MAE 2030 Lecture #26:

4/9/21

Today: ① Review

② Circular motion continued

Review:

- Laws of Mechanics (any system)

LMB: $\sum_{ext} \vec{F}_i = \begin{cases} \sum_i m_i \vec{a}_i \\ \int \vec{a}\, dm \end{cases}$

$AMB_{/c}$: any pt c $\sum_{ext} \vec{M}_{/c} = \begin{cases} \sum_i \vec{r}_{i/c} \times m_i \vec{a}_{/\mathcal{F}} \\ \int \vec{r}_{/c} \times \vec{a}\, dm \end{cases}$

$\vec{a}_{/\mathcal{F}}$

can do any math problem with these

Shortcuts:

LMB $\Longrightarrow$ $\sum_{ext} \vec{F} = m_{tot} \vec{a}_G$

any system

any motion

$AMB_{/c}$ $\Longrightarrow$ For a single rigid object with no rotation

$$\sum_{ext} \vec{M}_{/c} = \vec{r}_{/c} \times m_{tot} \vec{a}_G$$

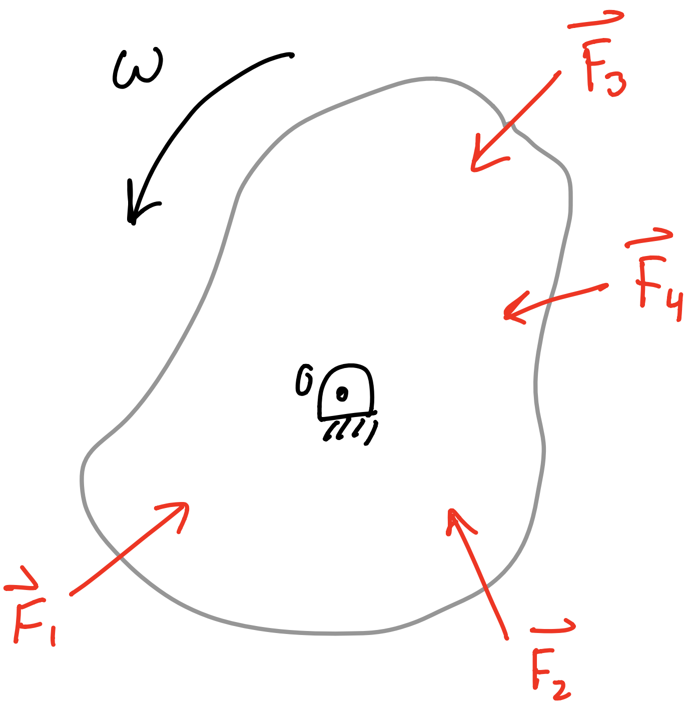
$\Longrightarrow$ For a 2D rigid object rotating about O

$$\sum \vec{M}_{/0} = I^o \dot{\omega} \hat{k} \qquad \text{"} M = I\alpha \text{"}$$

$I^o = \int |\vec{r}_{/0}|^2 dm$

just rotation about a point

Power for simple rotation :



What is net power of forces?

$$P = \sum_i \vec{F}_i \cdot \vec{v}_i$$

$$= \underbrace{\sum_i \vec{F}_i \cdot \vec{\omega} \times \vec{r}_{i/O}}_{\text{"mixed triple product"}}$$

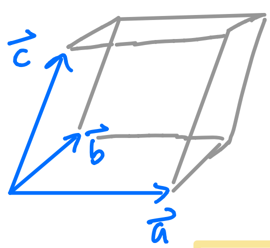
$$P = \sum \vec{r}_{i/O} \times \vec{F}_i \cdot \vec{\omega}$$

$$= \vec{\omega} \cdot \underbrace{\sum_i \vec{r}_{i/O} \times \vec{F}}_{\vec{M}_{/O}}$$

$$\boxed{P = \vec{\omega} \cdot \vec{M}_{/O}}$$

Aside: $\vec{a} \cdot \vec{b} \times \vec{c} = \det\left[[\vec{a}]_{xyz} \middle| [\vec{b}]_{xyz} \middle| [\vec{c}]_{xyz} \right]$

