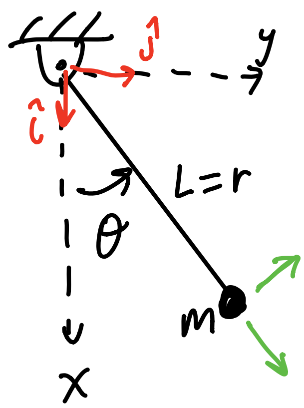
MAE 2030 Lecture #22:

3/31/21

Today: ① Circles continued

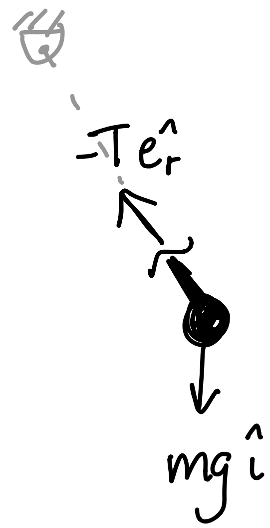
Pendulum:

recall:



$\downarrow g$

FBD:



LMB: $\sum \vec{F} = m\vec{a}$

$$\left\{ -\underbrace{T\hat{e}_r}_{} + mg\hat{i} = m(r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r) \right\} \quad (*)$$

does not spark joy :(

$$\{\ \}\cdot\hat{e}_\theta \Rightarrow mg\underbrace{\hat{i}\cdot\hat{e}_\theta}_{-\sin\theta} = mr\ddot{\theta}$$

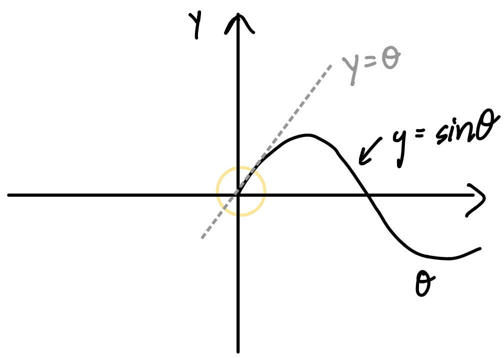
$$\Rightarrow -\cancel{m}g\sin\theta = \cancel{m}L\ddot{\theta}$$

$$\boxed{\ddot{\theta} = -\frac{g}{L}\sin\theta}$$

if $\underbrace{\theta \ll 1}_{\text{small angle approximation}} \longrightarrow \boxed{\ddot{\theta} = -\frac{g}{L}\theta}$ harmonic oscillator

Aside: small angle approx

1)



2) $\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} \dots$

$\approx \theta \qquad \theta \ll 1$

What about T?

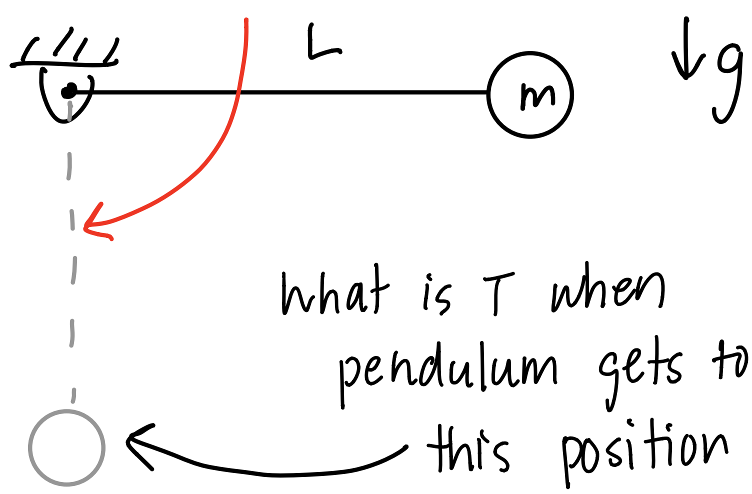
$\{(*)\} \cdot \hat{e}_r$

$$\Rightarrow -T\underbrace{\hat{e}_r \cdot \hat{e}_r}_{1} + mg\underbrace{\hat{i} \cdot \hat{e}_r}_{\cos\theta} = m\left[r\ddot{\theta}\underbrace{\hat{e}_\theta \cdot \hat{e}_r}_{0} - r\dot{\theta}^2\underbrace{\hat{e}_r \cdot \hat{e}_r}_{1}\right]$$

$$\Rightarrow -T + mg\cos\theta = -mr\dot{\theta}^2$$

$$\boxed{T = mg\cos\theta + mr\dot{\theta}^2} \quad (**)$$

ex)



What is T when pendulum gets to this position?

