boxed{\mu N_A \hat{i} + (N_A + N_B - mg)\hat{j} = m a_G \hat{j}}$$

unknowns: N_A, N_B, a_G

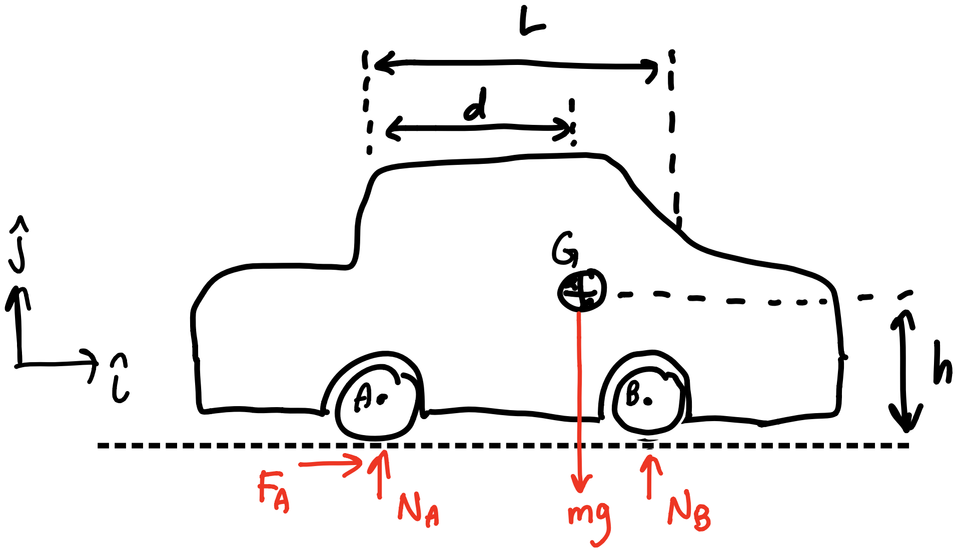
knowns: μ, d, L, h, m, g

Need another eqn $\rightarrow$ $AMB_{/G}$ or $AMB_{/A}$ or $AMB_{/B}$

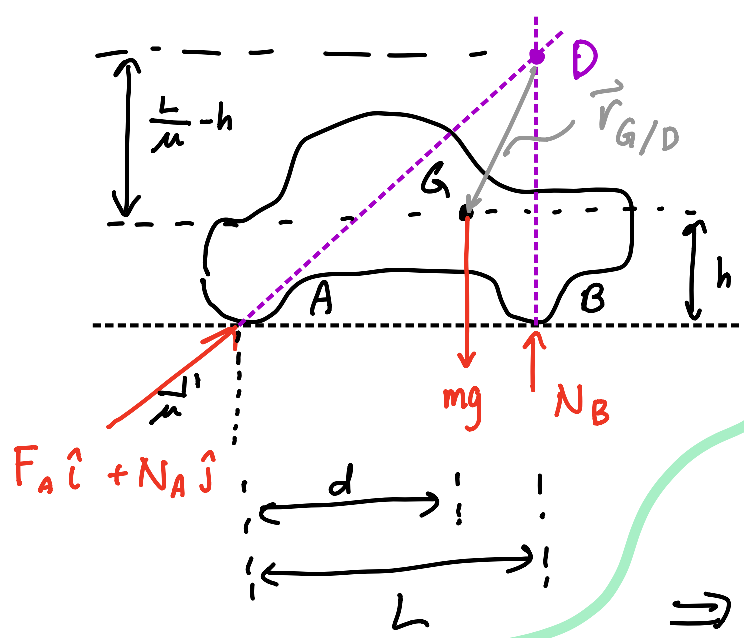
or $AMB_{/\sim}$ whatever point you like

use any point $\Rightarrow$ 3 eqns & 3 unknowns

$\Rightarrow N_A, N_B, F_A = \mu N_A, a_G$



$\rightarrow$ can we kill of the things that don't matter?



