****On the top of every page, write the following****
—Please print clearly (for computer text recognition)—

Your netID (xyz*lmn*) & name	Problem number and page number of that problem

Cornell ME 2030

No calculators, books or notes allowed.

Prelim 1 **Solutions**

Thursday Feb 4, 2021, 8:30 - 10:00 PM$^+$

3 Problems, 90 minutes (+ 90 minutes extra time)

How to get the highest score?

Please do these things:

Scans. Start each problem on a clean sheet. Only write on side.
Put ***your name, *net ID, *problem number** and ***page number, strarting with 1 for each problem** on the top of every side of every sheet. **At the end:**
—Scan your exam, including both sides of this sheet,& check it for completeness and quality;
—Filename should be netID-first-last.pdf (*e.g.,* alr3-Andy-Ruina.pdf);
— Upload each problem separately to Canvas —Check on Canvas that it has been received and approved before leaving the exam;

Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.

Use correct **vector notation**.

A+ Be (I) neat, (II) clear and (III) well organized.

□ TIDILY REDUCE and $\boxed{\text{box in}}$ your answers (Don't leave simplifiable algebraic expressions).

>> Make appropriate `Matlab code` clear and correct.
You can use shortcut notation like "$\phi_7 = 2\pi$" instead of, say, "`phi(7) = 2*pi;`".
Small syntax errors will have small penalties.

Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \hat{\boldsymbol{e}}_r, \hat{\boldsymbol{e}}_\theta, \hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{n}} \ldots$) and signs ($\pm$) with sketches, equations or words.

Justify your results so a grader can distinguish an informed answer from a guess. If you quote a fact that a grader might doubt your understanding of, explain it. Especially if it is not commonly used.

If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).

$\approx$ Work for **partial credit** (from 60–100%, depending on the problem)

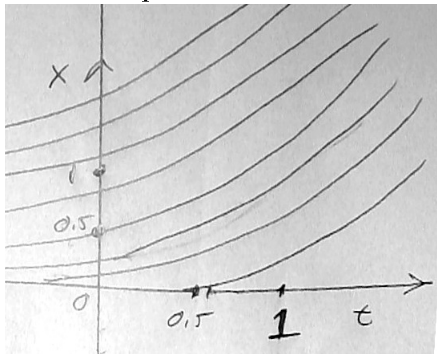
- Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
- Reduce the problem to a clearly defined set of equations to solve.
- Provide Matlab code which would generate the desired answer, and explain the nature of the output (unless specifically precluded).

Extra sheets. If live, Ask for more extra paper if you need it. Put your name, net ID, problem number and page number on each extra sheet, label it clearly place it in order with it's associated problem.

1) ODE solutions. A MATLAB function

```
function zdot = myrhs(t,z)
```

defines some differential equation. It has been written and is in your workspace. It is a first order ODE in the scalar variable x. Several exact solutions of that equation are shown.



Sketch two copies of this figure.

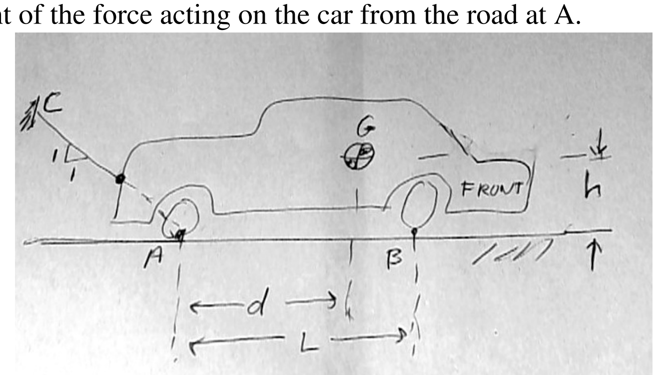
a) On one copy show, as accurately and clearly as you can (relative to your figure), a single Euler step of an approximate numerical solution with $x(0) = 0.5$ and step size $h = 1$.

b) On the other copy, do the same as above, with the midpoint method.

c) Assume `h` has been defined and that `znow` and `tnow` are known. Write the MATLAB code to find `znew` using Euler's method. This just one time step, not a complete solution.

d) Same as above, this time using the midpoint method.

2) Car. A rear wheel drive car is trying to pull down a wall with a cable. The cable is along the line CA. The front wheel is rolling freely. The rear wheel is skidding while trying and failing to pull the wall down. The coefficient of friction with the road is $\mu = 1$. The mass of the car is m. Gravity is g. In terms of some or all of m, g, d, L and h find the vertical component of the force acting on the car from the road at A.



