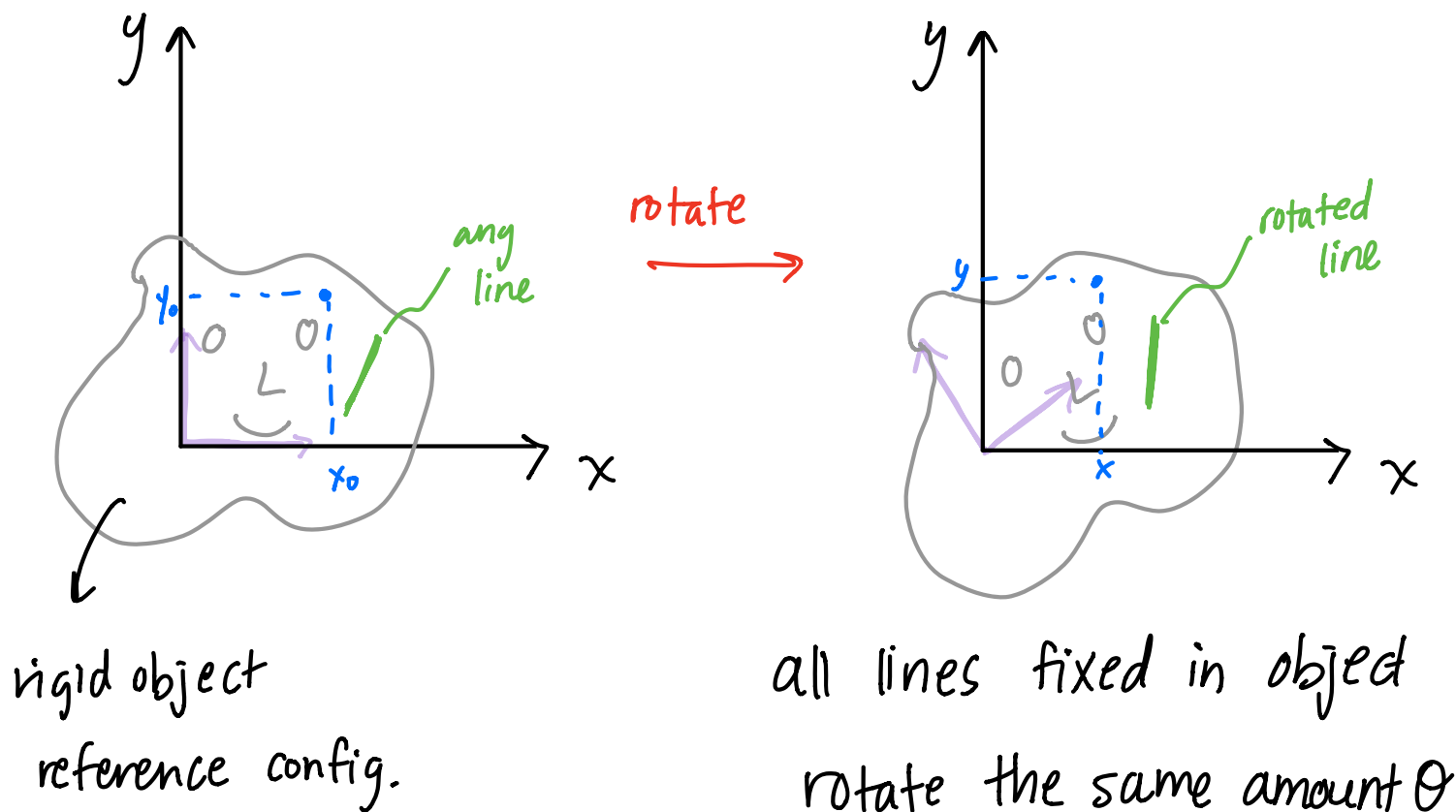


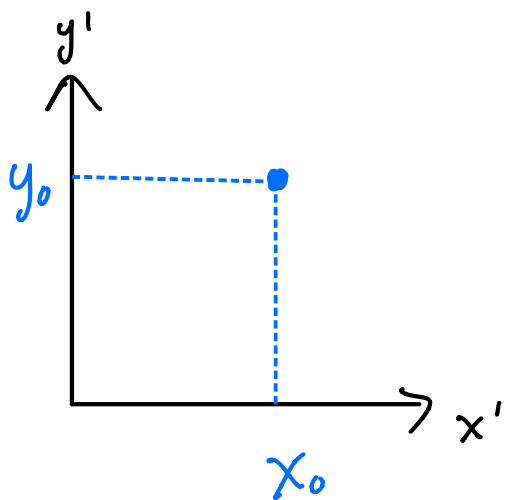
Today: ① Rotation

Rotation of a Rigid Object (2D):