$$\sum \vec{M}_{/D} = \dot{\vec{H}}_{/D}$$

$$(L-d)mg\hat{k} = \sum_{i=\text{every atom}} \vec{r}_{i/D} \times m_i\vec{a}_i$$

$$\vec{a}_i = \vec{a}_G$$

$$\Rightarrow (L-d)mg\hat{k} = \sum_i \vec{r}_{i/D} \times m_i\vec{a}_G$$

$$= \underbrace{\left(\sum_i \vec{r}_{i/D} m_i\right)}_{\vec{r}_{G/D}\cdot m_{tot}} \times \underbrace{\vec{a}_G}_{a_G\hat{i}}$$

$$\Rightarrow (L-d)mg\hat{k} = \underbrace{\vec{r}_{G/D}}_{-(L-d)\hat{i}-\left(\frac{L}{\mu}-h\right)\hat{j}} \times a_G\hat{i}\cdot \underbrace{M}_{m_{tot}}$$

$$\left\{(L-d)\cancel{m}g\hat{k} = a_G\left(\frac{L}{\mu}-h\right)\hat{k}\cancel{m}\right\}$$

$$\{\ \}\cdot\hat{k} \Rightarrow (L-d)g = a_G\left(\frac{L}{\mu}-h\right)$$

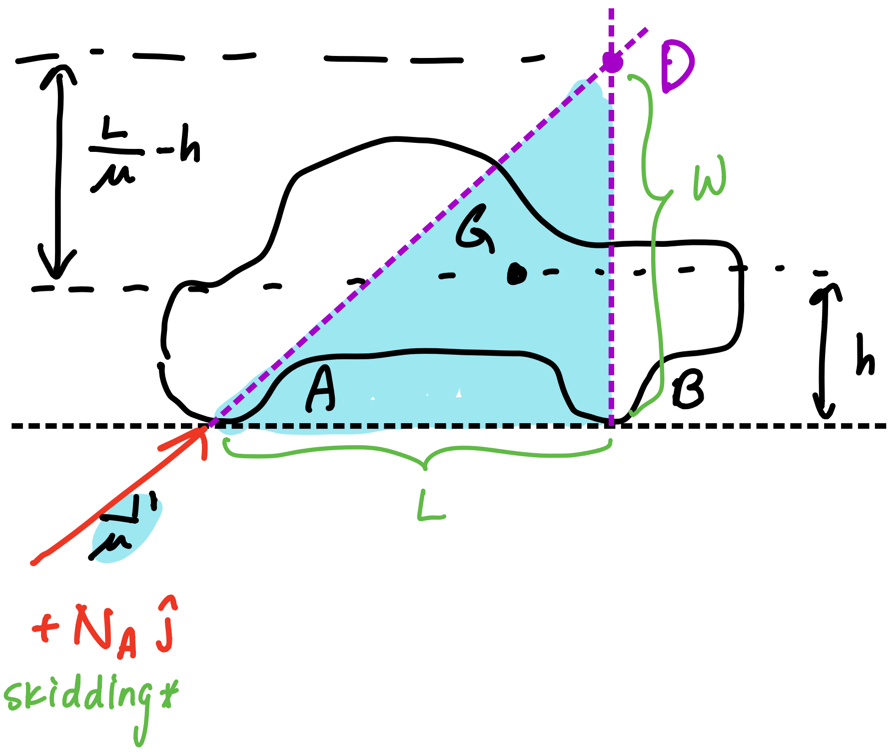
$$a_G = g\frac{L-d}{\left(\frac{L}{\mu}-h\right)} \longrightarrow \boxed{a_G = \frac{(1-d/L)}{\left(\frac{1}{\mu}-\frac{h}{L}\right)}g}$$

$$F_A = a_G m$$

$$N_A = F_A / \mu$$

$$N_A + N_B - mg = 0$$

Where did $\frac{L}{\mu} - h$ come from?



$$\frac{W}{L} = \frac{1}{\mu} \quad \text{(similar triangles)}$$

$$\Rightarrow W = \frac{L}{\mu}$$