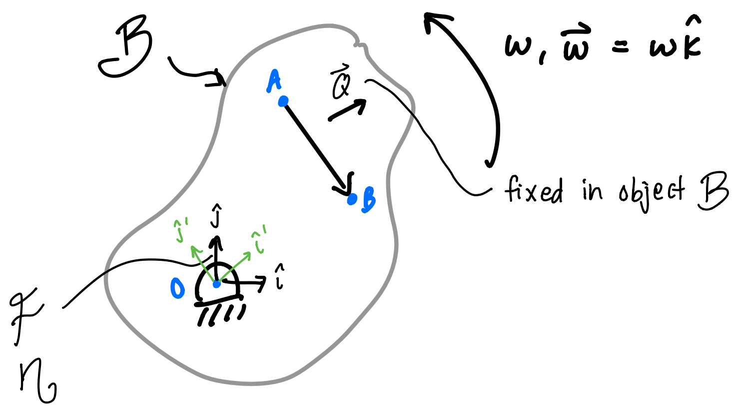
MAE 2030 Lecture #25:

4/7/21

Today: ① 2D rigid rotation continued

2D rotation, no translation:

recall kinematics:



What is $\vec{Q}$?

rotate with object

$$\vec{Q} = Q_{x'}\hat{\imath}' + Q_{y'}\hat{\jmath}'$$

$$\dot{Q}_{x'}, \dot{Q}_{y'} = 0$$

$$\dot{\vec{Q}} = \vec{\omega} \times \vec{Q}$$

ex) $$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$$

$$= \frac{d}{dt}\vec{r}_{B/A}$$

$$\boxed{\vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A}}$$

ex) $$\dot{\hat{\imath}}' = \vec{\omega} \times \hat{\imath}'$$

fixed in $\mathcal{B}$

$$\dot{\hat{\jmath}}' = \vec{\omega} \times \hat{\jmath}'$$

ex) $$\vec{a}_{B/A} = \frac{d}{dt}\left(\vec{v}_{B/A}\right)$$

$$= \frac{d}{dt}\left(\vec{\omega} \times \vec{r}_{B/A}\right)$$

$$= \dot{\vec{\omega}} \times \vec{r}_{B/A} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{B/A}\right)$$

$$\boxed{\vec{a}_{B/A} = \dot{\vec{\omega}} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}}$$

Back to dynamics:

For any system:

LMB: $\sum_{ext} \vec{F} = \sum m_i \vec{a}_i = m_{tot} \vec{a}_G$

AMB: $\sum_{ext} \vec{M}_{/c} = \sum \vec{r}_{i/c} \times m_i \vec{a}_i$

any point c

$\vec{a}_{i/\mathcal{F}}$

ex) 2D rigid object w/ no rotation

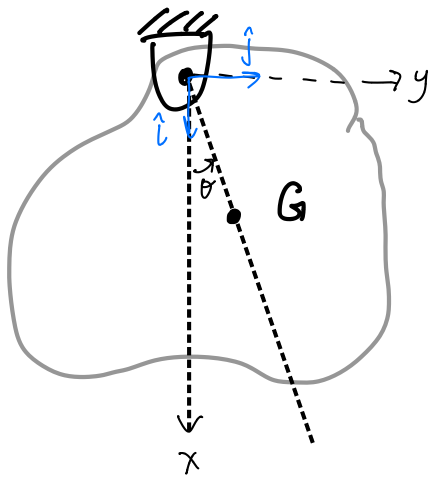
$$\sum \vec{M}_{/c} = \vec{r}_{G/c} \times m_{tot} \vec{a}_G$$

ex) AMB$_{/O}$ rotation of 2D object about O

(today's lecture)

2D object hinged at O:

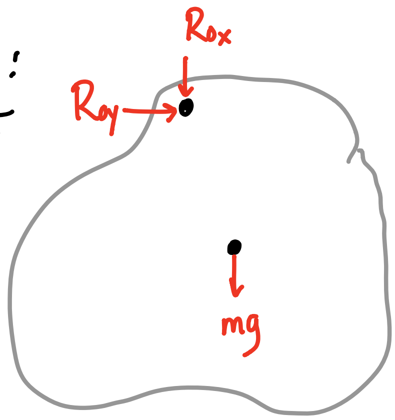
$\theta = 0$ at static equilibrium



How does it move?

