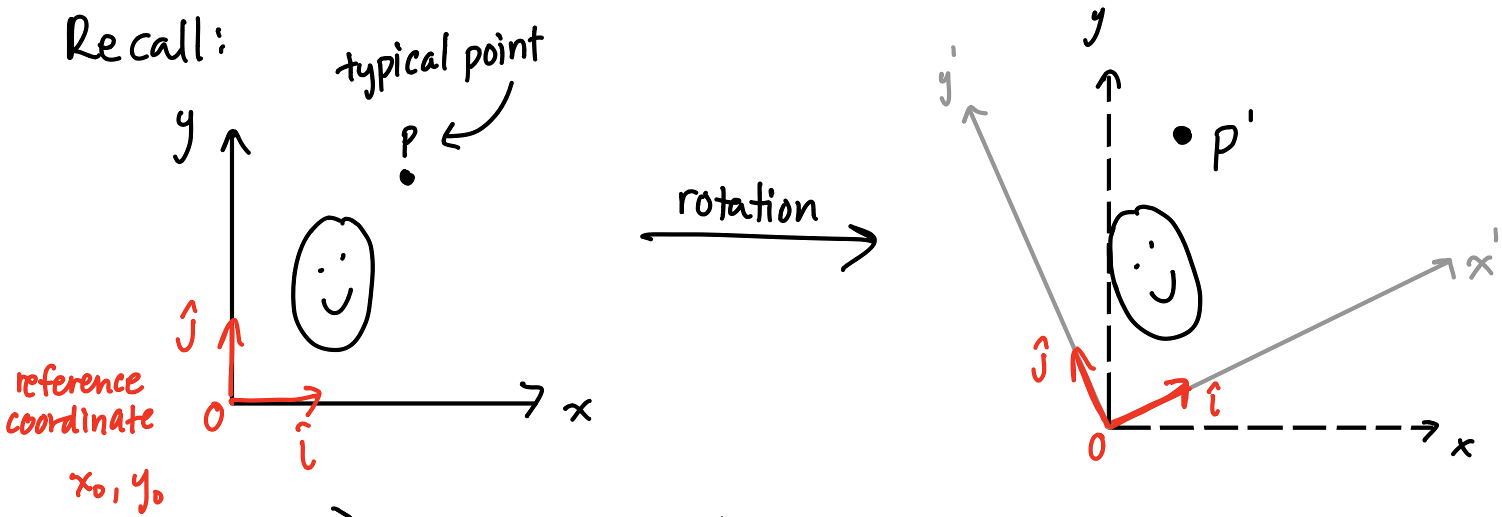
MAE 2030 Lecture # 24:

4/5/21

Today: ① Rotation continued
② Angular velocity $\omega, \vec{\omega}$

Recall:



$$\vec{r}_0 \xrightarrow{\text{rotate}} \vec{r}\,' = x'\hat{i}' + y'\hat{j}'$$

$$\vec{r}\,' = x_0\hat{i}' + y_0\hat{j}'$$

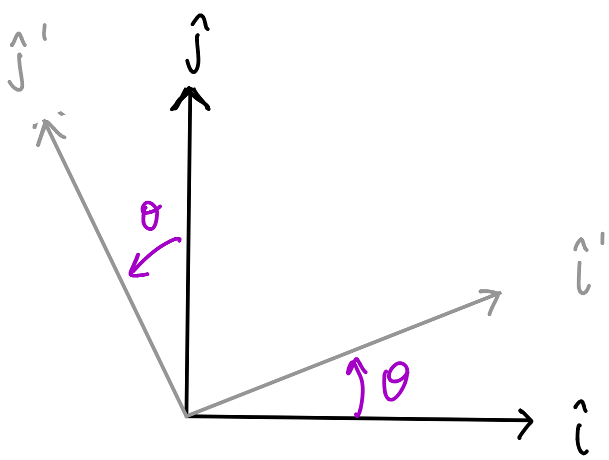
position of typical point in ref. config

