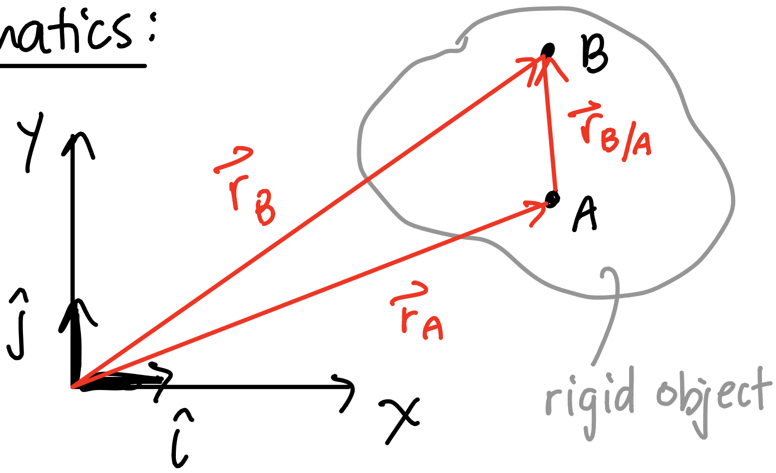
MAE 2030 Lecture #27:

4/12/21

Today: General motion of rigid objects in 2D

Kinematics:



often $A = G$

A, B fixed on object

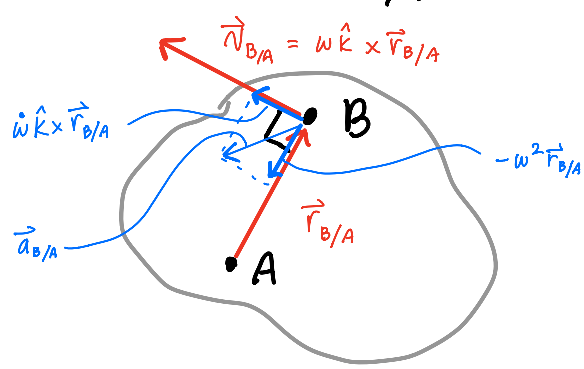
$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

$$\vec{v}_B = \dot{\vec{r}}_B = \dot{\vec{r}}_A + \underbrace{\dot{\vec{r}}_{B/A}}_{\omega \hat{k} \times \vec{r}_{B/A}}$$

$$= \vec{v}_A \times \omega \hat{k} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \dot{\vec{v}}_B = \ddot{\vec{r}}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$= \vec{a}_A + \dot{\omega} \hat{k} \times \vec{r}_{B/A} + -\omega^2 \vec{r}_{B/A}$$



Mechanics:

LMB $\Rightarrow$

$$\sum_{ext} F = \left[\begin{array}{l} \sum m_i \vec{a}_i \\ \int \vec{a} \, dm \end{array} \right.$$

absolute $\vec{a}$

absolute acceleration

$$= m_{tot} \vec{a}_G$$

CoM

AMB $\Longrightarrow$

any system

any motion

any point C

$$\sum \vec{M}_{/C} = \begin{cases} \sum \vec{r}_{i/C} \times m_i \vec{a}_i \quad \leftarrow \vec{a}_{i/\mathcal{F}} \\ \int \vec{r}_{/C} \times \vec{a}_{/\mathcal{F}}\, dm \end{cases}$$

$\mathcal{F} = \mathcal{N}$

Fixed or Newtonian reference frame

= no simple general form

General 2D motion of one rigid object:

AMB:

$$\boxed{\sum \vec{M}_{/C} = \vec{r}_{G/C} \times m\vec{a}_{/G} + I^G \dot{\omega} \hat{k}}$$