Method #1:

Conservation of energy:

$$E_K(0) + E_P(0) = E_K(t^*) + E_P(t^*)$$

t^* is when pendulum is vertical

datum $E_p = 0$

$$0 + 0 = \frac{1}{2} m v^{*2} + - mgL$$

$$\Rightarrow \boxed{v^{*2} = 2gL}$$

From (**) : $T = mg \underbrace{\cos\theta}_{t^* \Rightarrow 1} + m \underbrace{r\dot{\theta}^2}_{v^2/r}$

$$T = mg + \frac{m v^{*2}}{L} \rightarrow v^{*2} = 2gL$$

$$T = mg + 2mg$$

$$\boxed{T = 3mg}$$

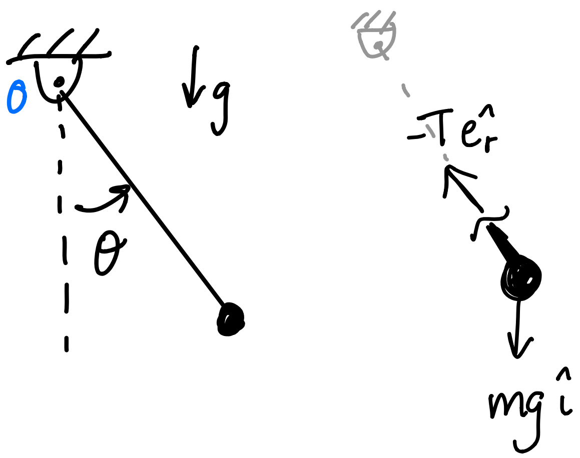
Method #2:

Note: no simple analytic sol'n to ODE

$$\ddot{\theta} = -\frac{g}{L}\sin\theta$$

$\Rightarrow$ use MATLAB "integrate"
use "events" then use v at event

Method #3:



AMB$_{/O}$: $\sum \vec{M}_{/O} = \dot{\vec{H}}_{/O}$

$$-mgL\sin\theta\,\hat{k} = \vec{r}_{/O} \times m\vec{a}$$

$\vec{r}_{/O}$: $L\hat{e}_r$

$\vec{a}$: $L\ddot{\theta}\hat{e}_\theta - \cancel{L\dot{\theta}^2\hat{e}_r}$

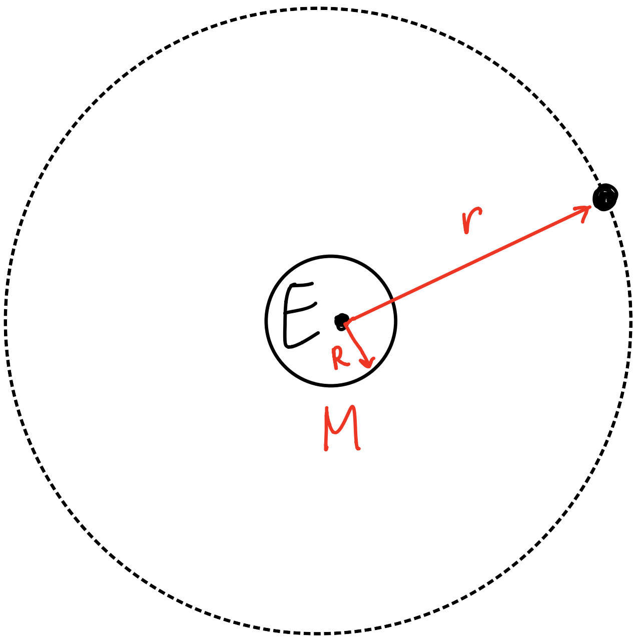
$\hat{e}_r \times \hat{e}_r = \vec{0}$

$$\{-\cancel{m}gL\sin\theta\,\hat{k} = \cancel{m}L^2\ddot{\theta}\,\hat{k}\}$$

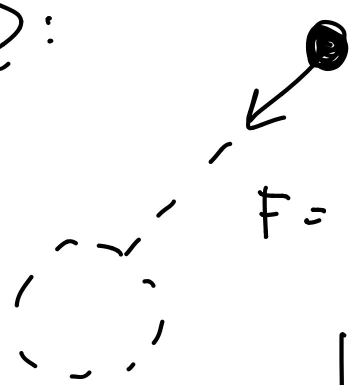
$$\{\ \}\cdot k \Rightarrow \boxed{\ddot{\theta} = -\frac{g}{L}\sin\theta}$$

ex) satellite & Earth

How fast is satellite?



FBD :



$$F = \frac{GMm}{r^2}$$

$$\Downarrow \quad r=R$$

$$F = \frac{GM\cancel{m}}{R^2} = \cancel{m}g$$

$$= \frac{GMR^2}{r^2R^2}m$$

$$\boxed{F = \frac{gR^2}{r^2}m}$$

LMB : $\sum \vec{F} = m\vec{a}$

$$\left\{ -\frac{gR^2}{r^2} m\,\hat{e}_r = m(r\ddot{\theta}\,\hat{e}_\theta - r\dot{\theta}^2\,\hat{e}_r) \right\}$$

$$\{\ \} \cdot \hat{e}_\theta \Rightarrow \boxed{\ddot{\theta} = 0}$$

$$\{\ \} \cdot \hat{e}_r \Rightarrow \frac{gR^2}{r^2}\cancel{m} = \cancel{m}r\dot{\theta}^2$$

$$= \frac{r^2\dot{\theta}^2}{r} = \frac{v^2}{r}$$

$$\Rightarrow \boxed{v^2 = \frac{gR^2}{r}}$$