$$\vec{r}_0 = x_0\hat{i} + y_0\hat{j}$$

$$[\vec{r}_0]_{xy} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$$[\vec{r}\,']_{xy} = x_0[\hat{i}']_{xy} + y_0[\hat{j}']_{xy}$$

$$= \underbrace{\left[\, [\hat{i}']_{xy} \;\middle|\; [\hat{j}']_{xy} \,\right]}_{R} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$



$$R = \left[\begin{array}{c|c} \text{components of} & \text{components of} \\ \hat{i}' \text{ in } x, y & \hat{j}' \text{ in } x, y \end{array}\right]$$

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$ ← as we got last lecture

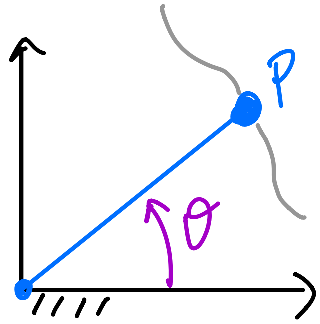
Angular velocity: w, $\vec{w} = w\hat{k}$

(2D)

1) Recall, for a particle: (Informal usage)

$\mathcal{F}$ fixed ref frame

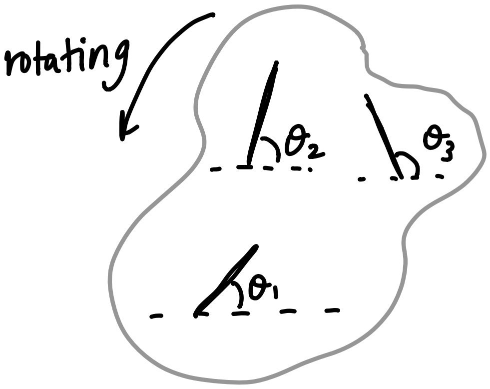
$\mathcal{N}$ Newtonian ref frame



$w = \dot{\theta}$

2) Angular velocity for rigid object

look at lines "marked" on object



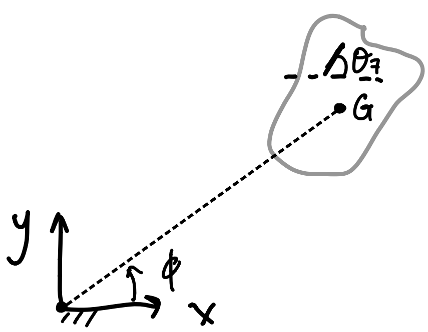
$\theta_1 \neq \theta_2 \neq \theta_3 \ldots$

$\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3 \ldots$

define $w = \dot{\theta}_1 = \dot{\theta}_2 = \ldots \dot{\theta}_n$

rate of change of orientation of every line on object

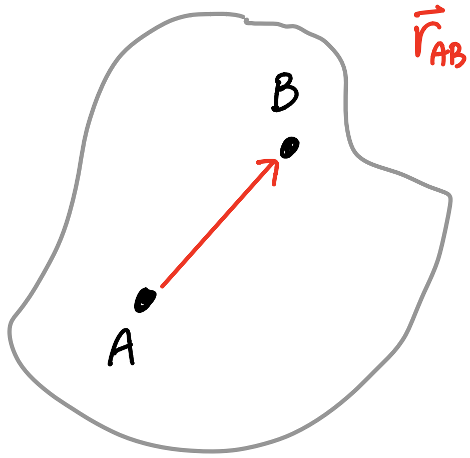
ex)



$w = \dot{\theta}_7$

$w \neq \dot{\phi}$

Main utility of $\vec{\omega}$:



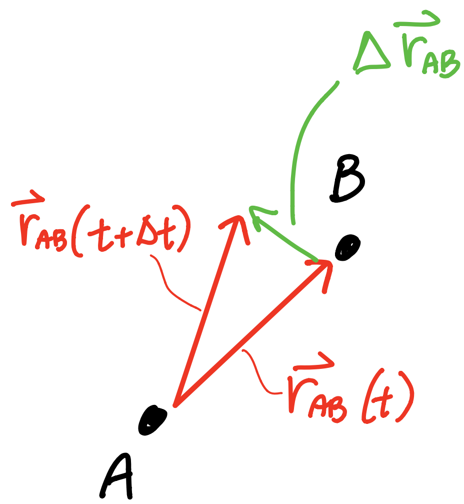
$\vec{r}_{AB} = \vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$

$\uparrow$

$\vec{r}_{B/0}$

independent of O

A & B are fixed to a rigid object



$\vec{r}_{AB}(t)$

$\dot{\vec{r}}_{AB} = ???$

a) $|\dot{\vec{r}}_{AB}| = |\vec{r}_{AB}||\dot{\theta}|$

b) direction is $\perp$ to $\vec{r}_{AB}$

$$\dot{\vec{r}}_{AB} = \lim_{\Delta t = 0} \frac{\Delta\vec{r}_{AB}}{\Delta t} = \frac{\Delta\vec{r}_{AB}}{\Delta\theta}\frac{\Delta\theta}{\Delta t}$$

$$\boxed{\dot{\vec{r}}_{AB} = \underbrace{\dot{\theta}\hat{k}}_{\omega} \times \vec{r}_{AB}}$$

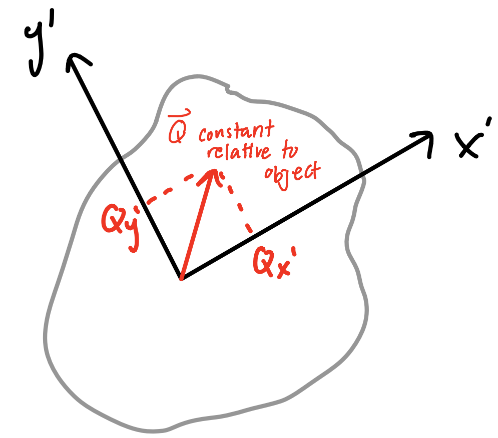
$$\boxed{\dot{\vec{r}}_{AB} = \omega \times \vec{r}_{AB}}$$

For any vector $\vec{Q}$ drawn on rigid object

$$\boxed{\dot{\vec{Q}} = \vec{\omega} \times \vec{Q}}$$

$\vec{Q}$ is drawn on object

$$\vec{Q} = Q_x \hat{i}' + Q_y \hat{j}'$$



$$\dot{\vec{Q}} = Q_{x'} \dot{\hat{i}}' + Q_{y'} \dot{\hat{j}}'$$

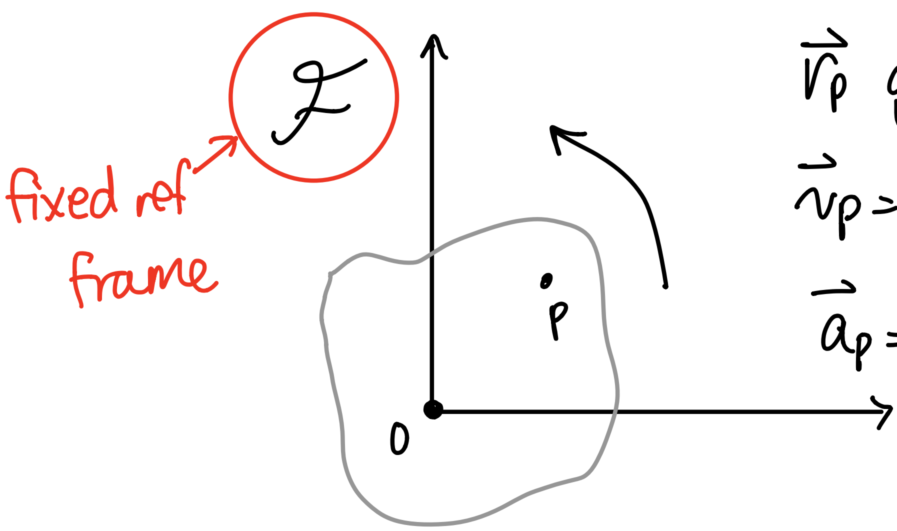
$$\dot{\hat{i}}' = \vec{\omega} \times \hat{i}$$

