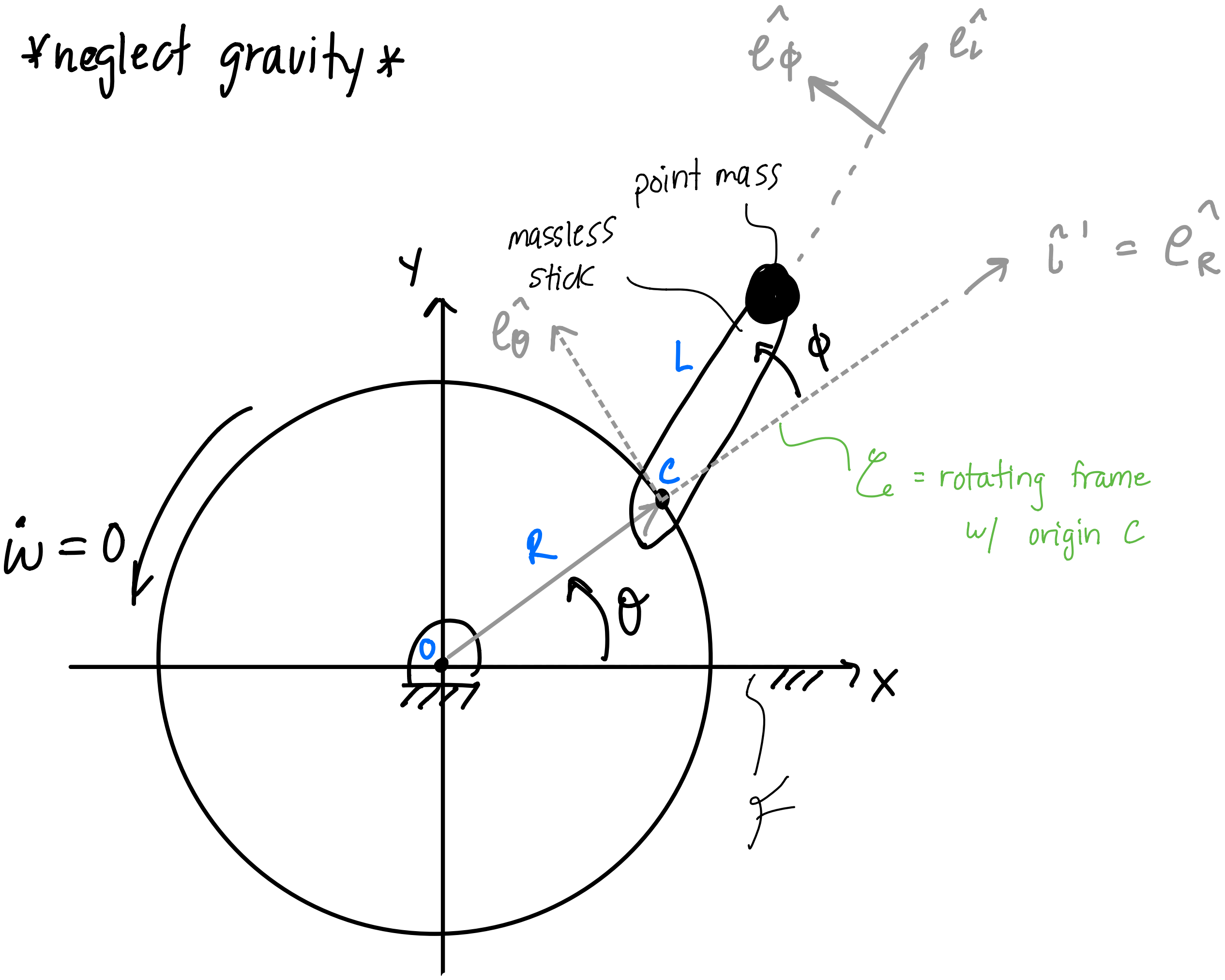
# MAE 2030 Lecture #34: 5/3/21

Today: Example of relative motion (continued)

*neglect gravity*

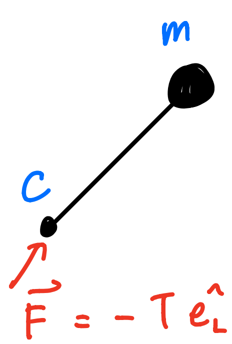



Given $\omega_{\mathcal{C}/\mathcal{F}} = \omega\hat{k}$, L, R, m, $\theta$, $\dot{\theta} = \omega$

