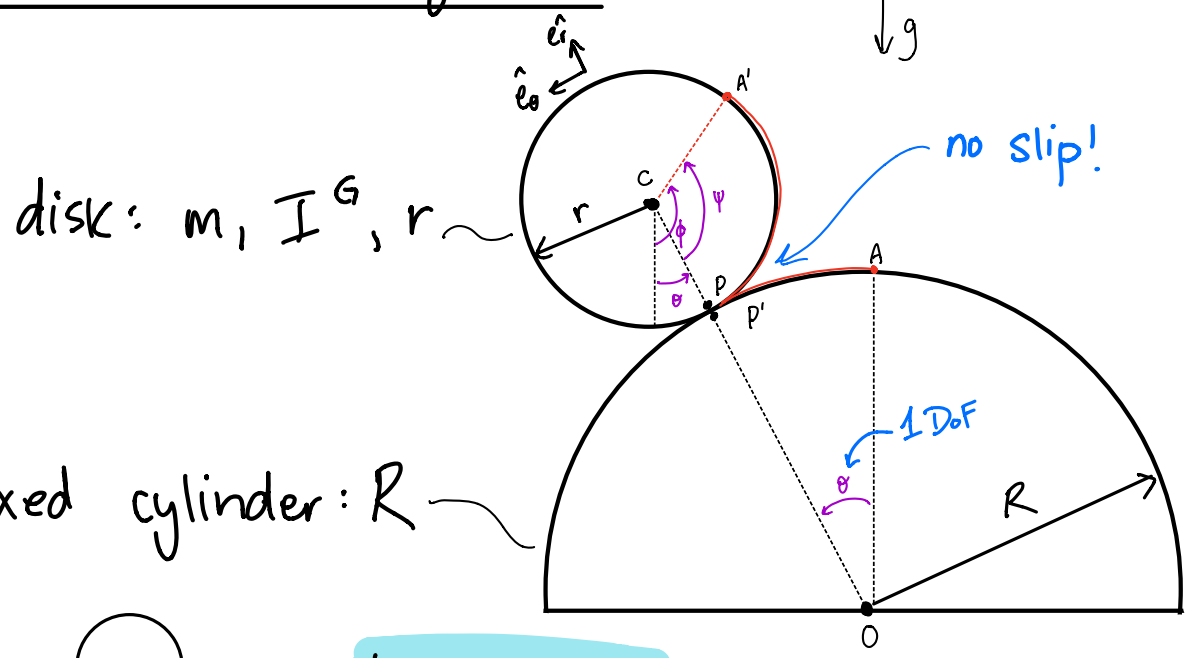
MAE 2030 Lecture #30: 4/19/21

Today: Rolling continued

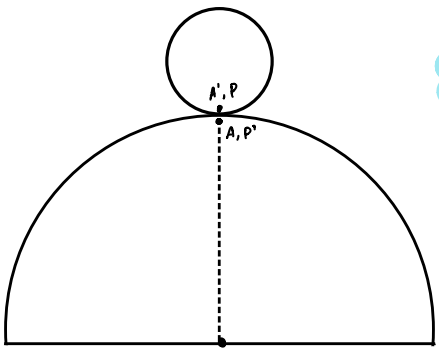
Disk rolls on fixed cylinder:



disk: m, I^G, r

fixed cylinder: R

ref state:



Kinematics:

rate of change of orientation

$$\vec{\omega} = \dot{\phi}\,\hat{k}$$

$$\vec{v}_{P'} = \vec{v}_P$$

$$0 = \vec{v}_C + \vec{v}_{P/C}$$

$$\vec{v}_{P/C} = \vec{\omega} \times \vec{r}_{P/C}, \quad \vec{\omega} = \dot{\phi}\hat{k}, \quad \vec{r}_{P/C} = -r\hat{e}_r$$

$$\vec{v}_C = (R+r)\,\dot{\theta}\,\hat{e}_\theta$$

$$0 = (R+r)\,\dot{\theta}\,\hat{e}_\theta + \dot{\phi}\hat{k} \times -r\hat{e}_r$$

$\hat{k} \times \hat{e}_r = \hat{e}_\theta$

$$\{0 = (R+r)\,\dot{\theta}\,\hat{e}_\theta - r\dot{\phi}\hat{e}_\theta\}$$

$$\{\} \cdot \hat{e}_\theta \Rightarrow (R+r)\dot{\theta} = r\dot{\phi}$$

$$\dot{\phi} = \frac{(R+r)\dot{\theta}}{r} \quad (1)$$

$$\vec{\omega} = \dot{\phi}\hat{k} = \frac{R+r}{r}\,\dot{\theta}\,\hat{k}$$

$$\dot{\vec{\omega}} = \ddot{\phi}\hat{k} = \frac{R+r}{r}\,\ddot{\theta}\,\hat{k}$$

Alternative Method to get (1):

$$|\text{Arc } AP'| = |\text{Arc } P'A'|$$

$$R\theta = \psi r$$

$\psi = \phi - \theta$

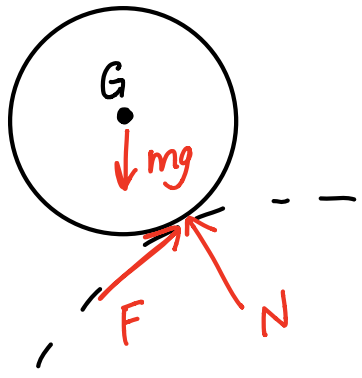
$$R\theta = (\phi - \theta) r$$

$$(R+r)\theta = r\phi$$

$$\phi = \frac{(R+r)}{r}\theta$$

$$\boxed{\dot{\phi} = \frac{R+r}{r}\dot{\theta}} \quad (1)$$

FBD:



3 main unknowns: F, N, $\ddot{\theta}$

How to solve?

Boring method:

LMB (2 scalar eqns) } 3 eqns

AMB/C (1 eqn) }

Spark Joy method:

AMB/P

$G = C$

$$AMB_{/P}: \quad \sum_{ext} \vec{M}_{/P} = \vec{r}_{G/P} \times m\vec{a}_G + I^G \ddot{\phi}\,\hat{k}$$

$\vec{r}_{G/P} = r\hat{e}_r$

$m\vec{a}_G = (R+r)\ddot{\theta}\,\hat{e}_\theta - (R+r)\dot{\theta}^2\,\hat{e}_r$

$$\left\{ r\sin\theta\, mg\,\hat{k} = r(R+r)\ddot{\theta}\, m\,\hat{k} + I^G \cdot \left(\frac{R+r}{r}\right)\ddot{\theta}\,\hat{k} \right\}$$

$$\{\ \} \cdot \hat{k} \Rightarrow r\sin\theta\, mg = \left[r(R+r)m + I^G \cdot \left(\frac{R+r}{r}\right) \right] \ddot{\theta}$$

$$\ddot{\theta} = \frac{r\sin\theta\, mg}{r(R+r)m + I^G \cdot \frac{R+r}{r}}$$