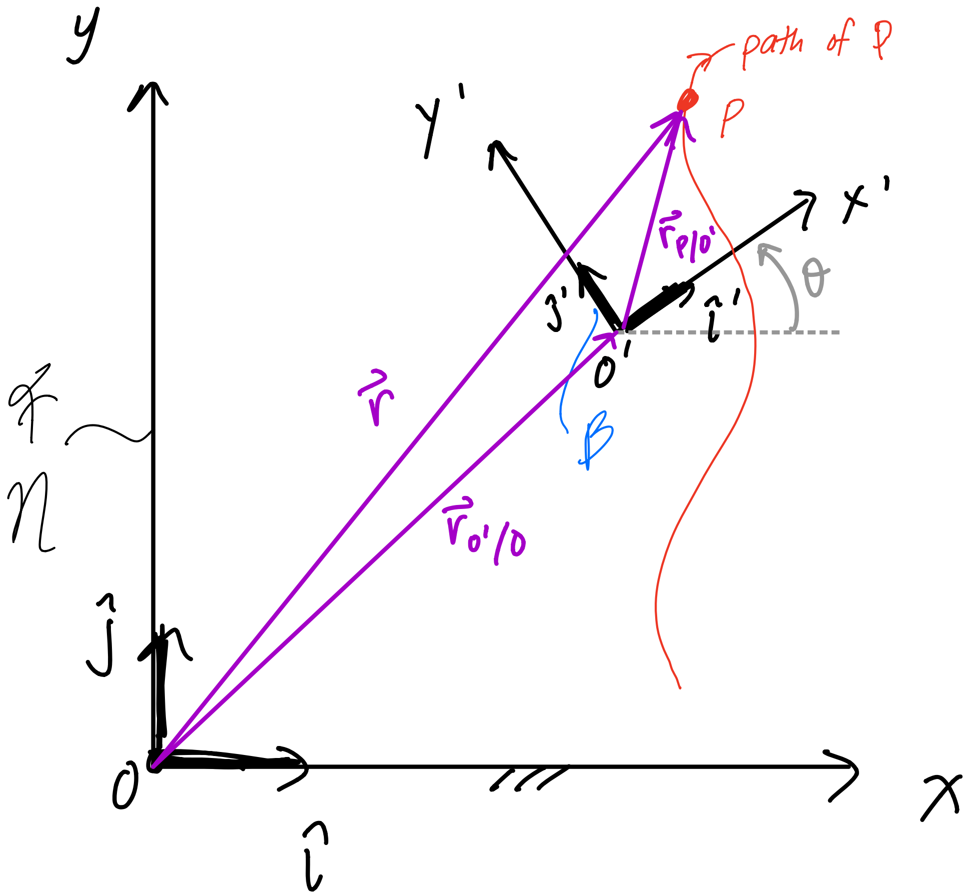
MAE 2030 Lecture #33:

4/30/21

Today: Relative motion continued

Goal: use information from moving observers to find

$$\vec{v}_P = \vec{v}_{P/\mathcal{F}} \quad \& \quad \vec{a}_P = \vec{a}_{P/\mathcal{F}}$$



For dynamics, we want

$$\vec{r} = \vec{r}_{P/O}$$

$$\vec{v} = \vec{v}_{P/\mathcal{F}} = \dot{x}\hat{i} + \dot{y}\hat{j}$$

$$\vec{a} = \vec{a}_{P/\mathcal{F}} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$$

We are given:

$$\vec{r}_{O'/O}, \ \vec{v}_{O'/O}, \ \vec{a}_{O'/O}$$

$\vec{v}_{O'/O}$: $\vec{v}_{O'/\mathcal{F}}$; $\vec{a}_{O'/O}$: $\vec{a}_{O'/\mathcal{F}}$

$$\vec{r}_{P/O'} = x'_{P/O'}\hat{i}' + y'_{P/O'}\hat{j}'$$

$$\vec{r}_{P/O'} = x_{P/O'}\hat{i} + y_{P/O'}\hat{j}$$

$$\boxed{\begin{aligned}\vec{v}_{P/\mathcal{B}} &\equiv \dot{x}'\hat{i}' + \dot{y}'\hat{j}' \\ \vec{a}_{P/\mathcal{B}} &\equiv \ddot{x}'\hat{i}' + \ddot{y}'\hat{j}'\end{aligned}}$$