Find $\ddot{\phi}$ in terms of $\phi$, $\dot{\phi}$ & $(\ldots)$

aka Find EoM

FBD:



$$\underline{\underline{AMB_{/c}}}: \quad \sum \vec M_{/c} = \vec r_{P/c} \times m\,\vec a_{P/\mathcal{F}} + \underbrace{\cancelto{0}{I^G}\,\dot\omega_{stick}\,\hat k}_{\vec 0}$$

$$\vec 0 = \underbrace{\vec r_{P/c}}_{L\hat e_L} \times m\,\vec a_{P/\mathcal{F}}$$

$$\cancelto{0}{\vec r_{c/c}} \times \vec F = \vec 0$$

$$\vec a_{P/\mathcal{F}} = \overset{(1)}{\underset{O'}{\vec a_c}} + \overset{(2)}{\vec a_{P/\mathcal{C}}} \overset{(3)}{- \omega_{\mathcal{C}}^2\,\vec r_{P/c}} + \overset{(4)}{\dot{\vec\omega}_{\mathcal{C}/\mathcal{F}} \times \vec r_{P/c}} + \overset{(5)}{2\vec\omega_{\mathcal{C}/\mathcal{F}} \times \vec v_{P/\mathcal{C}}}$$

Look at terms:

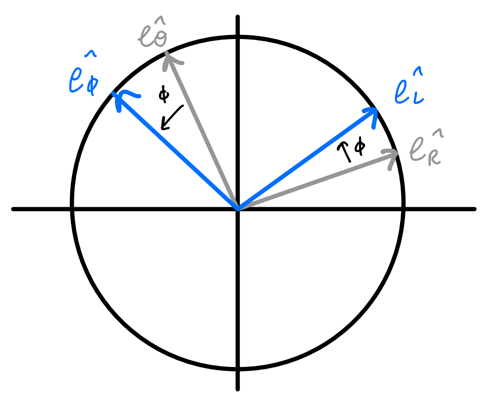
(1) $\Rightarrow \vec a_{c/\mathcal{F}} = -\omega^2 R\hat e_R$

(2) $\Rightarrow \vec a_{P/\mathcal{C}} = -\dot\phi^2 L\hat e_L + \ddot\phi L\hat e_\phi$

(3) $\Rightarrow -\omega_{\mathcal{C}}^2\,\vec r_{P/c} = -\omega^2 L\hat e_L$

(4) $\Rightarrow \dot{\vec\omega}_{\mathcal{C}/\mathcal{F}} \times \vec r_{P/c} = \dot\omega\hat k \times L\hat e_L = \dot\omega L\hat e_\phi = \vec 0$

unit circle



$$⑤ \Rightarrow 2\vec{\omega}_{e/f} \times \vec{v}_{P/e} = 2\omega\hat{k} \times \underbrace{L\dot{\phi}\hat{e}_{\phi}}_{\vec{v}_{P/e}} = -2\omega L\dot{\phi}\hat{e}_{L}$$

$$mL\hat{e}_L \times ① = \omega^2 mLR\sin\phi\,\hat{k}$$

$$mL\hat{e}_L \times ② = \vec{0} + mL^2\ddot{\phi}\hat{k}$$

$$mL\hat{e}_L \times ③ = \vec{0}$$

$$mL\hat{e}_L \times ④ = \vec{0}$$

$$mL\hat{e}_L \times ⑤ = \vec{0}$$

going back to $AMB_{/C}$ : $\sum\vec{M}_{/C} = \vec{r}_{P/C} \times m\vec{a}_{P/f}$

$$\left\{ \vec{0} = \omega^2 mLR\sin\phi\,\hat{k} + mL^2\ddot{\phi}\hat{k} \right\}$$

$$\{\ \} \cdot \hat{k} \Rightarrow \boxed{\ddot{\phi} = \frac{-\omega^2 R}{L}\sin\phi}$$

$$\boxed{\ddot{\phi} = \frac{-|\vec{a}_c|}{L}\sin\phi}$$