MAE 2030 Lecture #21:

3/29/21

Today: (1) Review
(2) Circular motion

Review:

Mechanics:

Given: some info about forces in motion
Find: other info about forces in motion

Dynamics:

Systems: particles & rigid bodies

* this course: mostly 2D (some 3D)

Tools: FBDs
Reaction & reaction
LMB & AMB
Force laws
Math: solve for desireable unknowns;
solve ODEs; analytically
MATLAB

Checks & shortcuts:

Energy, Power,
Conservation of $\vec{L}$
conservation of $\vec{H}_{/C}$

Organization: Progressive difficulty

particles → rigid objects

1D → 2D → 3D

one object → 2 or 3 objects

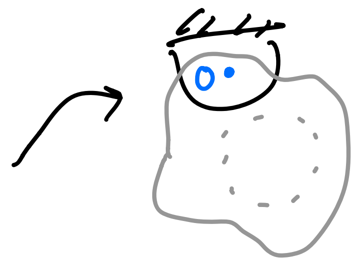
force laws + constraint forces

Circular Motion:

why is circular motion important?

→ important in engineering

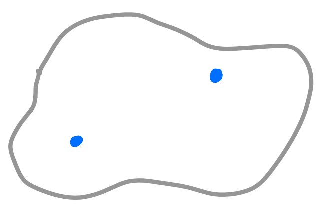
→ machines are full of hinges and all points in the object attached go in circles around the hinge (○)



→ machines are made of rigid parts

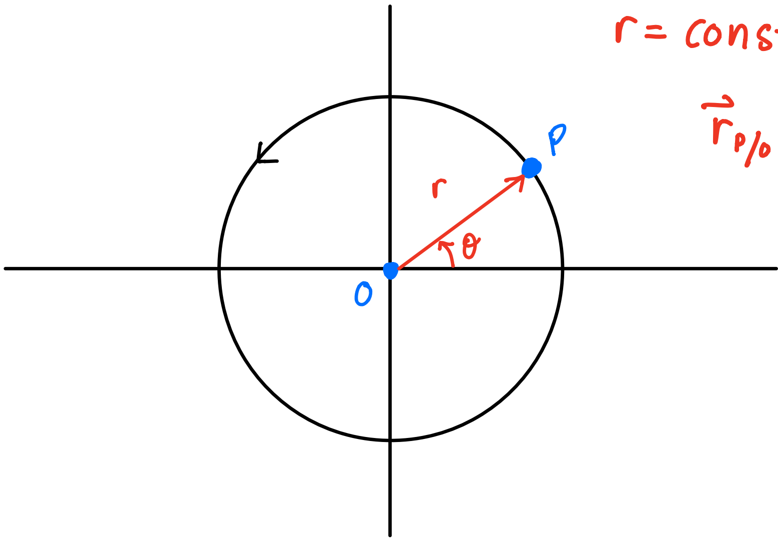
arbitrary 2D motion

↳ relative motion of any 2 points is circular motion



Kinematics of circular motion:

path of particle

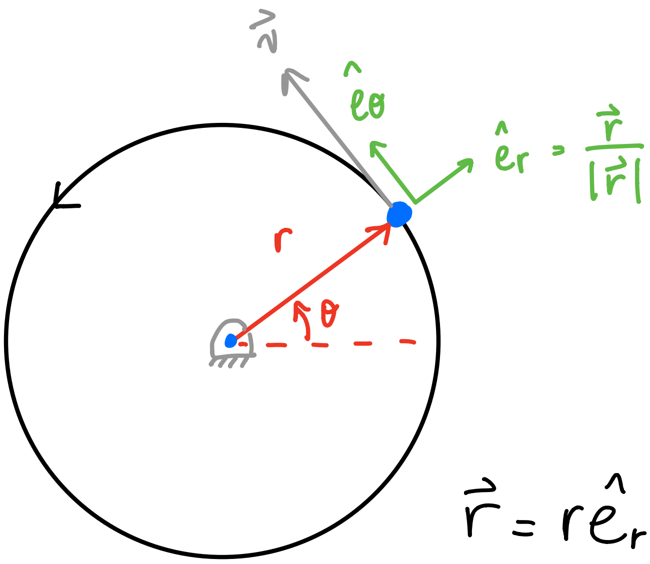


magnitude

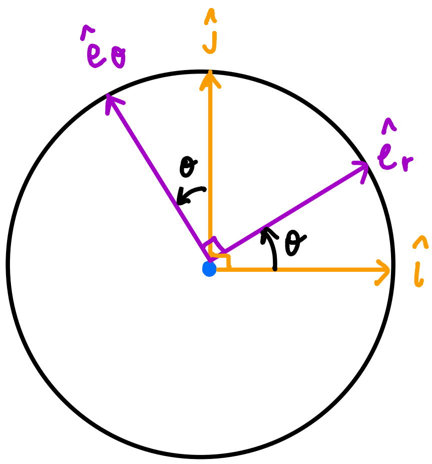
$r = \text{constant} = |\vec{r}|$

$\vec{r}_{P/O} = \vec{r} \neq \text{constant}$

unit vectors: $\hat{e}_r, \hat{e}_\theta$



unit circle:



$$\vec{r} = r\hat{e}_r$$

$$\vec{v} = r\dot{\theta}\,\hat{e}_\theta$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{\Delta\vec{v}}{\Delta t}$$

$\lim_{\Delta t \to 0}$

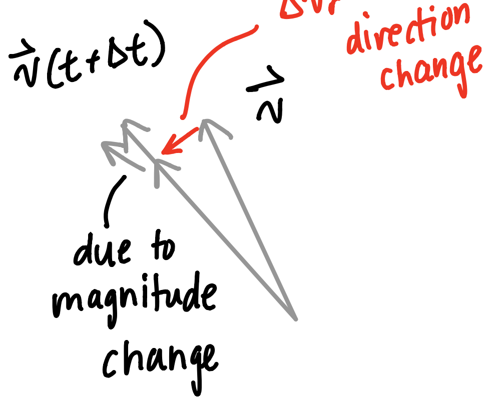
$$= -\dot{\theta} v \hat{e}_r + \dot{v}\,\hat{e}_\theta$$

$r\dot{\theta}$

$= -\dot{\theta}^2 r$

$= \frac{-v^2}{r}$

$(r\ddot{\theta})$



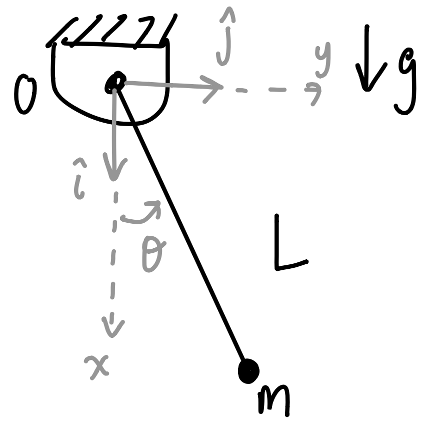
2 common ways to write $\vec{a}$:

$$\vec{a} = -\dot{\theta}^2 r \hat{e}_r + \ddot{\theta} r \hat{e}_\theta$$

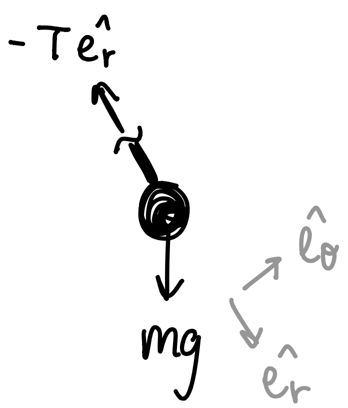
$$\vec{a} = -\frac{v^2}{r}\hat{e}_r + \dot{v}\hat{e}_\theta$$

demo: pendulum ~~problem~~ hypnosis session

pendulum:



FBD:



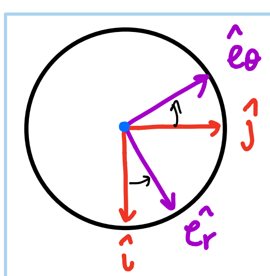
LMB: $\sum \vec{F} = m\vec{a}$

$$\{mg\hat{i} - T\hat{e}_r = m(\ddot{\theta} r \hat{e}_\theta - \dot{\theta}^2 r \hat{e}_\theta)\}$$

does NOT spark joy

∴ clean house → Marie Kondo is happy

$$\{\ \}\cdot\hat{e}_\theta \Rightarrow \cancel{m}g\underbrace{\hat{i}\cdot\hat{e}_\theta}_{-\sin\theta} = \cancel{m}\ddot{\theta} r$$



$$\rightarrow \boxed{\ddot{\theta} = -\frac{g}{r}\sin\theta}$$

L = r