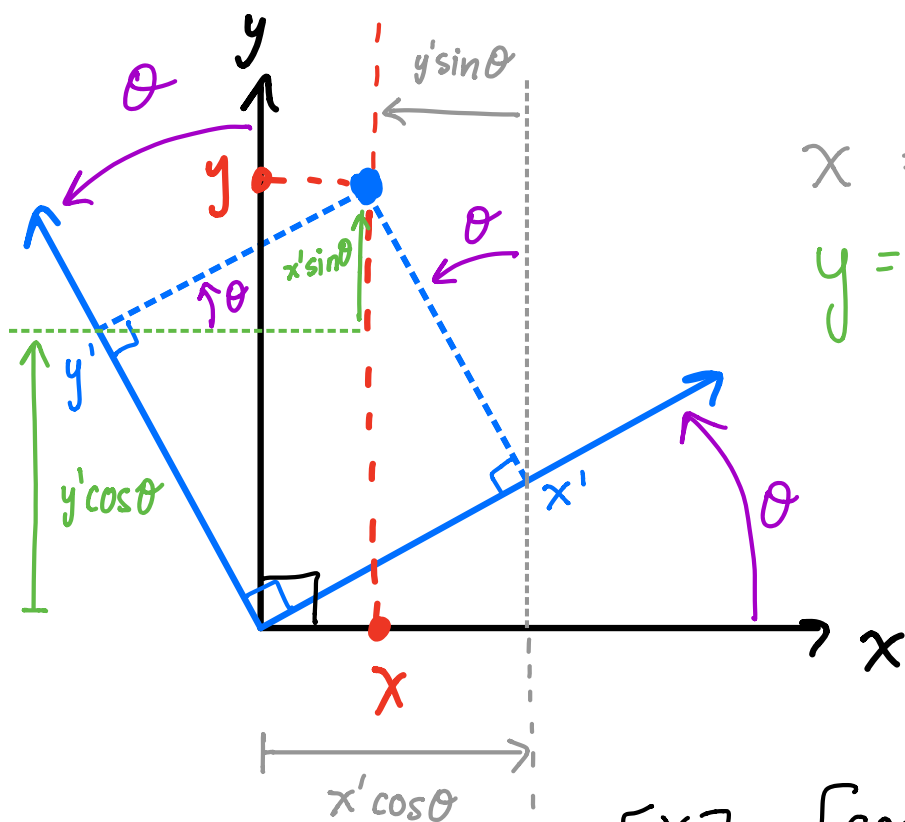
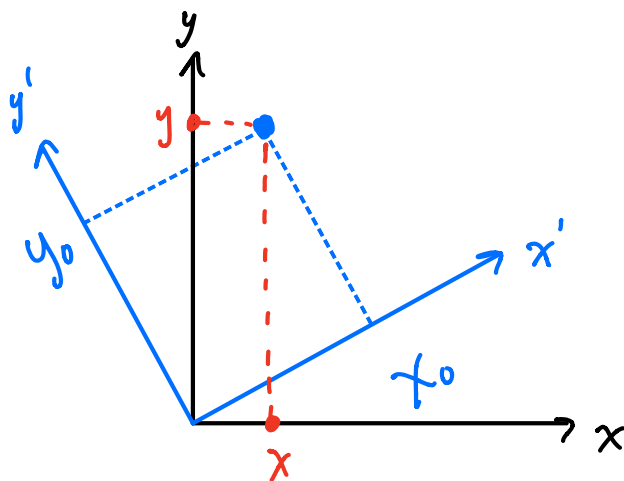


given coord of a point in reference config.,
find coordinates in rotated config.

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \xrightarrow{\text{rotate}} \begin{bmatrix} x \\ y \end{bmatrix}$$



$x' - y'$
is rotated
 $x - y$
 $x' = x_0$
 $y' = y_0$



$$x = x' \cos \theta - y' \sin \theta$$

$$y = x' \sin \theta + y' \cos \theta$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

R = Rotation Matrix

Two interpretations of R :

1) Change of coordinates:

given x', y' coordinates of a point in
a crooked coordinate system
(coord rotated by θ from x, y)

$$\text{Find } \begin{bmatrix} x \\ y \end{bmatrix} = R \cdot \begin{bmatrix} x' \\ y' \end{bmatrix}$$

2) Rotation:

given original x, y coords of a point $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$
and that point rotates to $\begin{bmatrix} x \\ y \end{bmatrix} = R \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$

Why are these the same?

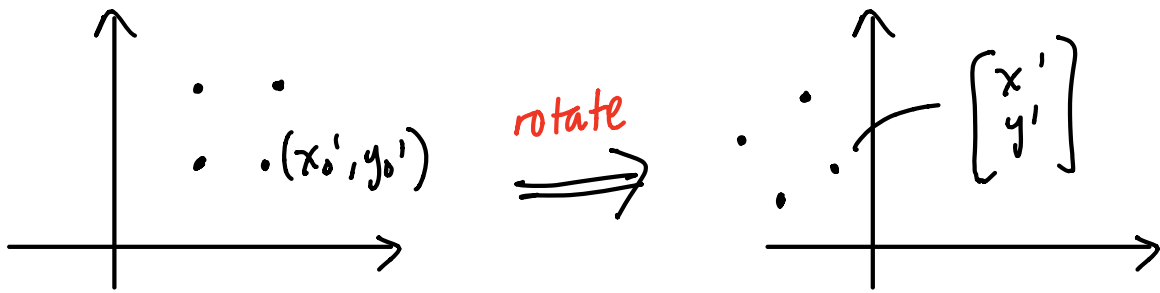
The $\begin{bmatrix} x' \\ y' \end{bmatrix}$ coords of a rotated point are $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$

Rotate a whole picture:

$$\begin{bmatrix} x_0' \\ y_0' \end{bmatrix} \xrightarrow{\text{rotate}} R \begin{bmatrix} x_0' \\ y_0' \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{bmatrix} x_0^2 \\ y_0^2 \end{bmatrix} \xRightarrow{\text{rotate}} R \begin{bmatrix} x_0^2 \\ y_0^2 \end{bmatrix} = \begin{bmatrix} x^2 \\ y^2 \end{bmatrix}$$

$$\begin{bmatrix} x_0^1 & x_0^2 & x_0^3 \\ y_0^1 & y_0^2 & y_0^3 \end{bmatrix} \xRightarrow{\text{rotate}} \begin{bmatrix} x^1 & x^2 & x^3 \\ y^1 & y^2 & y^3 \end{bmatrix} = R \begin{bmatrix} x_0^1 & x_0^2 & \dots \\ y_0^1 & y_0^2 & \dots \end{bmatrix}$$



demo: animate rotation of rigid objects
on MATLAB