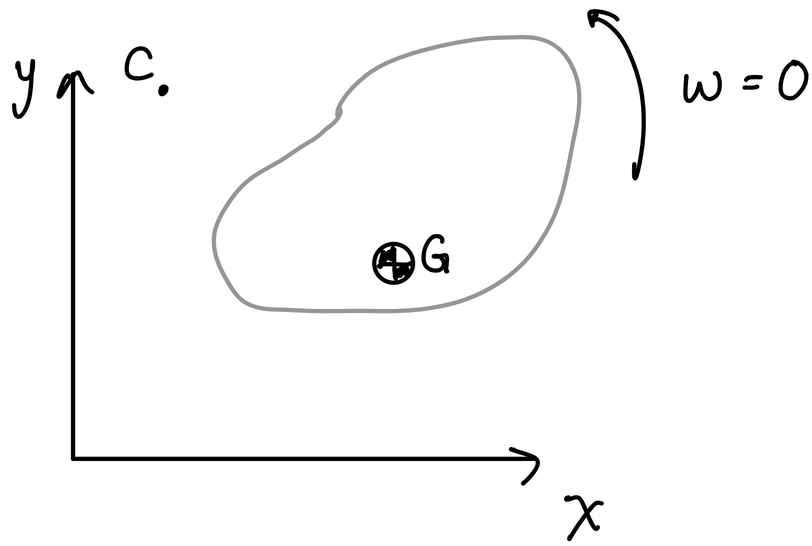
any pt c

$\vec{a}_{G/\mathcal{F}}$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

* has to be r×ma not max r

ex) object with no rotation

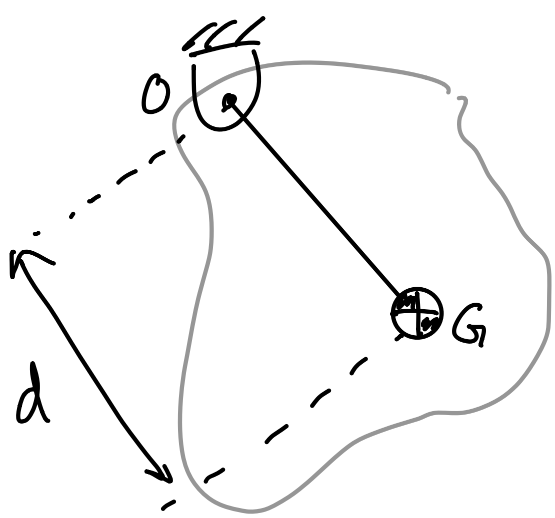


LMB: $\sum_{ext} F = m\vec{a}_G$

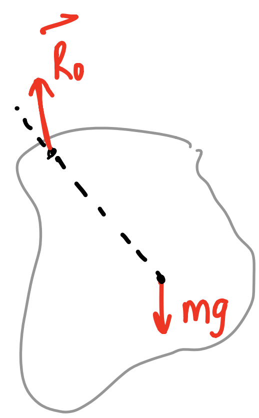
AMB: $\sum \vec{M}_{/c} = \vec{r}_{G/c} \times m\vec{a}_G + I^G \cancel{\dot{\omega}} \hat{k}$ $\quad \omega = 0$

$$\boxed{\sum \vec{M}_{/c} = \vec{r}_{G/c} \times m\vec{a}_G}$$

ex) object rotating about fixed point O



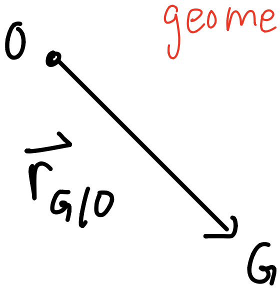
FBD:



LMB: $\sum_{ext} F = m\vec{a}_G$

$AMB_{/o}$: $\sum_{ext} M_{/o} = \vec{r}_{G/o} \times m\vec{a}_G + \underbrace{I^G}_{\int |\vec{r}_{/G}|^2 dm} \dot{\omega} \hat{k}$

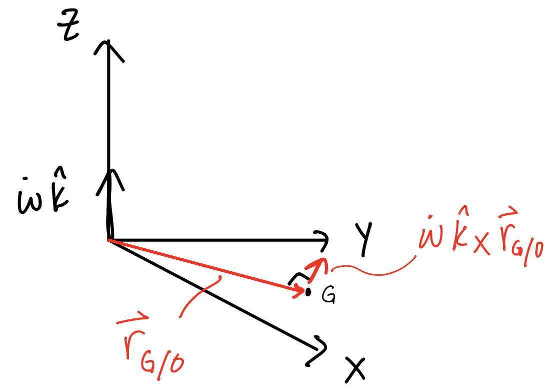
$$\sum \vec{M}_{/O} = \vec{r}_{G/O} \times \overbrace{\left(\dot{\omega}\hat{k} \times \vec{r}_{G/O} + -\omega^2 \vec{r}_{G/O}\right)}^{\vec{a}_G} m + I^G \dot{\omega}\hat{k}$$



$$\overset{\text{geometry}}{=} |\vec{r}_{G/O}|^2 \dot{\omega} m \hat{k} + I^G \dot{\omega}\hat{k}$$

$$= \underbrace{(d^2 m + I^G)}_{I^O = d^2 m + I^G} \dot{\omega}\hat{k}$$

(parallel axis theorem)