- include g

FBD:



AMB: $\sum \vec{M}_{/0} = \dot{\vec{H}}_{/0}$

equivalent

for particles $\Rightarrow \sum_i \vec{r}_{i/0} \times m_i \vec{a}_i$

for continuum $\Rightarrow \int \vec{r}_{/0} \times \vec{a}\, dm$

2D rigid rotation about 0 $\Rightarrow$ $\boxed{\sum \vec{M}_{/0} = \int \vec{r}_{/0} \times \left[(\dot{\vec{\omega}} \times \vec{r}_{/0}) + -\omega^2 \vec{r}_{/0} \right] dm}$

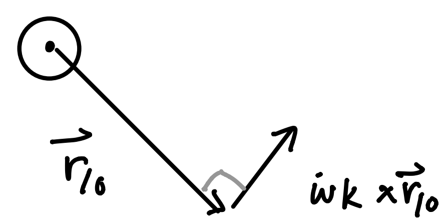
(acceleration simplifies to $\left[(\dot{\vec{\omega}} \times \vec{r}_{/0}) - \omega^2 \vec{r}_{/0} \right]$ in this special case)

(another special case)

Compare to 2D no rotation:

$$\sum \vec{M}_{/c} = \vec{r}_{G/0} \times m_{tot} \vec{a}_G$$

$\dot{\vec{\omega}} = \dot{\omega} \hat{k}$



$$\Rightarrow \sum \vec{M}_{/0} = \int \dot{\omega} |\vec{r}_{/0}|^2 \hat{k}\, dm + \vec{0}$$

$$\Rightarrow \left\{ -\underbrace{d}_{|\vec{r}_{G/0}|} \sin\theta\, mg\, \hat{k} = \int \dot{\omega} |\vec{r}_{/0}|^2 \hat{k}\, dm \right\}$$

$$\hat{k} \cdot \{\} \Rightarrow -mgd\sin\theta = \dot{\omega} \boxed{\int |\vec{r}_{/0}|^2\, dm}$$

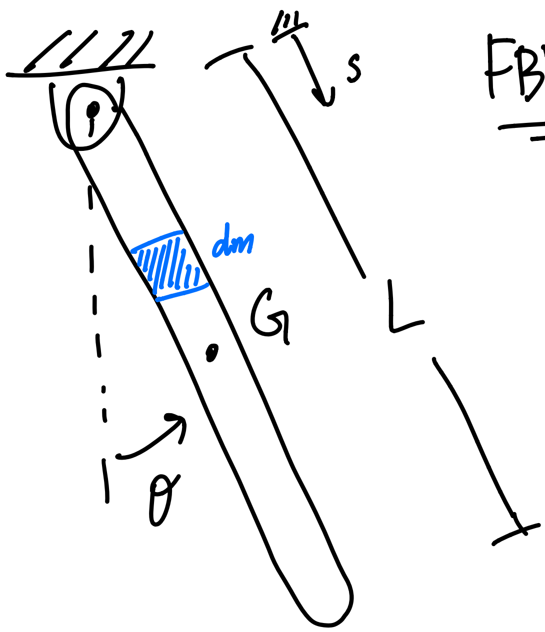
does not depend on θ

I^0 = moment of inertia about 0 (property of object)

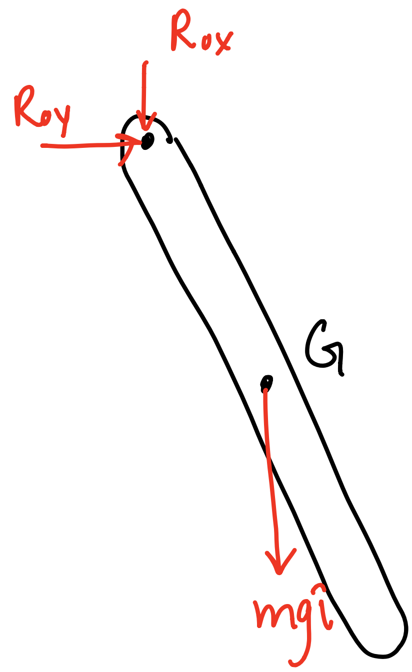
$$\Rightarrow \boxed{\sum \vec{M}_{/0} = I^0 \dot{\omega} \hat{k}}$$

$$I^0 \equiv \int |\vec{r}_{/0}|^2\, dm$$

ex)



FBD:



AMB/O: $\sum \vec{M}_{/O} = \dot{\vec{H}}_{/O}$

$$\left\{ -\frac{mgL\sin\theta}{2}\hat{k} = I^O \ddot{\theta}\hat{k} \right\}$$

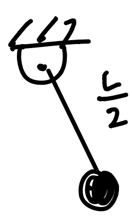
$$\hat{k}\cdot\{\} \Rightarrow \ddot{\theta} = -\frac{mgL\sin\theta}{2I^O} \Rightarrow \frac{-mgL}{2\left(\frac{mL^2}{3}\right)}\sin\theta$$

$$\boxed{\ddot{\theta} = -\frac{3g}{2L}\sin\theta}$$

$$I^O = \int_0^L s^2 \, dm$$

$$\underbrace{\left(\frac{m}{L}\right)}_{\text{density}} ds = \frac{m}{L}\int_0^L s^2\, ds = \frac{m}{L}\cdot\frac{L^3}{3}$$

recall: compare to point mass



$$\Rightarrow \boxed{\ddot{\theta} = -\frac{2g}{L}\sin\theta}$$