Volume of parallelapiped

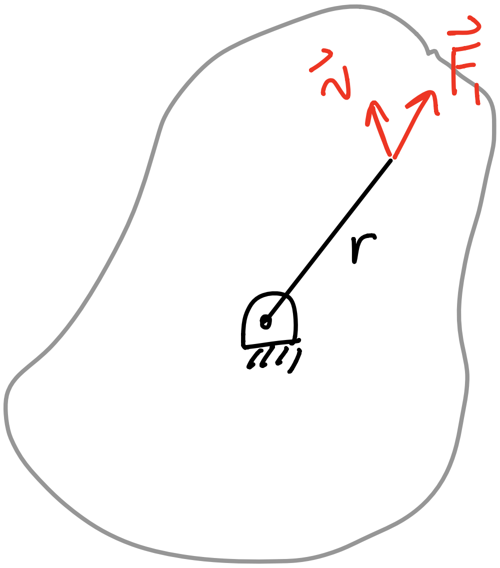


$$= \det\left[[\vec{c}]_{xyz} \middle| [\vec{a}] \middle| [\vec{b}] \right]$$

$$= \vec{c} \cdot \vec{a} \times \vec{b}$$

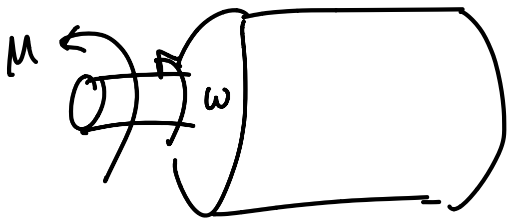
$$\boxed{\vec{a} \cdot \vec{b} \times \vec{c} = \vec{a} \times \vec{b} \cdot \vec{c}}$$

Look at just one force



$$P = \vec{F}_i \cdot \vec{v}$$

$$= \dot{\theta} r F_{proj} = \dot{\theta} M$$



$$\boxed{\text{Shaft power} = M\omega}$$

What about E_K?

$$E_K = \frac{1}{2} \sum_i v_i^2 m_i = \frac{1}{2} \sum_i \left[(\vec{\omega} \times \vec{r}_i) \cdot (\vec{\omega} \times \vec{r}_i) \right] m_i$$

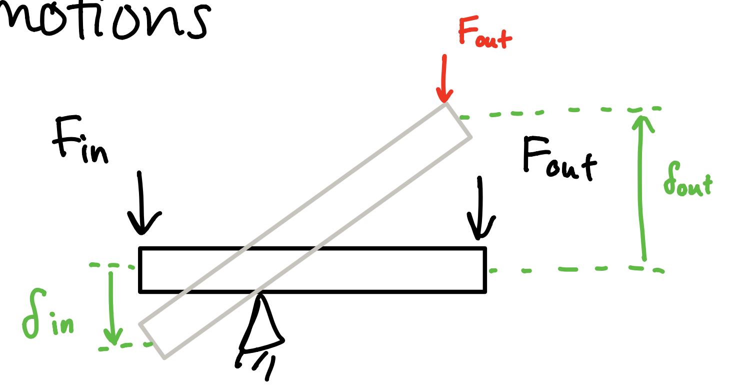
rigid object rotating about O

$$= \frac{1}{2} \omega^2 \sum_i |\vec{r}_i|^2 m_i$$

$$\boxed{E_K = \frac{1}{2} \omega^2 I^0}$$

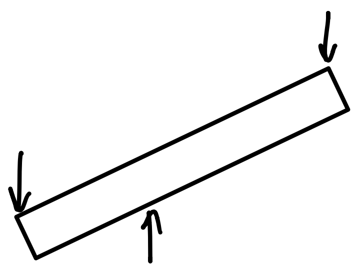
Transmissions: Passive machines which pass through force & motions

examples: **Lever**



$$P_{in} = F_{in}\dot{\delta}_{in} \qquad P_{out} = F_{out}\dot{\delta}_{out}$$

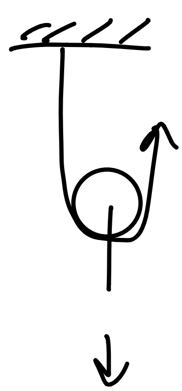
FBD:



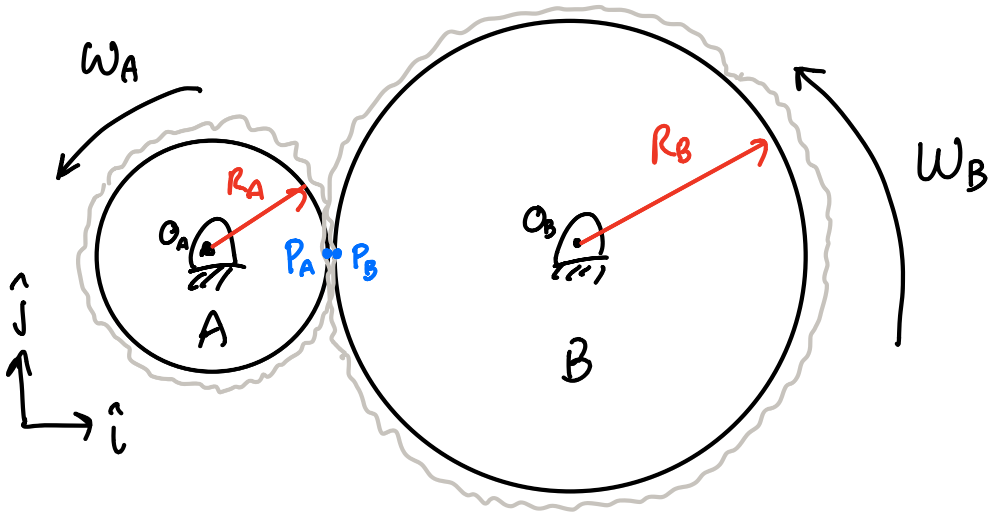
no dissipation
&
no internal sources or sinks
&
negligible kinetic energy

$$\boxed{P_{in} = P_{out}}$$

Pulleys



Gears (kind of lever)



$$\vec{v}_{P_A} = \vec{v}_{P_B}$$

$$\vec{\omega}_A \times \vec{r}_{P_A/O_A} = \vec{\omega}_B \times \vec{r}_{P_B/O_B}$$

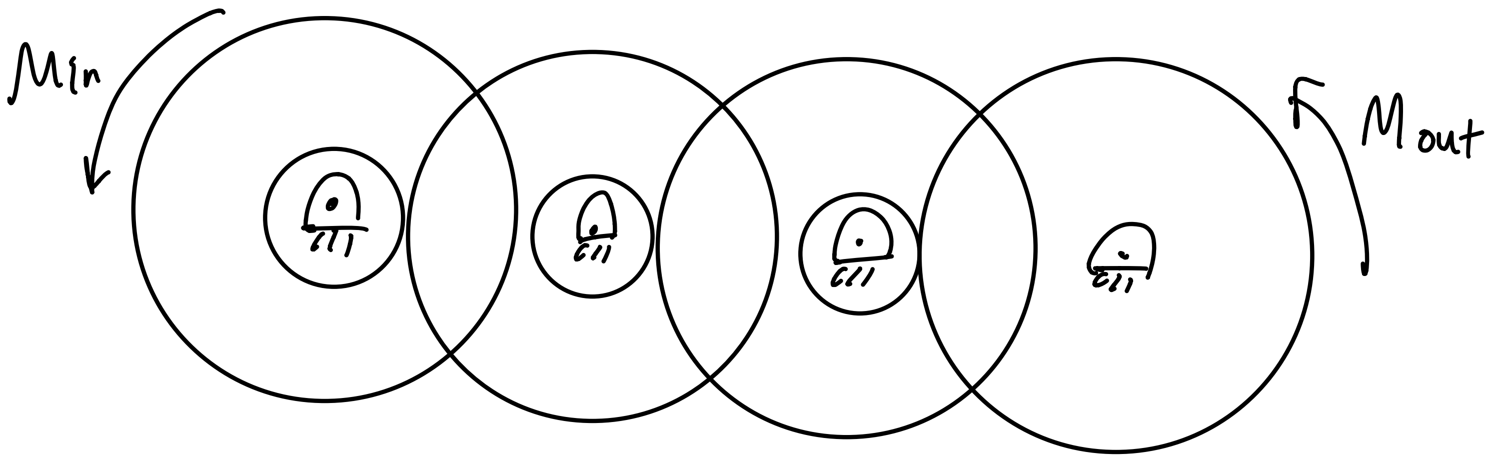
$$\omega_A \hat{k} \times R_A(\hat{i}) = \omega_B \hat{k} \times R_B(-\hat{i})$$

$$\{\omega_A R_A \hat{j} = -\omega_B R_B \hat{j}\}$$

$$\{\ \} \cdot \hat{j} \Rightarrow \quad \omega_A R_A = -\omega_B R_B$$

$$\boxed{\omega_B = -\frac{R_A}{R_B}\omega_A}$$

How to get a huge torque:



glue two gears together

→ Big stepdown transmission

$$|M_{out}| = \left(\text{radial ratio}\right)^3 |M_{in}|$$

→ Big gear reduction

→ Force multiplier

→ very "low" system