3) Particle in viscous fluid. A particle with mass m is released from rest at $x = 0$ in a viscous fluid which resists it's motion with a force $F_d = cv$. Ignore buoyancy. Gravity g acts on the particle.. Assume x is positive for downwards motion.

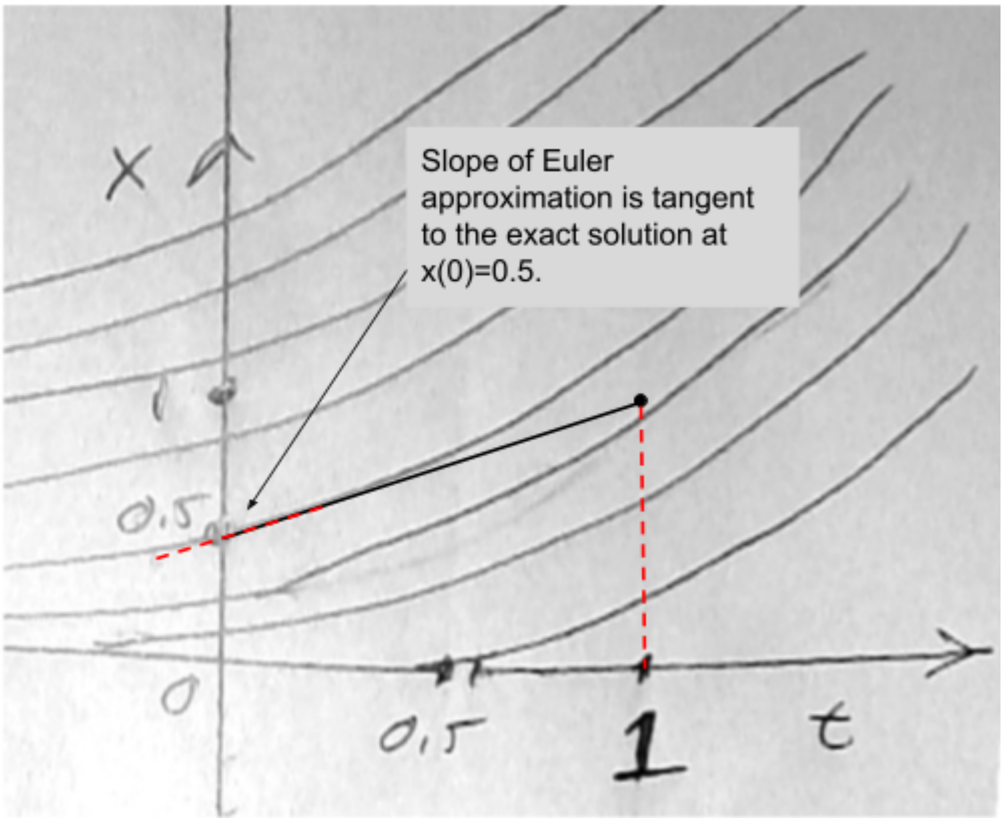
a) Find a formula for $x(t)$.

b) On three separate plots plot $x(t), v(t) = \dot{x}(t)$ and $a(t) = \ddot{x}(t)$. Label and evaluate any intercepts, slopes or asymptotes that you can.

Question 1

1a. Problem Statement: Show, accurately and clearly a single Euler step of an approximate numerical solution with x(0)= 0.5 and step size h = 1.

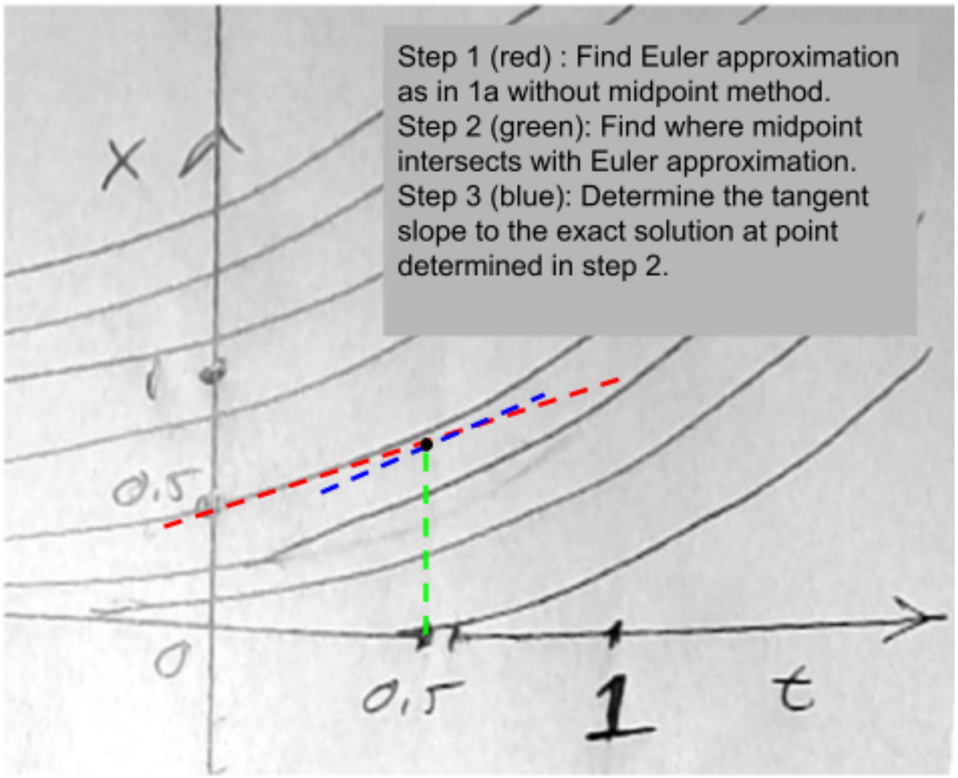
Solution:

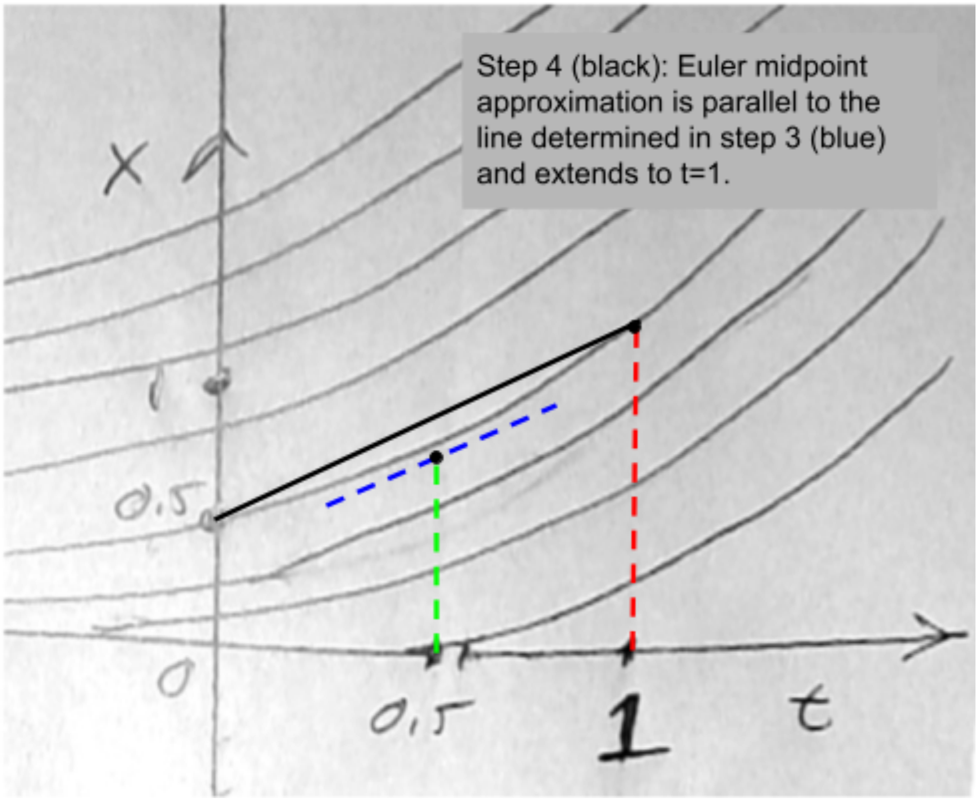


Line starts at (0, 0.5), slope is tangent at (0,0.5) and extends until endpoint lines up with t=1.

1b. Problem Statement: Show, accurately and clearly a single Euler step of an approximate numerical solution with x(0)= 0.5 and step size h = 1 using the midpoint method.

Solution:





1c. Problem Statement: Assume h has been defined and that znow and tnow are known. Write the MATLAB code to find znew using Euler's method.

