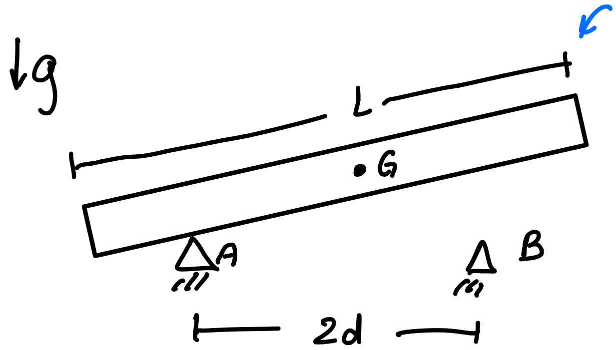
MAE 2030 Lecture #31: 4/21/21

Today: ① Collision example
② polar coordinates

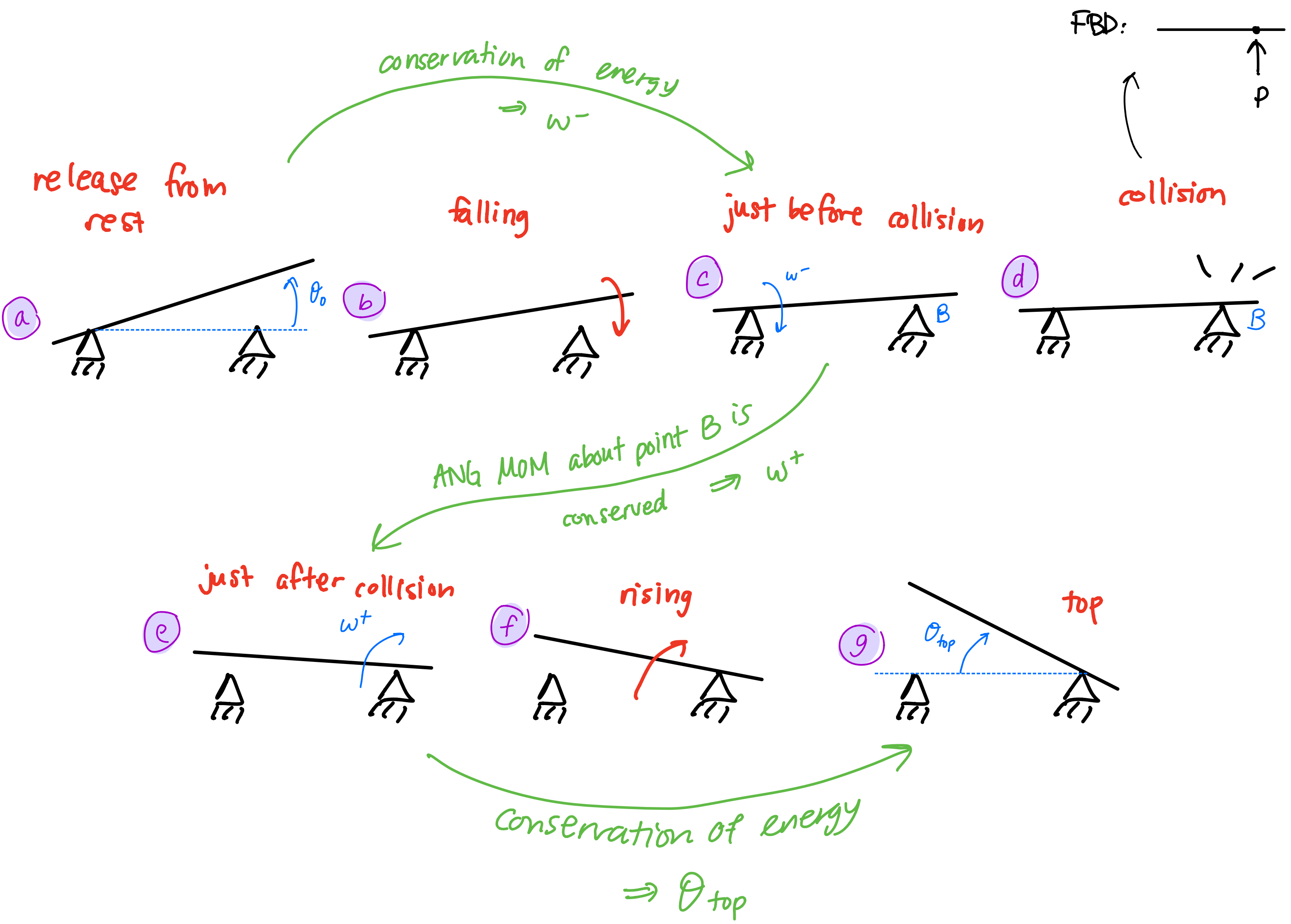
demo: bar rocking on two point supports

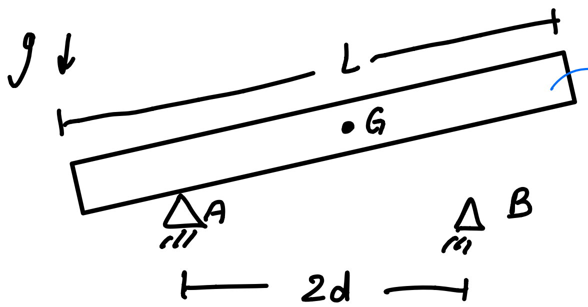
Rocking ruler:



release from rest

what happens?