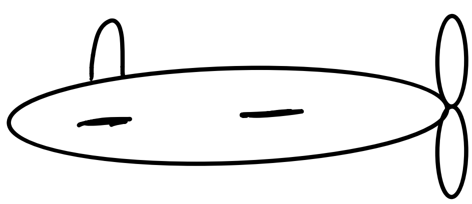


$$\sum \vec{M}_{/O} = \begin{cases} I^O \dot{\omega}\hat{k} \\ \vec{r}_{G/O} \times m\vec{a}_G + I^G \dot{\omega}\hat{k} \end{cases}$$

equivalent for rotation about fixed point O

When is this useful (a 2D object):

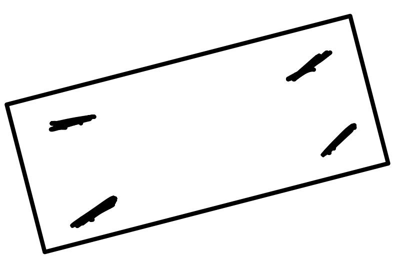
ex)



flight dynamics

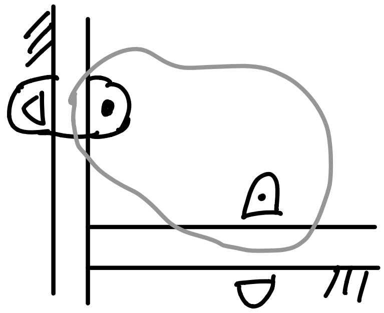
side view

ex)



car: top view

ex) machine parts w/ more complicated motion

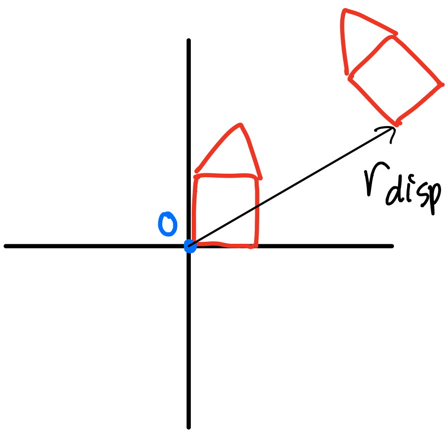


Animation:

Given picture:

$$r_{picture} = \begin{bmatrix} x_1 & x_2 & x_3 & \dots \\ y_1 & y_2 & y_3 & \dots \end{bmatrix}$$

reference



$r_{disp} = x_{disp} + y_{disp}$

Net motion is rotation about O & translation

$$\text{New pic} = R \cdot (\text{ref pic}) + \begin{bmatrix} x_{disp} \\ y_{disp} \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Dynamics Problem:

1) Draw FBD

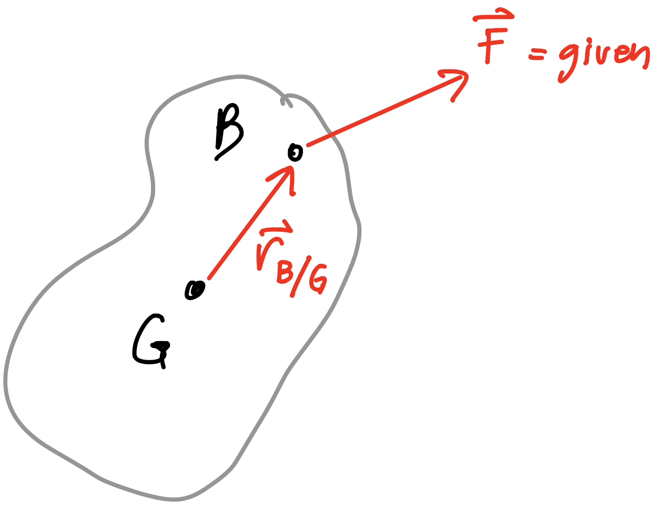
2) Pick motion coords

(most common: x_G, y_G, θ)

3) LMB & AMB to get

$$\left.\begin{matrix} \underbrace{\begin{aligned} \dot{x}_G &= v_{xG} \\ \dot{y}_G &= v_{yG} \\ \dot{\theta} &= \omega \end{aligned}}_{\text{kinematics}} & \quad & \underbrace{\begin{aligned} \dot{v}_{xG} &= F_{x\,tot}/m \\ \dot{v}_{yG} &= F_{y\,tot}/m \\ \dot{\omega} &= |\vec{M}_{/G}|/I^G \end{aligned}}_{\text{mechanics}} \end{matrix}\right\} \text{6 1st order ODEs}$$

ex) object with single force



$$\Rightarrow \vec{a} = \vec{F}/m$$

$$\dot{\omega} = \vec{r}_{B/G} \times \vec{F}/I^G$$