Solution:

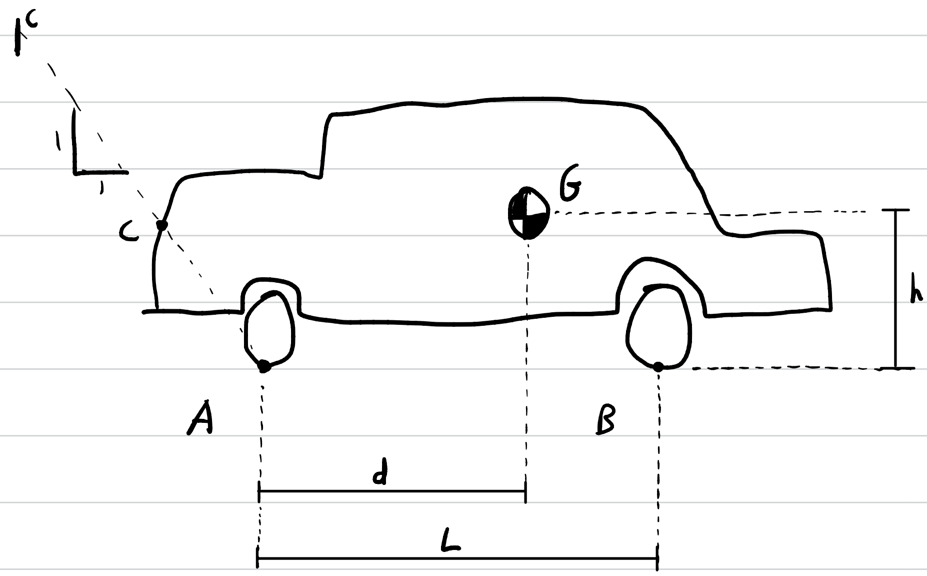
```
zdot = myrhs( tnow , znow );
znew = znow + (h*zdot);
```

1d. Problem Statement: Assume h has been defined and that znow and tnow are known. Write the MATLAB code to find znew using Euler's midpoint method.

Solution:

```
zdot = myrhs( tnow , znow );
tmid= tnow + (h/2);
zmid= znow + ((h/2)*zdot);
zdotmid=myrhs( tmid , zmid );
znew = znow + (h*zdotmid);
```

Question 2

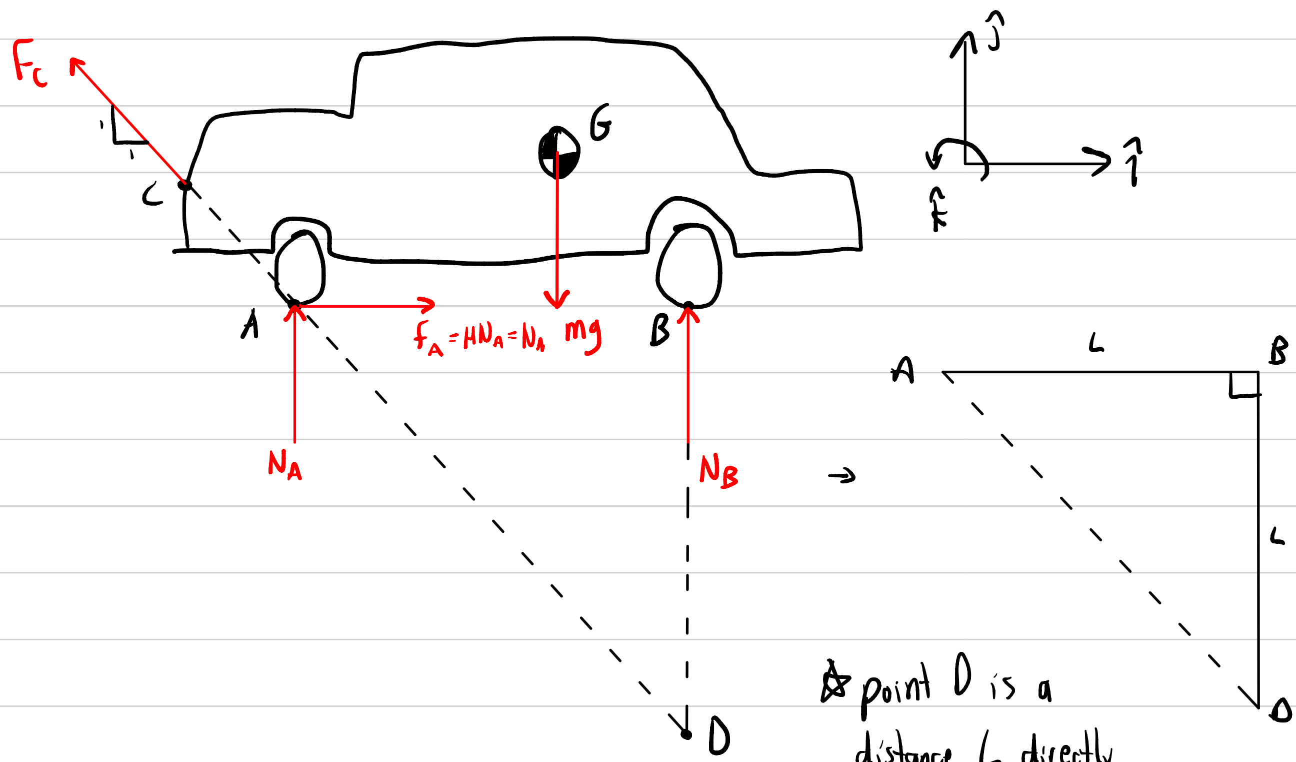


Given: $\mu = 1$, m, g, d, L, h

Find: vertical component of the force acting on car at A

SOLUTION 1: ΣM about imaginary point D, in line with F_c and below B

FBD:



☆ point D is a distance L directly below B ☆

only mg vector, $\mu \vec{N}_A$, and $\vec{N}_A$ cause moment about D: 1 equation, 1 unknown

$$\sum M_D = 0 \quad (+\circlearrowleft) \quad \rightarrow \text{ car is not rotating}$$

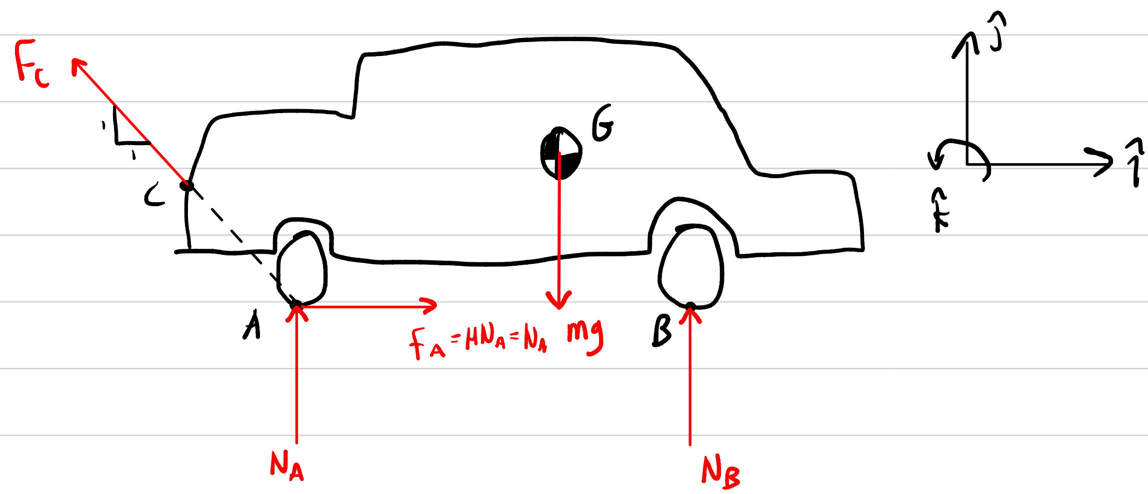
$$\sum M_D = (L-d)mg - L\mu N_A - L N_A = 0$$

$$(L-d)mg = LN_A(1+\mu) = 2LN_A \quad \text{since } \mu = 1$$

$$\boxed{N_A = \frac{mg}{2}\left(1 - \frac{d}{L}\right)}$$

SOLUTION 2: $\sum M$ about point A + LMB

FBD:



$$AMB_{/A}: \sum M_A = 0 \quad (+\circlearrowleft)$$

$$\sum M_A = -mgd + N_B L = 0$$

$$N_B = \frac{mgd}{L} \quad ①$$

$$LMB \ \hat{i}: \ \frac{\sqrt{2}}{2} F_C = \mu N_A \rightarrow N_A = \frac{\sqrt{2}}{2} F_C \quad ②$$

$$LMB \ \hat{j}: \ N_A + N_B - mg + \frac{\sqrt{2}}{2} F_C = 0 \quad ③$$

$$N_A + \frac{mgd}{L} - mg + \frac{\sqrt{2}}{2}F_c = 0 \quad ④$$ sub ① into ③

$$\frac{\sqrt{2}}{2}F_c + \frac{mgd}{L} - mg + \frac{\sqrt{2}}{2}F_c = 0 \quad ⑤$$ sub ② into ④

$$\sqrt{2}F_c + \frac{mgd}{L} - mg = 0$$ simplify

$$F_c = \frac{1}{\sqrt{2}}\left(mg - \frac{mgd}{L}\right) \longrightarrow \boxed{N_A = \frac{mg}{2}\left(1 - \frac{d}{L}\right)}$$

relationship from ②

☆ Both solutions 1 and 2 lead to the same result. ☆

☆ IN GENERAL: use AMB / LMB to produce 3 linearly independent equations that ☆ solve for N_A, N_B, and F_c.