$m,\ I^G = \int |r_{/G}|^2 \underbrace{dm}_{\rho\, dL}$

$= \int_{\frac{L}{2}}^{-\frac{L}{2}} x^2 \frac{m}{L}\, dx$

$\boxed{I^G = \frac{mL^2}{12}}$ for uniform rod

$I^B = I^G + md^2$

Step 1:

a → c

$$E_K = \underbrace{\tfrac{1}{2} m v^2}_{\text{translation}} + \underbrace{\frac{I^G \omega^2}{2}}_{\text{rotation}}$$

$$E^a_{tot} = E^c_{tot}$$

$$\cancelto{0}{E_K^a} + E_p^a = E_K^c + \cancelto{0}{E_p^c}$$

$$mg \underbrace{y_0}_{(d\sin\theta)} = \left[\begin{array}{l} \omega_c^2 I^A/2 \\ \omega_c^2 I^G/2 + (v_G^c)^2 m/2 \end{array} \right.$$

$(\omega_c d)^2$

$$\Longrightarrow \boxed{\omega_c = \text{algebraic mess}}$$

Step 2:

c → e

assume $\omega^c > 0$

CW

just before

just after

$$\vec{H}^c_{/B} = \vec{H}^e_{/B}$$

c = "−"

e = "+"

$$I^G(-\omega^c \hat{k}) + \vec{r}_{G/B} \times m\vec{v}_G^c = I^G(-\omega^e \hat{k}) + \vec{r}_{G/B} \times m\vec{v}_G^e$$

$-d\hat{i}$ $\quad -\omega^c d\hat{j}$ $\quad -d\hat{i}$ $\quad \omega^e d\hat{j}$

$$\Longrightarrow \boxed{\omega^e = \ldots\ldots}$$

Step 3:

e → g

Energy conservation again

$$\Rightarrow \boxed{\theta_{top} = \dots}$$

Time varying base vectors:

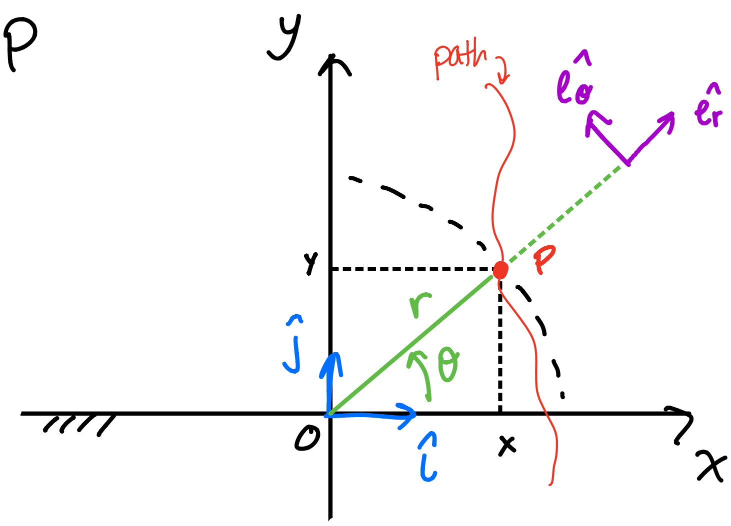
(moving reference frames)

ex) Polar coordinates

→ look at one particle P

given $r(t)$, $\theta(t)$

what are $\vec{v}$ & $\vec{a}$?



$$\vec{r}_{P/O} = \vec{r} = r\hat{e}_r$$

$$\vec{v} = \dot{\vec{r}} = \dot{r}\hat{e}_r + r\dot{\hat{e}}_r$$

$\dot{\theta}\hat{e}_\theta$

$$\boxed{\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$$

$\dot{\theta}\hat{e}_\theta$ $\qquad\qquad$ $-\dot{\theta}\hat{e}_r$

$$\vec{a} = \dot{\vec{v}} = \ddot{r}\hat{e}_r + \dot{r}\cancel{\dot{\hat{e}}_r} + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\cancel{\dot{\hat{e}}_\theta}$$

$$\boxed{\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta}$$