definitions of velocity & acceleration relative to a moving frame

use givens to find $\vec{v}_{P/\mathcal{F}}$ & $\vec{a}_{P/\mathcal{F}}$

$$\vec{r} = \vec{r}_P = \vec{r}_{O'/O} + \vec{r}_{P/O'}$$

$$= \left(x_{O'/O}\,\hat{i} + y_{O'/O}\,\hat{j}\right) + \left(x'_{P/O'}\,\hat{i}' + y'_{P/O'}\,\hat{j}'\right)$$

Recall: $\dot{\hat{i}}' = \vec{\omega} \times \hat{i}$

$\dot{\theta}\hat{k} = \vec{\omega}_{P/\mathcal{F}}$

$\dot{\hat{j}}' = \vec{\omega} \times \hat{j}$

$$\vec{v}_{P/\mathcal{F}} = \vec{v}_P = \frac{d}{dt}\left(\vec{r}_P\right) = \frac{d}{dt}\left(\vec{r}_{O'/O}\right) + \frac{d}{dt}\left(\vec{r}_{P/O'}\right)$$

$$\vec{v}_{P/\mathcal{F}} = \vec{v}_{O'/\mathcal{F}} + \underbrace{\left(\dot{x}'\hat{i}' + \dot{y}'\hat{j}'\right)}_{\vec{v}_{P/\mathcal{B}}} + \underbrace{\left(x'\dot{\hat{i}}' + y'\dot{\hat{j}}'\right)}_{\vec{\omega}\times\vec{r}_{P/O'}}$$

$$\boxed{\vec{v}_{P/\mathcal{F}} = \vec{v}_{O'/\mathcal{F}} + \vec{v}_{P/\mathcal{B}} + \vec{\omega}\times\vec{r}_{P/O'}}$$

* listen to lecture to understand breakdown of all 3 terms

* $\vec{v}_{P/O'} = \vec{v}_P - \vec{v}_{O'} \neq \vec{v}_{P/\mathcal{B}}$

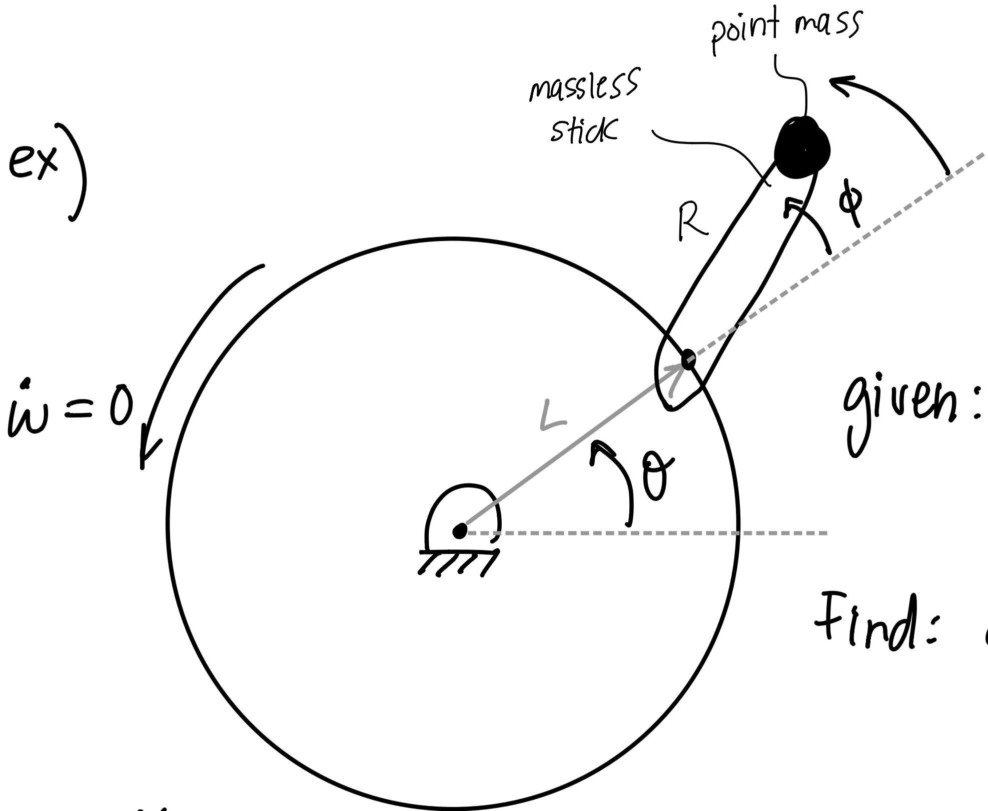
$$\vec{a}_{P/\mathcal{F}} = \frac{d}{dt}\left(\vec{v}_{P/\mathcal{F}}\right)$$