☆ $N_A = \frac{mg}{2}\left(1 - \frac{d}{L}\right)$ is half of what N_A would be if the car was ☆ just sitting there, not pulling on anything.

initial conditions

$x(0) = 0$

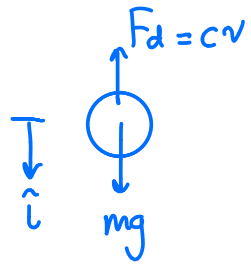
$v(0) = 0$

3) Particle in viscous fluid. A particle with mass m is released from rest at $x = 0$ in a viscous fluid which resists it's motion with a force $F_d = cv$. Ignore buoyancy. Gravity g acts on the particle.. Assume x is positive for downwards motion.

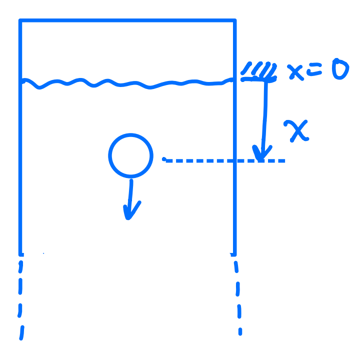
a) Find a formula for $x(t)$.

b) On three separate plots plot $x(t)$, $v(t) = \dot{x}(t)$ and $a(t) = \ddot{x}(t)$. Label and evaluate any intercepts, slopes or asymptotes that you can.

a) FBD:



particle released from rest @ $x=0$



LMB:

$$\sum \vec{F} = m\vec{a}$$

$$\{\ \}\cdot\hat{i} \Rightarrow mg - cv = ma$$

$$\dot{v}$$

$$\Rightarrow m\dot{v} + cv = mg$$

$$v = v_p + v_h$$

guess: v_p = constant

$$\Rightarrow \cancel{m\dot{v}_p} + cv_p = mg \qquad 0$$

$$v_p = mg/c$$

guess: $v_h = Ae^{\lambda t}$

$$m\dot{v}_h + cv_h = 0$$

$$m\lambda \cancel{e^{\lambda t}} + c\cancel{e^{\lambda t}} = 0$$

$$m\lambda + c = 0$$

$$\Rightarrow \lambda = -c/m$$

$$\Rightarrow v_h = Ae^{-\frac{ct}{m}}$$

$$v = v_p + v_h$$
$$= \frac{mg}{c} + Ae^{-\frac{ct}{m}}$$

→ initial conditions: $v(0) = 0$

$$\frac{mg}{c} + A \cdot 1 = 0$$
$$A = -\frac{mg}{c}$$
$$\Rightarrow v(t) = \frac{mg}{c}\left(1 - e^{-\frac{ct}{m}}\right) \quad *$$
$$\dot{x} = v = \left\{ \underbrace{\frac{mg}{c}\left(1 - e^{-\frac{ct}{m}}\right)} \right\}$$
$$\int_0^t \{\ \} \cdot dt' \Rightarrow x = \frac{mg}{c}\left(t + \frac{m}{c} e^{-\frac{ct}{m}}\right) + B$$

$x(0) = 0$

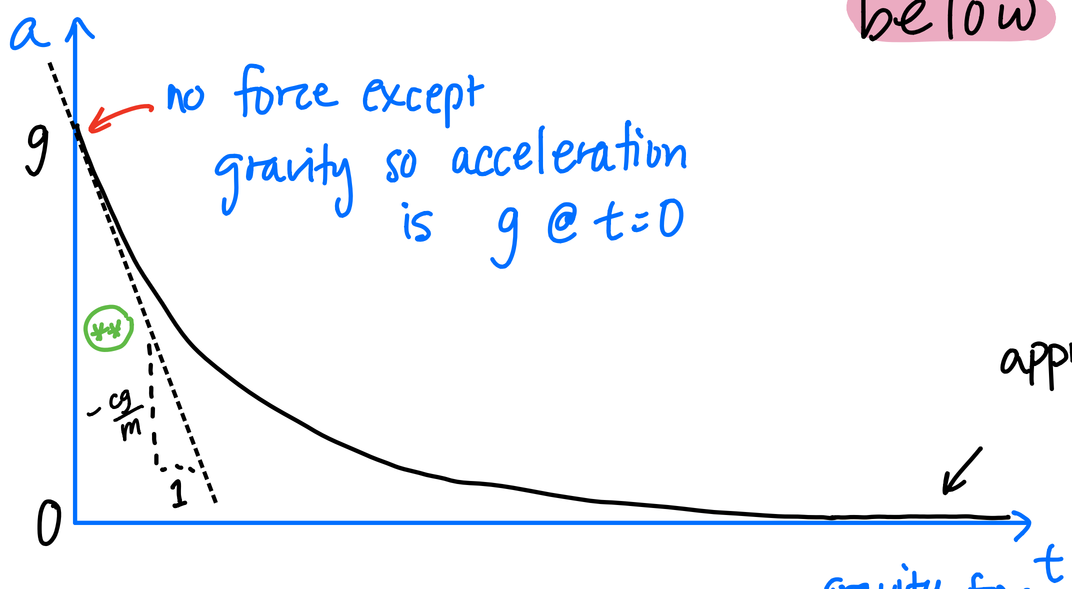
$$\frac{mg}{c}\left(\frac{m}{c}\right) + B = 0$$
$$\frac{m^2 g}{c^2} + B = 0$$
$$B = -\frac{m^2 g}{c^2}$$

