Recitation, week of 3/29

Setup. A particle P has position $\vec{r}_{0/\mathrm{P}}$ which we call, simply, $\vec{\boldsymbol{r}}$. For all of this problem

$$\vec{\boldsymbol{r}} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}} \qquad \text{(position)}$$

$$\begin{aligned} \vec{\boldsymbol{v}} = \dot{\vec{\boldsymbol{r}}} &= v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}} \qquad \text{(velocity)} \\ &= \dot{x}\hat{\boldsymbol{\imath}} + \dot{y}\hat{\boldsymbol{\jmath}} \end{aligned}$$

$$\begin{aligned} \vec{\boldsymbol{a}} = \dot{\vec{\boldsymbol{v}}} = \ddot{\vec{\boldsymbol{r}}} &= a_x\hat{\boldsymbol{\imath}} + a_y\hat{\boldsymbol{\jmath}} \qquad \text{(acceleration)} \\ &= \dot{v}_x\hat{\boldsymbol{\imath}} + \dot{v}_y\hat{\boldsymbol{\jmath}} \\ &= \ddot{x}\hat{\boldsymbol{\imath}} + \ddot{y}\hat{\boldsymbol{\jmath}} \end{aligned}$$

and

$$\begin{aligned} &x = r\cos\theta, \qquad y = r\sin\theta \\ &\theta = \theta(t), \quad \text{and} \quad r = \text{constant}. \end{aligned}$$

Assume $\theta(t)$ is given and is such that $\theta(t)$ starts at $\theta(0) = 0$ and grows to much bigger than 2π.

Questions.

For this set of problems do not use polar coordinate formulas you remember. (Well you *can* use them after you derive them using what is given above.) You must derive everything clearly.

a) Draw the path of the particle.

b) Pick a random point on the path and park a point representing the particle.

c) Draw the vector $\vec{\boldsymbol{r}}$.

d) Define a vector $\hat{\boldsymbol{e}}_r \equiv \vec{\boldsymbol{r}}/|\vec{\boldsymbol{r}}|$. Find $\hat{\boldsymbol{e}}_r$ in terms of $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$ and θ. Draw $\hat{\boldsymbol{e}}_r$, with the tail just beyond the tip of $\vec{\boldsymbol{r}}$.

e) The unit vector $\hat{\boldsymbol{k}} = \hat{\boldsymbol{\imath}} \times \hat{\boldsymbol{\jmath}}$. Define the unit vector $\hat{\boldsymbol{e}}_\theta \equiv \hat{\boldsymbol{k}} \times \hat{\boldsymbol{e}}_r$. Find $\hat{\boldsymbol{e}}_\theta$ in terms of $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$ and θ. Draw $\hat{\boldsymbol{e}}_\theta$, with its tail at the tail of $\hat{\boldsymbol{e}}_r$.

f) Find $\vec{\boldsymbol{v}}$ in terms of $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$, r, θ and $\dot{\theta}$. [Hint: use the chain rule.]

g) Find $\vec{\boldsymbol{v}}$ in terms of $\hat{\boldsymbol{e}}_\theta$, r and $\dot{\theta}$. Draw $\vec{\boldsymbol{v}}$ on the main drawing. Put it on top of $\hat{\boldsymbol{e}}_\theta$ with its tail on top of $\hat{\boldsymbol{e}}_\theta$'s tail.

h) Find $\dot{\hat{\boldsymbol{e}}}_r$ in terms of $\hat{\boldsymbol{e}}_\theta$ and $\dot{\theta}$. [Hint, differentiate the result from (d) and compare with (e).]

i) Note that $\vec{\boldsymbol{r}} = r\hat{\boldsymbol{e}}_r$. Differentiate that expression to find another derivation of (g) above.

j) Using any mixture of the methods above, find $\vec{\boldsymbol{a}} \equiv \dot{\vec{\boldsymbol{v}}}$ in terms of $\hat{\boldsymbol{e}}_r$, $\hat{\boldsymbol{e}}_\theta$, θ, $\dot{\theta}$, $\ddot{\theta}$ and r. [Hint: You will have to use the chain rule and the product rule.]

k) Define the speed $v = r\dot{\theta}$. Find $\vec{\boldsymbol{a}}$ in terms of $\hat{\boldsymbol{e}}_r$, $\hat{\boldsymbol{e}}_\theta$, r, v and $\dot{v}$.