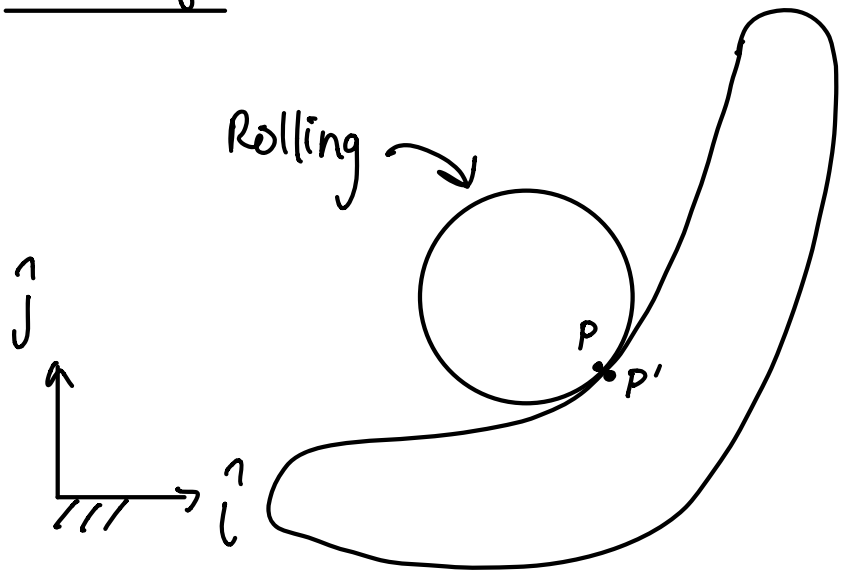
MAE 2030 Lecture #29:

4/16/21

Today: Rolling

Rolling:

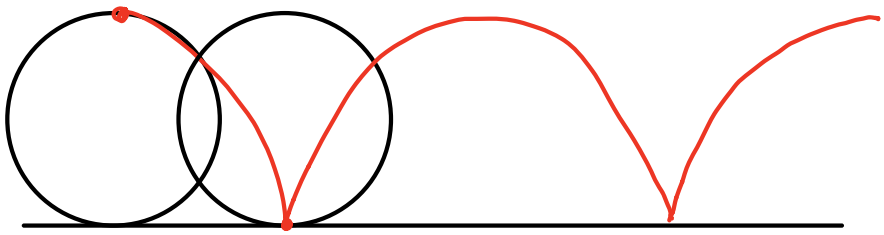


Key ideas:

$$\vec{r}_P = \vec{r}_{P'}$$
$$\vec{v}_P = \vec{v}_{P'}$$
$$\vec{a}_P \neq \vec{a}_{P'}$$

demo: disc rolling along path

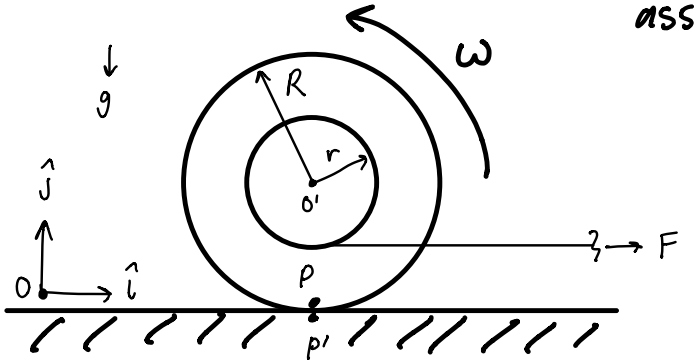
trajectory:



ground is still

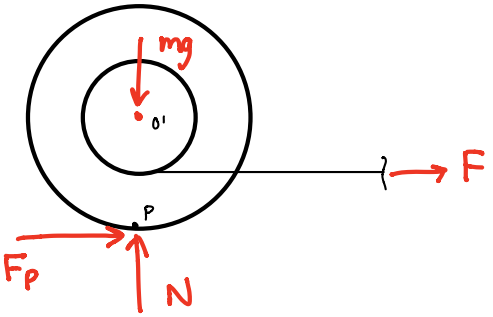
$\vec{v}_P = \vec{0}$ for point on ground

ex) Heat treatment of Al



assume: roll without slip
(not pulling too hard)

FBD :



Kinematics :

* O' doesn't go up or down

$$\vec{r}_{O'/O} = x_{O'/O}\,\hat{i} + R\hat{j}$$

$$\vec{v}_{O'} = \dot{x}_{O'}\,\hat{i}$$

$$\vec{a}_{O'} = \ddot{x}_{O'}\,\hat{j}$$

* rolling constraint

$$\vec{v}_P = \vec{v}_{P'}$$

$$\vec{v}_{O'} + \vec{v}_{P/O'} = \vec{0}$$

$$\dot{x}_{O'}\,\hat{i} + \omega\hat{k} \times \vec{r}_{P/O'} = \vec{0}$$

$$\vec{r}_{P/O'} = -R\hat{j}$$

$$\{\dot{x}_{O'}\,\hat{i} + \omega R\,\hat{i} = \vec{0}\}$$

$$\{\} \cdot \hat{i} \Rightarrow \dot{x}_{O'} + \omega R = 0$$

$$\dot{x}_{O'} = -\omega R$$

$$\omega = -\frac{\dot{x}_{O'}}{R}$$

$$\frac{d}{dt}\left(\dot{x}_{O'} = -\omega R\right)$$

$$\Rightarrow \ddot{x}_{O'} = -\dot{\omega} R$$

Mechanics:

$\underline{AMB/P}$: $\sum \vec{M}_{/P} = \vec{r}_{G/P} \times \vec{a}_G m + I^G \omega \hat{k}$

$$\{ -F(R-r)\hat{k} = (R\hat{j}) \times (m\ddot{x}_{o'}\hat{i}) - I^G \frac{\ddot{x}_{o'}}{R}\hat{k} \}$$

$$-R\ddot{x}_{o'} m \hat{k}$$

$$\{\} \cdot \hat{k} \Rightarrow +F(R-r) = +R\ddot{x}_{o'} m + I^G \frac{\ddot{x}_{o'}}{R}$$

$$\ddot{x}_{o'} = \frac{F(R-r)}{I^G/R + Rm}$$

(*) $$\ddot{x}_{o'} = \frac{F(R-r)/R}{I^G + mR^2} > 0 \quad (R > r)$$

$\Rightarrow$ pull to right ($F>0$) $\Rightarrow$ goes to right

$\ddot{x}_{o'} > 0$

Find Forces?

$\underline{LMB}$: $\sum_{ext} \vec{F} = m\vec{a}$

$$\{ (N - mg)\hat{j} + (F + f_p)\hat{i} = m\ddot{x}_{o'}\hat{i} \}$$

$\{\} \cdot \hat{j} \Rightarrow \boxed{N = mg}$

$\{\} \cdot \hat{i} \Rightarrow F + F_p = m\ddot{x}_{o'}$

(*)

F_p = some other mess but we know it's < 0 based on intuition