$$\boxed{x(t) = \frac{mg}{c}\left(t + \frac{m}{c} e^{-\frac{ct}{m}}\right) - \frac{m^2 g}{c^2}}$$

expanded: $x(t) = \frac{mg}{c} t + \frac{m^2 g}{c^2} e^{-\frac{ct}{m}} - \frac{m^2 g}{c^2}$

b) plot of $a(t) = \ddot{x}(t)$

*see calculation below



no force except gravity so acceleration is g @ $t=0$

approaches zero as $t \to \infty$

gravity force balances out F_d so $\vec{a}=0$ @ $t \to \infty$

mathematically: $v(t) = \frac{mg}{c}\left(1 - e^{-\frac{c}{m}t}\right)$

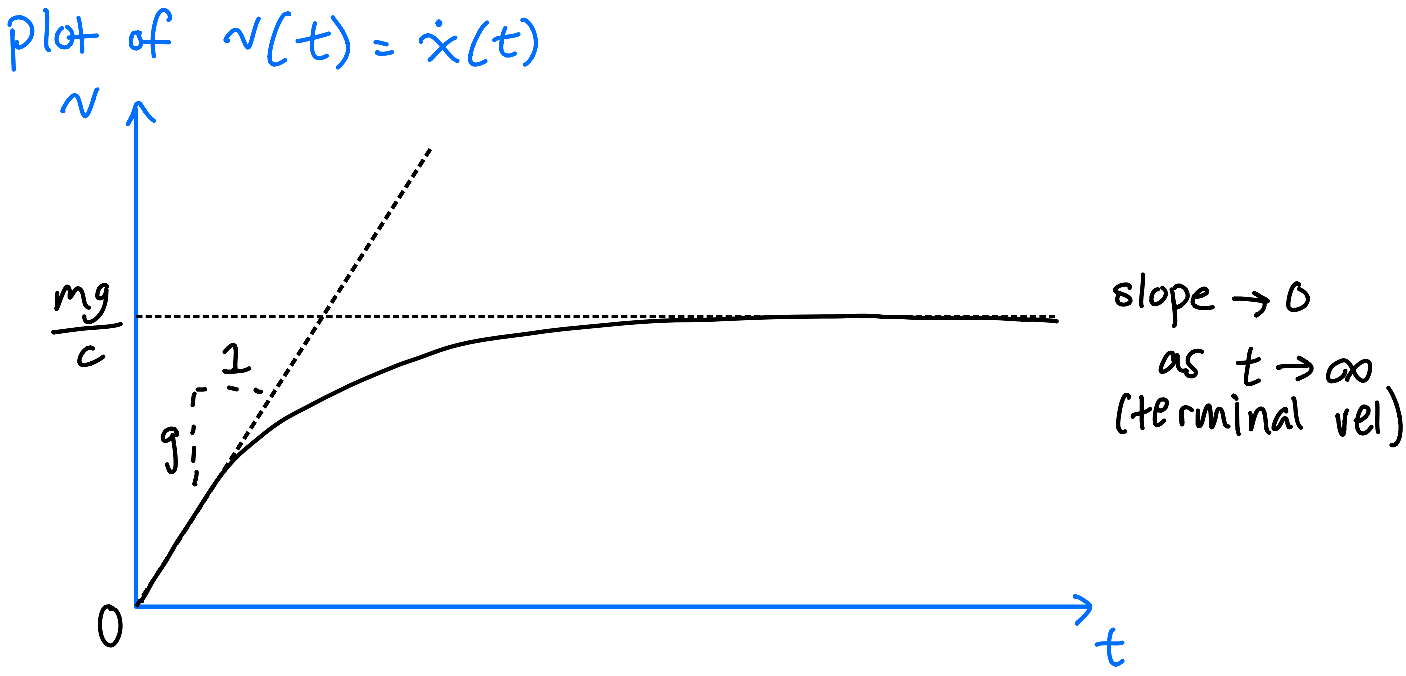
$$a(t) = \frac{dv}{dt} = \frac{mg}{c}\left[-\frac{c}{m}e^{-\frac{c}{m}t}\right]$$