* read texbook for full derivation

$$\boxed{\vec{a}_{P/\mathcal{F}} = \vec{a}_{O'/O} + \vec{a}_{P/\mathcal{B}} + -\omega^2 \vec{r}_{P/O'} + \dot{\vec{\omega}} \times \vec{r}_{P/O'} + 2\vec{\omega} \times \vec{v}_{P/\mathcal{B}}}$$

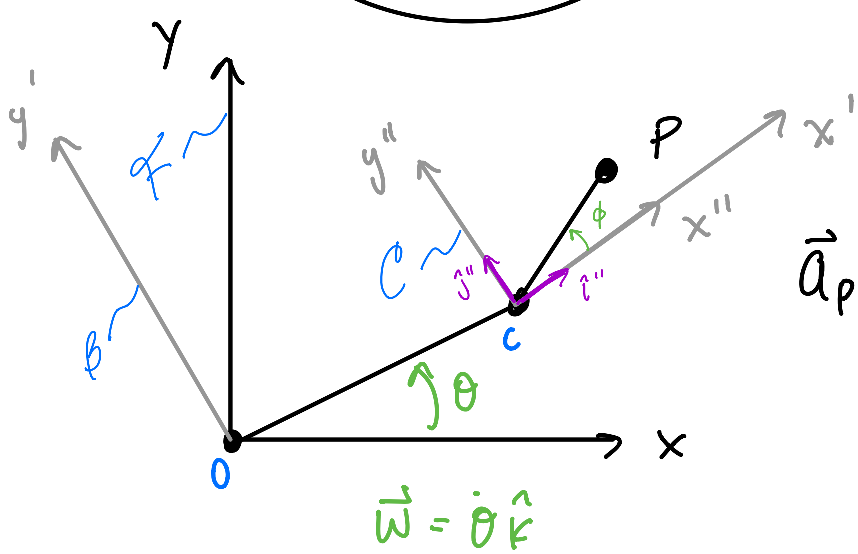
"The five term acceleration formula" *listen to lecture to understand breakdown of all 5 terms

ex)



given: L, R, θ, ϕ, m

Find: $\ddot{\phi} \iff$ find EoM



$$\vec{a}_P = \vec{a}_C + \vec{a}_{P/\mathcal{C}} + -\omega^2 \vec{r}_{P/C}$$
$$+ \dot{\vec{\omega}} \times \vec{r}_{P/C} + 2\vec{\omega} \times \vec{v}_{P/\mathcal{C}}$$

$$\vec{a}_{c/o} = -\omega^2 \vec{r}_{c/o} = -\omega^2 \cdot L\hat{i}$$

$$\vec{a}_{P/C} = \ddot{\phi}\hat{k} \times \vec{r}_{P/C} + -\dot{\phi}^2 \vec{r}_{P/C}$$

$$-\omega^2 \vec{r}_{P/C} = -\omega^2 \vec{r}_{P/C}$$

$$\dot{\vec{\omega}} \times r_{P/C} = \dot{\vec{\omega}} \times \vec{r}_{P/C}$$

$$2\vec{\omega} \times \vec{v}_{P/C} = 2\vec{\omega} \times (\dot{\phi}\hat{k} \times r_{P/C})$$

put it all together to get $\ddot{\phi} = \ldots$