$$\dot{\hat{j}}' = \vec{\omega} \times \hat{j}$$

$$\boxed{\dot{\vec{Q}} = \vec{\omega} \times \vec{Q}}$$

for any $\vec{Q}$ which is constant in object

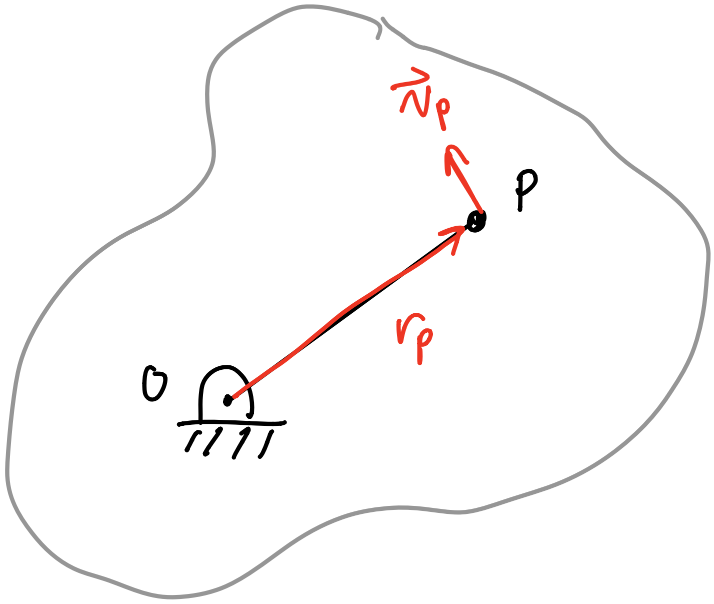
Consider object rotation about Q



$\vec{r}_p$ given

$\vec{v}_p = ?$

$\vec{a}_p = ?$



$$\vec{v}_p = \vec{v}_{P/O} = \vec{\omega} \times \vec{r}_p$$

same $\vec{\omega}$ for all points on object

$$\vec{a}_p = \frac{d}{dt}\vec{v}_p = \frac{d}{dt}\left(\vec{\omega} \times \vec{r}_p\right)$$

$$= \dot{\vec{\omega}} \times \vec{r}_p + \vec{\omega} \times \underbrace{\dot{\vec{r}}_p}_{\vec{v}_p = \vec{\omega} \times \vec{r}}$$

$$= \underbrace{\dot{\vec{\omega}}}_{\dot{\omega}\hat{k}} \times \vec{r}_p + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_p\right)$$

recall polar coord

$$\vec{a} = -r\dot{\theta}^2 \hat{e}_r + \ddot{\theta} r \hat{e}_\theta$$

looks similar!

$$\boxed{\vec{a}_p = \dot{\omega}\hat{k} \times \vec{r}_p - \omega^2 \vec{r}_p}$$

Laws of Mechanics for arbitrary system:

LMB:

$$\sum_{\text{all ext}} \vec{F}_i = \sum m_i \vec{a}_i = m_{tot}\vec{a}_G$$

$$\boxed{\vec{H}_{/c} = \sum \vec{r}_{i/c} \times m_i \vec{v}_{i/c}}$$

AMB:

any pt c

$$\boxed{\sum_{\substack{\text{all ext} \\ \text{torques}}} \vec{M}_{/c} = \sum \vec{r}_{i/c} \times m\vec{a}_{/f} \overset{\text{if C is a fixed point or CoM}}{=} \frac{d}{dt}\left(\vec{H}_{/c}\right)}$$

relative to fixed frame