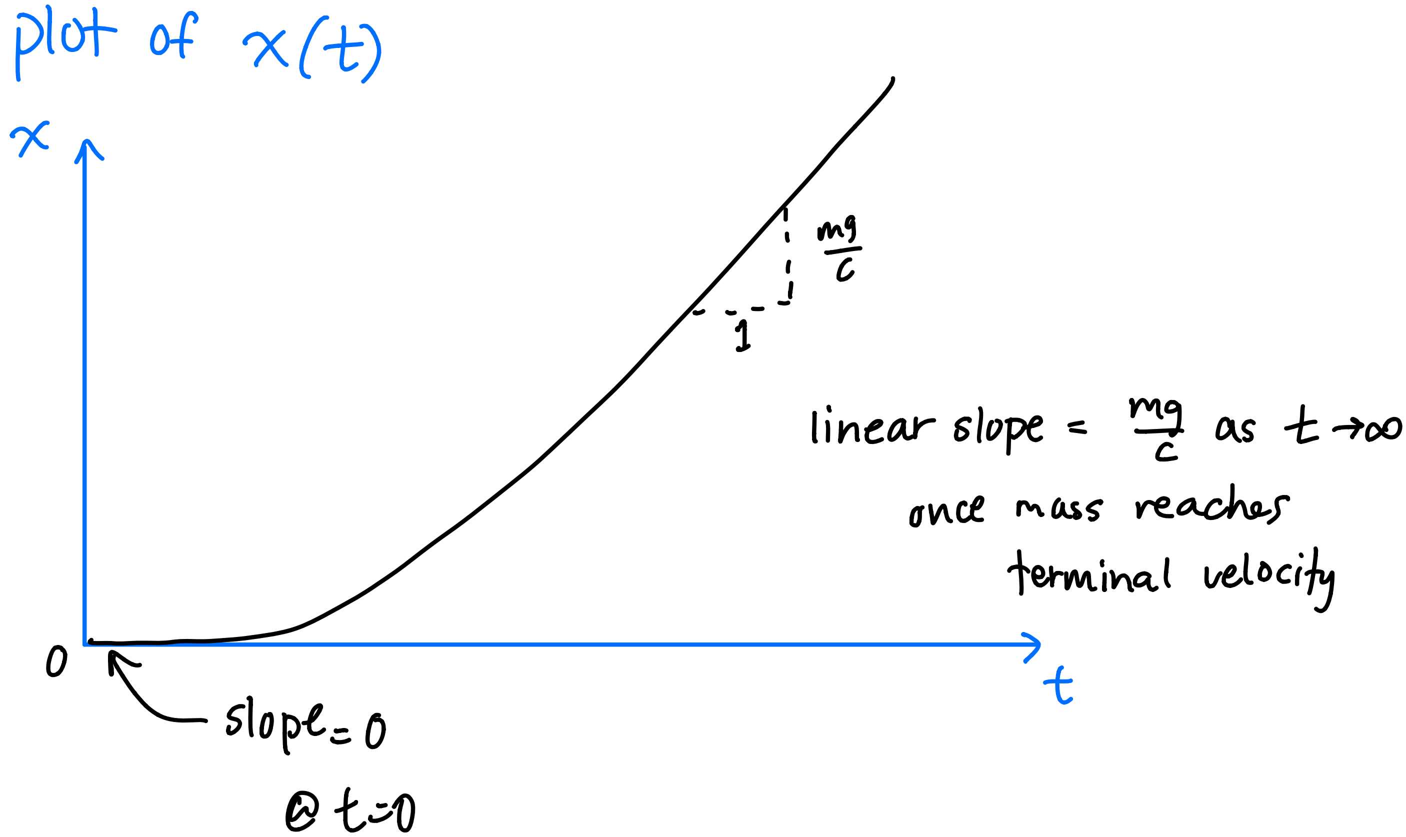
$$\boxed{a(t) = g e^{-\frac{c}{m}t}}$$

(**) Initial slope of graph:

$$\frac{d}{dt}a(t) = -\frac{cg}{m}e^{-\frac{c}{m}t}$$

@ $t=0 \to -\frac{cg}{m}$


plot of $v(t) = \dot{x}(t)$
v
$\frac{mg}{c}$
1
g
0
t
slope → 0
as $t \to \infty$
(terminal vel)


plot of $x(t)$
x
$\frac{mg}{c}$
1
linear slope = $\frac{mg}{c}$ as $t \to \infty$
once mass reaches
terminal velocity
0
t
slope = 0
@ t = 0